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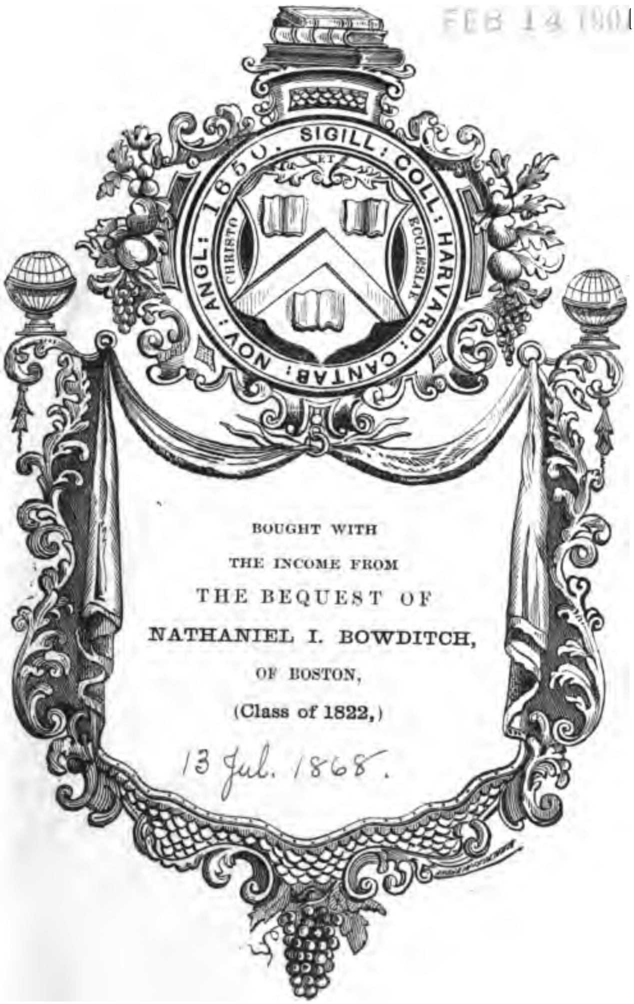

PLATE I.


*Fig. 1.* SATURN AND HIS RINGS, FEBRUARY 2nd, 1862.
*Fig. 2.* SATURN AND HIS RINGS, NOVEMBER 3rd, 1858.
*Fig. 3.* SATURN AND HIS RINGS, MARCH 23rd, 1856.
[AS SEEN IN AN INVERTING TELESCOPE.]

London: Longman & Co.

# SATURN AND ITS SYSTEM:

CONTAINING DISCUSSIONS OF

THE MOTIONS (REAL AND APPARENT) AND TELESCOPIC APPEARANCE OF THE PLANET SATURN, ITS SATELLITES, AND RINGS; THE NATURE OF THE RINGS; THE 'GREAT INEQUALITY' OF SATURN AND JUPITER; AND THE HABITABILITY OF SATURN.

TO WHICH ARE APPENDED

NOTES ON CHALDÆAN ASTRONOMY, LAPLACE'S NEBULAR THEORY, AND THE HABITABILITY OF THE MOON; A SERIES OF TABLES WITH EXPLANATORY NOTES; AND EXPLANATIONS OF ASTRONOMICAL TERMS.

BY

RICHARD A. PROCTOR, B.A.

*Late Scholar of St. John's College, Cambridge, and King's College, London.*

ILLUSTRATED BY FOURTEEN ENGRAVINGS IN STEEL AND COPPER.

THE HEAVENS DECLARE THE GLORY OF GOD,
THE FIRMAMENT SHOWETH HIS HANDYWORK.

LONDON:
LONGMAN, GREEN, LONGMAN, ROBERTS, & GREEN.
1865.

1868. July 13.
Bowditch Fund.

Heav'n
Is as the book of God before us set,
Wherein to read His wondrous works, and learn
His seasons, hours, or days, or months, or years.

MILTON.

This I say, and would wish all men to know and lay to heart, that he who discerns nothing but Mechanism in the Universe, has in the fatallest way missed the Secret of the Universe altogether.

CARLYLE.

MICROFILMED
AT HARVARD

# PREFACE.

I HAVE endeavoured in this work to give a complete account of the phenomena presented by the planet Saturn and its system. It might appear, at first sight, that a single planet, however interesting or elaborate the scheme of which it is the centre, should rather be made the subject of a chapter than of a volume, even of the moderate dimensions of the present. It will be found, however, that much that is contained in these pages, is applicable, with suitable changes in matters of detail, to all the members of the solar system.

The inquiry into the nature of the rings, in Chapter V., deals with a subject not uninteresting, I think, on its own account, but which gathers an additional interest from its bearing on the speculations of Laplace. It is not altogether impossible that in the variations perceptibly proceeding in the Saturnian ring-system a key may one day be found to the law of development under which the solar system has reached its present condition.

Certain points of resemblance between the relations of Saturn and our earth, as respects the variations of their seasons, have induced me to devote somewhat more space to the consideration of the celestial phenomena presented to the Saturnians than the nature of the subject might appear to warrant. These features of resemblance—singular in planets that differ so widely in all other respects—may be thus presented :—

The vernal equinoxes of the northern hemispheres of Saturn and the earth occur when the heliocentric longitudes of those planets are respectively 171° 43′ 35″·1, and 180°; the axes of Saturn and the earth are inclined 26° 49′ 27″·87 and 23° 27′ 24″·69, respectively, to the respective orbital planes of the two planets; and the north pole of the Saturnian celestial sphere is separated by an arc of only 7° 7′ 24″ from the north pole of our heavens: again, the longitudes of the perihelia of the orbits of Saturn and the earth are respectively 90° 23′ 36″·4, and 100° 38′ 1″·8; and Saturn and the earth are in perihelion after passing over 98° 40′ 1″·3 and 100° 38′ 1″·8, respectively, in longitude, from the autumnal equinoxes of their respective northern hemispheres.

Thus the variations in the Saturnian seasons more directly illustrate the corresponding variations in those of the earth than might, at first sight, be supposed. It will be seen from the note on page 117 that even the examination of the appearance of the rings to the Saturnians is not without its bearing on terrestrial phenomena.

The connection between the subjects treated of in the notes forming Appendix I., and the main subject of the work, will appear on perusal. The reader is reminded that these are notes, not essays; they contain merely the heads of arguments supporting opinions expressed in the body of the work.

The Tables numbered VII., VIII., X. and XI. now appear for the first time: parts of the other tables, also, are original. The sources from which the tables have been derived, or the formulæ from which they have been calculated, are given in the explanations of the Tables, which I have endeavoured to make as complete as possible. The explanations of astronomical terms extend only to terms actually used in the present work.

In the body of the work I have adopted for the sun's equatorial horizontal solar parallax (the earth at her mean distance) Professor Hansen's determination, namely, 8″·9159. More than ten years have elapsed since M. Hansen first pointed out that the solar parallax (8″·5776) deduced by Encke from the transits of Venus in 1761 and 1769, requires to be increased, to correspond with the observed extent of the moon's parallactic inequality. The above value, M Hansen's later calculation, corresponds very closely with the results obtained by Leverrier and others. In the tables of Appendix II., two values are given (corresponding to the above values of the solar parallax) of every element whose determination depends upon the sun's distance.

I have endeavoured to make the engravings represent as accurately as possible what they are meant to illustrate. This is to be understood of all the figures throughout the work, unless it is expressly stated in the text that general principles only are illustrated (as in the first six figures of Plate X.); or that a part of any figure is purposely exaggerated (as the rings in fig. 1, Plate VIII.)

In the figures of Plate I., the outlines of the planet and rings, and of their shadows, have been determined from calculations founded on the dimensions of the planet and rings adopted in Tables III. and IV.; such details have then been introduced as have been noted by the best observers. The slope of the figures is, to a certain extent, a matter of indifference, since it varies with the hour of observation; but to preserve uniformity I have adopted the following rule:—The horizontal line through the centre of the disc in the figures of Plates I., VII., IX., and XIII. represents the planet's heliocentric path, the direction of the planet's motion being from left to right in the figures of Plate I. (which are supposed to be seen through an inverting telescope), and from right to left in all the other figures.

The star-maps in Plates II. and III. are on the gnomonic projection.* The stars in Plate III. have been taken (with a correction for precession) from the maps of the Society for the Diffusion of Useful Knowledge. The positions of the stars in Plate II. have been corrected for the precession of the equinoxes, but not for the proper motions, since we have no means of learning whether the proper motions of the stars are constant for long periods. The path of Saturn in Plate III. has been taken from the 'Nautical Almanacs' for the years 1859—1866; the path in Plate II. has been calculated for an undisturbed elliptic orbit, corrected from Saturn's present orbit for variations in the position of the nodal line and perihelion: thus the minor irregularities in the path on Plate III. (chiefly due to the disturbing attractions of Jupiter) are wanting in Plate II. The figures of the constellations have been slightly altered from the Society's maps.

* The gnomonic projection seems the natural mode of projection for star-maps, since the eye of an observer viewing the celestial sphere actually occupies the 'point of sight' of the projection—that is the centre of the sphere. The object to be sought in all star-maps, is that the greatest portion possible of the celestial sphere should be visible at a single view with as little distortion as possible. In the Society's maps, the celestial sphere is projected from the centre upon the circumscribing cube; thus forming six maps considerably distorted near the angles. I am preparing a series of star-maps on a plan which appears to offer greater advantages. The celestial sphere is projected from the centre upon the circumscribing dodecahedron; thus forming twelve pentagonal maps much less distorted near the angles than the Society's maps: and further, by presenting the six maps of each hemisphere in a single plate in their proper relative positions—that is, as five pentagons upon the five sides of the polar pentagonal map—the relative positions of the northern or of the southern constellations are seen at a glance. A slight addition to the outer maps brings the whole length of the equator into each sextuple map.

The orbit of Nysa in fig. 3, Plate VI., has been derived from the elements given in Nichol's 'Cyclopædia of the Physical Sciences,' and Mitchel's 'Popular Astronomy' (both published in 1860). I think it little probable that these elements are even approximately correct. The direction of the planetary motions is indicated in this figure, in fig. 1, Plate VIII., and in fig. 7, Plate X., by the Zodiacal signs outside the orbit of Saturn; that is, the motions are supposed in these figures to take place in a direction contrary to that in which the hands of a watch move.

The dimensions of the satellites Mimas, Enceladus, and Hyperion, in fig. 1, Plate VII., are slightly exaggerated. These satellites would scarcely be visible on the scale of that figure.

The figures of Plate XIV. are derived from woodcuts in Layard's 'Nineveh and Babylon' and 'Nineveh and its Remains.'

Collingwood Villas, Stoke, Devon:
*May*, 1865.

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# SATURN AND ITS SYSTEM.

ERRATA.

In Table XI., page 224, heading of first row; in page 234, line 19; and in page 235, line 1, for *saturnicentric* read *saturnigraphical*, understanding the word in a sense corresponding to that of the word *geographical* in the expression *geographical latitude*.

In page 243, lines 18—24, the first parts of the definitions of *geocentric* and *geographical latitude* are correct; the parts from 'in other words' to the end must be interchanged.

Proctor's Saturn.

manner, with his ruddy but brilliant light, and swift motions amongst the fixed stars, was probably recognised as a wandering orb at a very remote period. Mercury, on the other hand, twinkling like a fixed star, and never visible but when near the horizon, and when his lustre is dimmed by the glory of the rising or setting sun, probably escaped the notice of astronomers, or was at least not recognised as a planet* till long after even Saturn's dull orb had been discovered, and his slow movements traced upon the celestial sphere.

Let us consider in what manner the astronomers of old must have attained to the discovery of Saturn's planetary nature, and

* Mercury, though probably the last-discovered planet of the ancient system of astronomy, was included in the stellar worship practised under the Lachmites by the Asedites.

# SATURN AND ITS SYSTEM.

## CHAPTER I.

### DISCOVERY OF SATURN—THE SIMPLER ELEMENTS OF HIS ORBIT.

No account, historical or traditional, has been handed down to us of the discovery of the planet Saturn. Though he is included in the list of wandering stars in the earliest astronomical systems whose records have reached us, there can be little doubt that Venus, Jupiter, and Mars were discovered long before Saturn—the most distant planet known to the ancients. The two first, surpassing in splendour the brightest of the fixed stars, must at a very early period have attracted the notice of astronomical observers, who could not fail soon to perceive that those orbs were changing their positions on the celestial sphere. Mars, in like manner, with his ruddy but brilliant light, and swift motions amongst the fixed stars, was probably recognised as a wandering orb at a very remote period. Mercury, on the other hand, twinkling like a fixed star, and never visible but when near the horizon, and when his lustre is dimmed by the glory of the rising or setting sun, probably escaped the notice of astronomers, or was at least not recognised as a planet* till long after even Saturn's dull orb had been discovered, and his slow movements traced upon the celestial sphere.

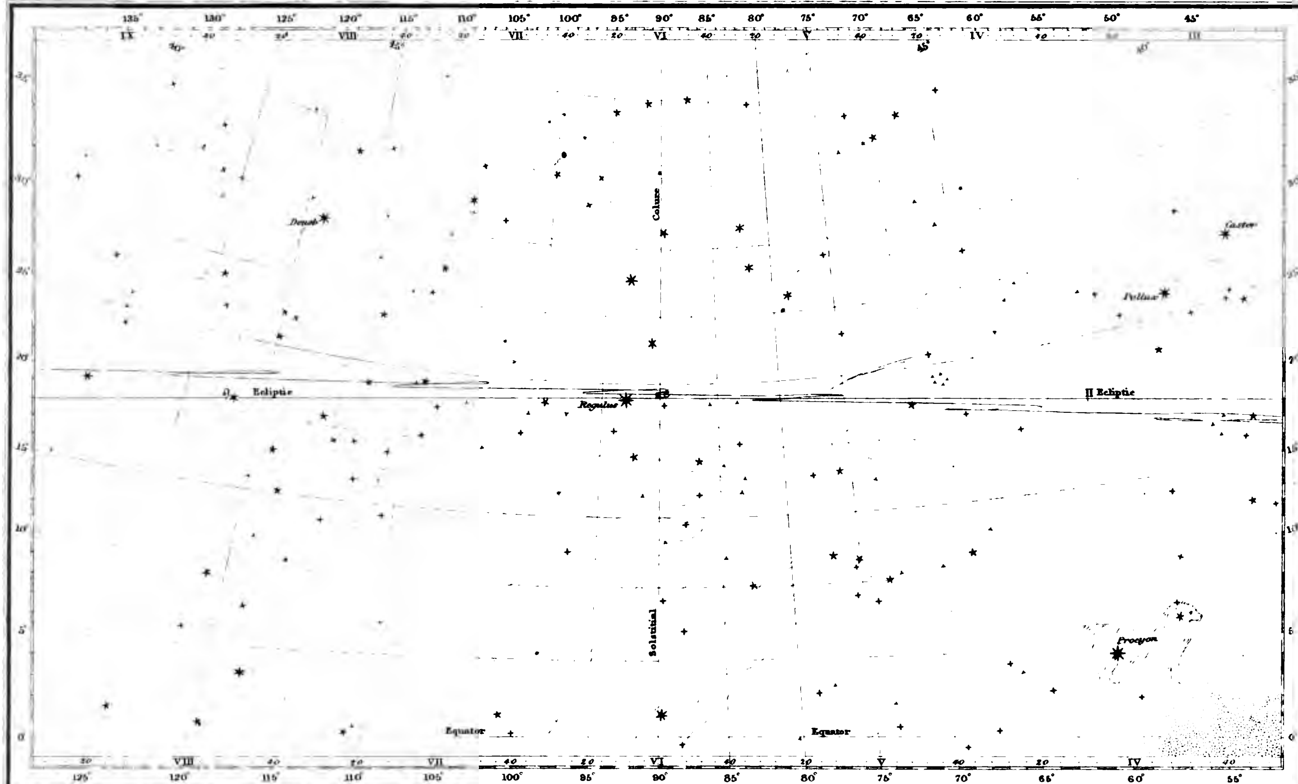
Let us consider in what manner the astronomers of old must have attained to the discovery of Saturn's planetary nature, and

* Mercury, though probably the last-discovered planet of the ancient system of astronomy, was included in the stellar worship practised under the Lachmites by the Asedites.

to the knowledge of the various facts respecting him with which we learn that they were acquainted. The time thus spent in retreading the paths by which past generations attained to knowledge would not be altogether wasted if we only learnt thence the lessons of patience and watchfulness. But such an inquiry has, in fact, a closer connection with modern science than might at first be supposed. In the brief paragraphs that announce each year the discoveries of new asteroids or of telescopic comets, we see the records of the same patient observation applied in the same manner as of old. It is true that the powerful and delicate instruments now used, and the application of modern methods of mathematical analysis, enable the astronomer to obtain in a few weeks results that formerly a life would have been insufficient to compass. An air of mystery, also, surrounds his researches, lying, as they do, chiefly in depths to which the far-seeing eye of the telescope alone penetrates. Yet the modern astronomer, like the observer of old, seeks among the celestial bodies the signs of change and motion, and when he has detected these, he tracks the wandering orb and calculates the extent and character of its orbit. To this work he must apply—besides his telescopes and his analyses—the old-fashioned instruments, patience, energy, and watchfulness.

At a very early period the ancients separated the stars into constellations. In all probability this arrangement was soon followed by the construction of rough maps, on which the positions of the principal stars were noted down. They marked, no doubt, with special care, the stars in the zodiac—that belt of the celestial sphere within which the sun, moon, and planets were observed to move. Let us consider how this process must have led to the discovery of Saturn.

Shining with a dull yellow light, and travelling amongst the fixed stars with a scarcely perceptible motion, the planetary nature of Saturn remains for a long time unnoticed. But his path on the celestial sphere brings him into very close (apparent) proximity with some of the brightest of the fixed stars, and astronomers must at length have become too well acquainted with the general configuration of the zoadical constellations not to notice the presence of a strange star in such a position. Once observed,



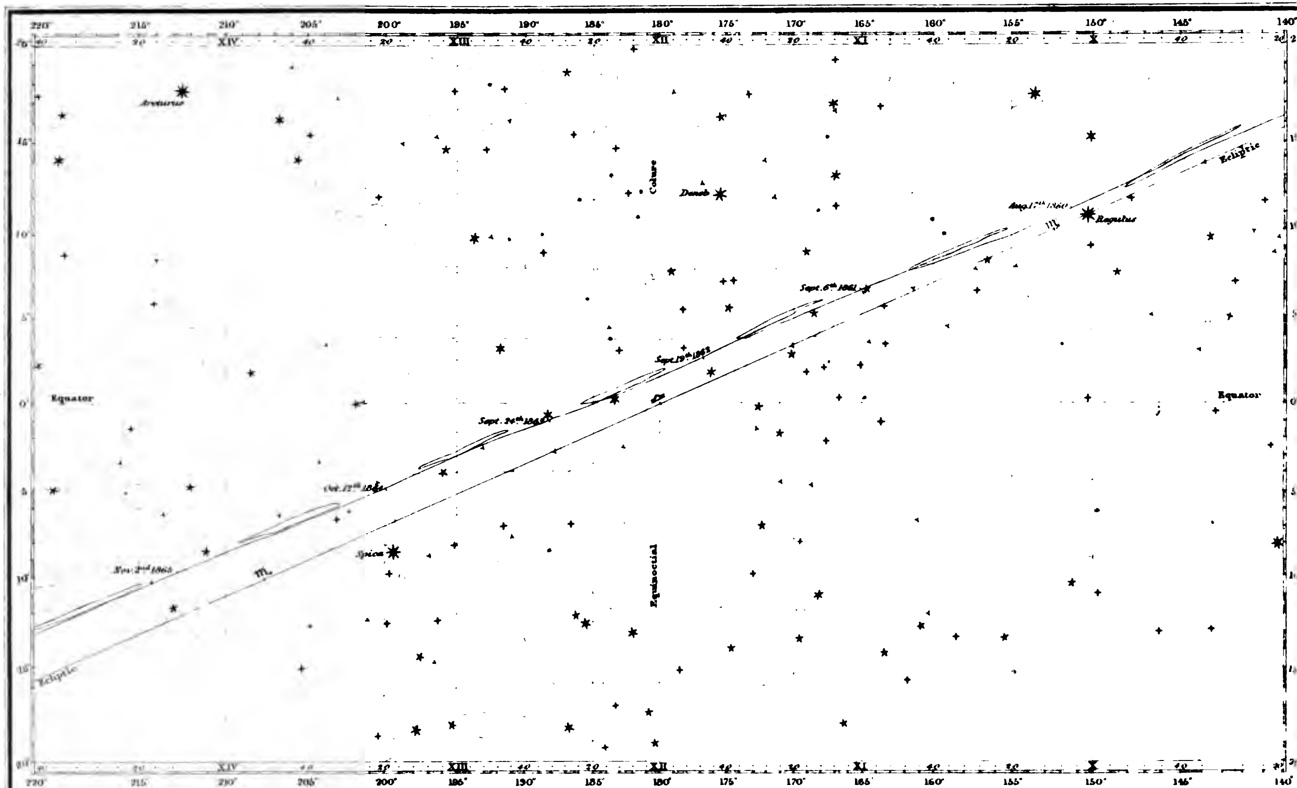

SATURN'S PATH ON THE CELESTIAL SPHERE
ABOUT 4000 YEARS AGO.

Stars of the 1st, 2nd, 3rd, 4th and 5th magnitudes,

Telescopic Star, Nebula, Moon's disc on Scale of Map, at centre. Moon's disc on Scale of Map, at angles.

H. Adlard Sc.

London: Longman & Co.




SATURN'S PATH ON THE CELESTIAL SPHERE

DURING THE YEARS 1859, 1860, 1861, 1862, 1863, 1864, 1865 and 1866.

Stars of the 1st, 2nd, 3rd, 4th and 5th magnitudes, Telescopic Star, Nebula, Moon's disc on Scale of Map at centre, Moon's disc on Scale of Map at angles.

R. A. Proctor del.

H. Ward Sc.

London: Longman & Co.

though they might remain in doubt for a few days as to his planetary nature, these doubts would soon be set at rest by obvious alterations in his bearing with respect to the neighbouring fixed stars.

Let us suppose, by way of illustration, that Saturn is describing that part of his orbit indicated in Plate II., and that he is first observed when in opposition near the bright star Regulus.* The time then is mid-winter.† The heavens—in the eastern clime of those first astronomers—are sparkling as if set with myriads of varied gems. The greater Lion, with the brilliant stars Deneb and Regulus, is in the south-east, slowly rising to the meridian. Near Regulus, about a degree and a half to the north, a strange star is seen, whose dull yellow light contrasts strangely with the dazzling white of the fixed star. As they rise together to culmination, and then sink towards the south-western horizon, the closest observation can detect no change in their relative positions.

On the following night the stranger is again seen close to Regulus. But now its position seems slightly changed:—it is no longer due north of Regulus, but has moved slightly westward. The change of position is, however, so small as to be scarcely perceptible. It is not until several days have elapsed that the wandering nature of the stranger is certainly established. It passes to the north-west of Regulus, and continues to move slowly westward.

As Mars and Jupiter both appear to move from east to west when in opposition, though their real motion is from west to east, it remains doubtful whether the new planet moves from east to west, as he appears to do, or, like the other planets, from west to east. Careful observation soon shows that Saturn's westward motion is gradually diminishing; yet, when six weeks (the time

* The bright star below the zodiac, near the centre of the map.

† Regulus now souths at midnight in the middle of March. Four thousand years ago he passed the meridian at midnight two months earlier. The position of the ecliptic was also different. Regulus, now nearly half a degree to the north, was then south of the ecliptic. Saturn's path on the celestial sphere has, in like manner, undergone several changes; for instance, the positions of the nodes and of the perihelion, and the inclination of the orbit to the ecliptic, have varied, and other changes have taken place which need not at present be dwelt upon.

in which Mars retrogrades after opposition) have passed, Saturn is still moving westward, nor has his retrograde motion ceased when two months (the corresponding period of Jupiter's retrogression) have elapsed. For yet another fortnight he retrogrades, and then begins to move slowly along his advancing arc. Although he has thus been retrograding for nearly two months and a half from opposition, he has passed over an arc of only three degrees on the celestial sphere.

Saturn's progressive motion, slow at first, gradually increases as he approaches conjunction, when, becoming an evening star, his light is dimmed, and finally lost, in the light of the sun. About a month after conjunction, he again becomes visible as a morning star. His apparent motion is still progressive, but gradually decreases until he becomes stationary. He then slowly retrogrades for nearly five months, passing through opposition; becomes stationary again, then advances, and so on continually, advancing during seven months and a half and retrograding during five, but on the whole slowly traversing the zodiac from west to east, or in the order of the signs.

Having ascertained that the strange orb is a planet, let us see how the ancient astronomers could approximately determine the distance and period of the new planet from its apparent motions. We may proceed on the supposition that they were acquainted with the true system of the world. It would obviously be a waste of time to consider, at any length, methods belonging to a false system; there are also good reasons for supposing that the true system was actually known to ancient astronomers.* For the sake of simplicity, the paths of Saturn and the earth are supposed to lie in the same plane, and to be circles about the sun as centre.

In the first place, what inferences may be deduced from Saturn's slow retrograde motion when in opposition, his long period of retrogression, and the small arc passed over by him in that period? To answer these questions it will be necessary to recall to the reader's mind the cause of the retrograde motion of a planet in opposition.

* See Note A, Appendix I., Chaldæan Astronomy.

PLATE IV.

*Fig. 1.*

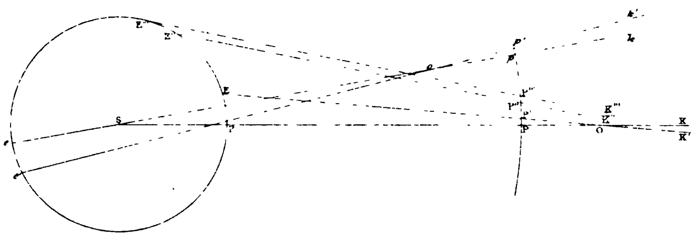



*Fig. 2.*

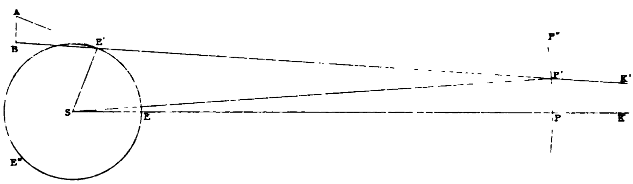



*Fig. 3.*

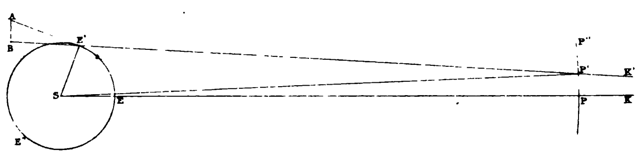



*Fig. 4.*

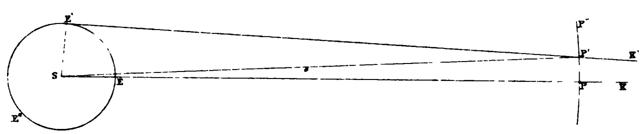
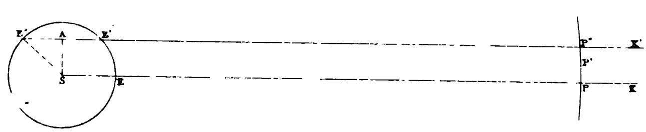



*Fig. 5.*

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Let S (fig. 1, Plate IV.) represent the sun, E E′E″ the earth's orbit, P P′P″ part of the orbit of a superior planet. When the earth is at E let the planet be at P, so that (S E P being a straight line) the planet is in opposition when at P. Starting from these positions, suppose that the earth and the planet, in the same interval of time, pass respectively over the arcs E E′ and P P′, E E′ being greater than P P′. Then it is obvious that the line E′P′ is inclined to the line E P, and that if these two lines are produced they will meet beyond P. Let them be produced, beyond their point of intersection O, to K and K′ respectively. Now the observer on the earth sees the planet in the direction E K when the earth is at E, and in the direction E′K′ when the earth is at E′; thus the planet appears to have moved in the direction K K′, while it has actually moved in the contrary direction, namely, from P to P′. The amount of the planet's retrograde motion during the interval is measured by the angle contained between the lines E P, E′P′, that is, by the angle E O E′ or K O K′; and, *vice versâ*, if the retrograde arc passed over by the planet on the celestial sphere be measured, the angle E O E′ becomes known with an exactness proportioned to the accuracy of the instruments used in effecting the measurement and the skill of the observer employing them.

Let us now carry the earth and planet forward in their orbits. It is obvious that the path of the earth becomes more and more inclined to the line of sight to the planet the farther the earth is carried on in the arc E E′E″; thus if E″, P″, and E‴, P‴, be respectively contemporaneous positions of the earth and planet, the angle between the line E‴P‴ and the arc E‴E″ is less than the angle between the line E″P″ and the arc E″E′, and this angle again is less than the angle between the line E′P′ and the arc E′E. Hence the effect of the earth's superior velocity, so far as it operates in changing the direction of the line of sight to the planet, gradually diminishes, until at length the earth reaches a position, as at E″, such that the effect of the direction of its motion exactly counterbalances its superior velocity, and the planet appears to be stationary. If E″E‴ and P″P‴ are small arcs passed over by the earth and planet in the same time at this period, the line E‴P‴ is parallel to the line E″P″, the superiority in length of the arc E″E‴ over the arc $P'''P''$ being compensated by the smallness of the angle at which $E''E'''$ is inclined to the line of sight $E'''P'''$, compared with the inclination of $P''P'''$ to the same line. Thus the planet is seen in the same direction at the end of this interval as at the beginning, or is stationary.*

After this, it is plain that the planet will appear to advance, for at first the earth's path becomes inclined at a smaller angle to the line of sight to the planet, till it coincides with that line; and afterwards, as the earth passes on to conjunction, its motion (considered with reference to the line of sight at any instant) is in a contrary direction to that of the planet, and therefore adds to the planet's apparent motion on the celestial sphere.

If the planet were visible when in conjunction, its motion would appear swifter then than at any other time. Thus let $e\,s\,p$ be the line along which conjunction takes place, the earth being at $e$ and the planet at $p$; and $e'$, $p'$ the positions of the earth and planet after a short interval of time: then $e\,p$ is the direction in which the planet would be seen when in conjunction, $e'p'$ that in which it would be seen at the end of the interval of time. Now $e\,p$ and $e'p'$ meet within the orbit of the planet at $o$; the angle $e\,o\,e'$ or $p\,o\,p'$ measures the arc on the celestial sphere passed over by the planet, and the whole motions both of the planet and the earth conspire to increase this angle; whereas in any other positions, either the difference of these motions, or only parts of them, affect the angle between the lines of sight at the beginning and end of any corresponding interval of time. And although the angle is diminished when a superior planet is in conjunction through the effect of increased distance ($e\,p$ plainly exceeding $E\,P$ by twice the radius of the earth's orbit, or by twice $S\,E$), yet, under the actual relations of the velocities and distances of the planets,† increase prevails over decrease, and a superior planet, if

* It must be remembered that the arcs $E''E'''$ and $P''P'''$ are supposed to be very small. A planet is not actually stationary during any finite period of time; retrograde motion merges into progressive, progressive into retrograde, at a definite instant, before which (by however small an interval of time) the motion is of one kind, afterwards of the contrary; it is only *at* that instant that there is no motion, progressive or retrograde.

† If $d$, $D$ be respectively the mean distances of the earth and of a superior planet

visible, would appear to move more rapidly when in conjunction than at any other time. A planet is, however, always invisible when at or near conjunction, since it then occupies the same region of the celestial sphere as the sun, and is therefore lost in his superior light. When the planet next becomes visible, however, the effect of its swifter motion in conjunction is seen in the change of its position among the zodiacal constellations.

Passing from conjunction to opposition, the planet goes through the same changes in a reverse order. Its progressive motion gradually diminishes till it becomes stationary; thence it retrogrades through opposition to its next station; and so on continually, the total result of its motion in each synodical revolution being a progression from east to west, or in the order of the signs of the zodiac.

Let us now consider what inferences may be drawn from the nature of Saturn's apparent motions on the celestial sphere. The first point to be noticed is the slowness of his retrogression when in opposition. Now, referring to fig. 1, Plate IV., it becomes clear that this must arise from one of two causes. If P P′ were very nearly equal to E E′, E′P′ would be very nearly parallel to E′P′, or, in other words, the angle E O E′ would be very small. Hence the slowness of Saturn's retrogression *might* arise from his velocity being very nearly equal to that of the earth. But again,

from the sun; $p$, P their respective periods; $v$, V their respective mean angular velocities about the sun. We have from Kepler's third law—

$$p : \mathrm{P} :: d^{\frac{3}{2}} : \mathrm{D}^{\frac{3}{2}};$$

$$\text{but } v : \mathrm{V} :: \frac{d}{p} : \frac{\mathrm{D}}{\mathrm{P}} :: \frac{1}{d^{\frac{1}{2}}} : \frac{1}{\mathrm{D}^{\frac{1}{2}}}$$

$$\text{therefore } v - \mathrm{V} : v + \mathrm{V} :: \mathrm{D}^{\frac{1}{2}} - d^{\frac{1}{2}} : \mathrm{D}^{\frac{1}{2}} + d^{\frac{1}{2}};$$

now, on the supposition of circular orbits, the retrograde velocity of a superior planet in opposition would be proportional to $\frac{v-\mathrm{V}}{\mathrm{D}-d}$, and the progressive velocity in conjunction would be proportional to $\frac{v+\mathrm{V}}{\mathrm{D}+d}$; but from the proportion deduced above, it follows that

$$\frac{v-\mathrm{V}}{\mathrm{D}-d} : \frac{v+\mathrm{V}}{\mathrm{D}+d} :: \frac{\mathrm{D}^{\frac{1}{2}}-d^{\frac{1}{2}}}{\mathrm{D}-d} : \frac{\mathrm{D}^{\frac{1}{2}}+d^{\frac{1}{2}}}{\mathrm{D}+d} :: \mathrm{D}+d : (\mathrm{D}^{\frac{1}{2}}+d^{\frac{1}{2}})^2;$$

that is, (since $(\mathrm{D}^{\frac{1}{2}}+d^{\frac{1}{2}})^2 = \mathrm{D} + d + 2\mathrm{D}^{\frac{1}{2}}d^{\frac{1}{2}}$, or is greater than $\mathrm{D} + d$,) the apparent velocity in conjunction is greater than the apparent velocity in opposition.

if (the rest of the figure remaining unchanged) we make the orbit P P′P″ very large, and suppose the planet's velocity in this large orbit to be no greater than in the smaller one, the angle E O E′ would obviously become very small, for the farther P P′ is removed from E E′ the more nearly will E′P′ and E P approach to parallelism. Thus, then, Saturn's slow motion in opposition *might* arise from the fact that his orbit is very large compared with that of the earth. Hence, within a week of Saturn's discovery, enough might be known to show that either his velocity in his orbit is very nearly equal to that of the earth in hers, or that he must move at an immense distance from the earth.

Another fact revealed by observation points to the true cause of Saturn's slow retrograde motion. After opposition he retrogrades for nearly two months and a half. In this time the earth has completed nearly a quarter of her orbit, and therefore her path has become inclined at a very small angle to the line of sight to Saturn. Since, then, it is only when the earth's path is thus inclined that her superior velocity is so far compensated by inclination that Saturn appears to be stationary, it is clear that the earth's motion must be much swifter than Saturn's. Hence we have only one possible explanation left of the slowness of Saturn's retrograde motion; namely, that it is due to his vast distance from the earth.

Having arrived at this conclusion, let us see how the ancient astronomers might apply the results of observations of the new planet (even those taken during only the first few months after discovery) to obtain more definite notions of his distance, and thence to determine his period.

Let a small circle E E′E″ (fig. 2, Plate IV.) be described to represent the orbit of the earth about S the sun; and with the same centre let P P′, part of a large circle, be described to represent Saturn's orbit, of which as yet nothing is supposed to be known but that it is large compared with the earth's orbit. Let E, P be the positions of the earth, and Saturn when the latter is in opposition, so that S E P is a straight line. Let E′ be the position of the earth when Saturn is stationary, that is, two months and a half after opposition; thus E S E′ is an angle of about 75°. Through E′ let the straight line E′P′ be drawn, inclined to S P at an angle containing the same number of degrees, minutes, and seconds, as the arc on the celestial sphere through which Saturn has been observed to move during the two months and a half following opposition: thus E′K′ is the line of sight from the earth to the planet when the earth is at E′; and P′, the point in which E′K′ meets the planet's orbit, must be the position of the planet at that time. Produce P′E′ to a convenient distance from E′, to B; through B draw B A in a direction perpendicular to S P, and draw E′A touching the circle E E′E″ in E′. We have, then, the following facts to guide us:—the earth and Saturn both lie in the line B K′; the earth is leaving this line in the direction E′A; Saturn is leaving it in the direction of the tangent to P P′P″ at P′, a direction approximately parallel to B A.* Now the planet appears stationary when at P′—in other words, the rates of departure of Saturn and the earth from the line E′P′ are exactly equal. If, then, we suppose a point to move from B in direction B A (which is parallel to Saturn's line of motion at P′) with Saturn's velocity, and another point to start from E′ at the same moment in direction E′A with the earth's velocity, then, since the rates of departure of these two points from the line B E′ are exactly equal, they would arrive at the point A at the same instant. Hence the velocities of these moving points—which velocities are, by our supposition, the velocities of Saturn and the earth respectively—are respectively proportional to B A, E′A, the spaces they pass over in equal times. We arrive, then, at the important result, that Saturn's velocity in his orbit : the earth's velocity in hers :: the line B A : the line E′A, very approximately. If the figure is constructed with proper care, we have only to measure the lines B A and E′A to determine the value of this proportion; or we can employ a very simple trigonometrical calculation for this purpose.† Either method leads to the result

* The tangent at P is parallel to B A; since, then, the arc P P′ is very small, the tangent at P′, which is perpendicular to S P′, is inclined at a very small angle to the tangent at P, and is therefore very nearly parallel to B A, and for our purpose may be considered as actually parallel to B A.

† In the triangle A B E′ the angle B A E′ is equal to the known angle E S E′, and the angle A E′B is the complement of the angle B E′S, which is the sum of two known angles; viz.

that A E′ is about $3\frac{1}{11}$ times as large as B A, or Saturn's velocity is to that of the earth in the proportion of 11 to 34, very nearly.

We can now determine Saturn's distance in either of two ways. The orbit P P′P″, assigned to Saturn in the above investigation, was simply a circle, large compared with E E′E″, and it is to be observed that the dimensions of this circle had nothing to do with the formation of the triangle A B E′, on which the determination of Saturn's velocity was made to depend; except that, knowing the planet's orbit to be large, we were able to assert that the direction of its motion at P′ was very nearly parallel to B A. But we can apply the result just obtained to see whether P P′P″ correctly represents Saturn's orbit. For the arc P P′ passed over by Saturn should bear to the arc E E′ passed over, in the same time, by the earth, the proportion, above determined, of 11 to 34. In our figure P P′ does not bear this proportion to E E′, being too large. The radius S P is therefore too small, and we must select such a radius in place of S P that the arc intercepted between S P and E′P′ may be of the requisite length, viz. $\frac{11}{34}$ths of E E′. It will be found that for this purpose S P should be about $9\frac{1}{2}$ times as great as S E.

We may confirm the correctness of this result by applying a second method to determine Saturn's distance. Observation shows that, when near opposition, Saturn retrogrades daily over an arc of about 4′ 43″ on the celestial sphere. Now Saturn is advancing from P with $\frac{11}{34}$ths of the velocity with which the earth advances from E. Therefore Saturn's motion, as observed from the earth, is the same as if he were retrograding with $\frac{23}{34}$ths of the earth's velocity. If, then, he were at the same distance from the earth as the sun is, he would appear to pass daily over an arc equal to $\frac{23}{34}$ths of the arc passed over daily by the sun, since the sun's apparent motion is due to *the whole* of the earth's motion. Now the sun passes daily over an arc of about 59′ 8″.* Thus Saturn, if he were at the sun's distance, would pass over nearly 40′ 0″ daily. But Saturn actually passes over 4′ 43″, or about $\frac{2}{17}$ths

the angle E S E′ and the angle of inclination of E′P′ to S P. Thus the proportion that B A bears to A E′ can be determined.

* The arc passed over daily by the sun is $\frac{1}{365\cdot24}$ of 360° approximately.

of the arc he would pass over if he were at the same distance as the sun from the earth. Hence his distance from the earth when in opposition must be greater than the sun's distance from the earth in the proportion of 17 : 2; that is, E P is $8\frac{1}{2}$ times as great as S E; and therefore S P is $9\frac{1}{2}$ times as great as S E.

Having thus ascertained Saturn's distance approximately, another figure may be constructed (as fig. 3, Plate IV.) in the same manner, in which the orbits of Saturn and the earth are more correctly proportioned, and a new triangle A B E′ may be drawn, in which B A, instead of being at right angles to S P, is parallel to the tangent at P′. The proportion that B A bears to A E′ in this triangle will more correctly represent the proportion that Saturn's velocity bears to the velocity of the earth than the corresponding proportion in the original triangle. Thence we can arrive at a new and more exact determination of Saturn's distance. This might be again applied to correct the triangle A B E′; but the repetition of this approximative process would be useless after a second or third construction, since the errors of observation, and those due to the supposition of circular orbits, are far more important than the corrections that would be obtained from a fourth or fifth construction.

Having determined the proportion that the distance of Saturn from the sun, and his velocity in his orbit, bear to the distance and velocity respectively of the earth, Saturn's period follows at once. The path he describes in completing one revolution round the sun is $9\frac{1}{2}$ times as great as the corresponding path of the earth, while his velocity is only $\frac{11}{34}$ths of the earth's velocity; therefore the time he occupies in completing a revolution : the corresponding time occupied by the earth (that is, a year) as $\frac{19}{2} \times \frac{34}{11} : 1$, or as $29\frac{1}{2} : 1$, very nearly. Thus Saturn's year contains about $29\frac{1}{2}$ of our years.

Soon after passing his stationary point, Saturn arrives at another important position. When in opposition, he passes the meridian at midnight—that is, twelve hours after the sun. After this he souths earlier every night—until, when nearly three months have elapsed, he passes the meridian six hours after midday: in other words, Saturn in opposition was 180° from the sun, but is now

90° from the sun.* Let fig. 4, Plate II., represent the sun, Saturn, and the earth, in this position at S, P′, and E′, respectively. The angle P′E′S′ is a right angle, and the angle E S E′ very nearly a right angle, E E′ being the arc passed over by the earth in three days less than a quarter of a year. Now P′E′ is a tangent to the circle E E′E″, since it is at right angles to S E′. Hence the earth when at E′ is moving directly from Saturn at P′, and the earth's motion therefore produces no modifying effect whatever upon Saturn's apparent motions on the celestial sphere: these are due to Saturn's own motion only. It is easily seen that, under these circumstances, Saturn's motion, viewed from the earth at E′, is exactly the same in amount as it would appear if viewed from the sun at S. For though E′P′ is less than S P′, and Saturn's path at P′ not inclined at a right angle to E′P′ as it is to S P′, yet the two errors introduced by these causes act in opposite ways, and, being exactly equal, destroy each other. For if Saturn were viewed from S at a distance E′P′, his motion would be greater than it would appear at a distance S P′, in the proportion of S P′ to E′P′; and again, if Saturn's motion were inclined to S P at the angle in which it is inclined to E P′, that motion would appear *less* than it would if at right angles to S P′, in the proportion of E′P′ to S P′;† hence Saturn's actual motion, when at P′, is exactly the same in amount, whether viewed from E′ or from S. Now it is found that Saturn's apparent daily motion at this time is slightly greater than 2′. His daily motion about the sun is therefore also slightly in excess of 2′; so that he completes his orbit about the sun in rather less than 360 × 30 or 10,800 days.‡

Owing to a cause presently to be explained, Saturn does not in his progressive path exactly retrace his former retrograde path on the celestial sphere. In the instance selected as an illustration of his movements, his advancing arc lies at first slightly to the south of his former retrograde path (see Plate II.); but in about four months and a half from opposition he returns almost to the exact

* When thus situated, a planet is said to be in *quartile*, or *quadrate* to the sun.

† The sine of the angle E′P′P″ is the ratio in this case; but the angle E′P′P″ is the complement of the angle E′P′S, and is therefore equal to the angle E′S P′, and the sine of E′S P′ is equal to the ratio of E′P′ to S P′.

‡ The true period is $10{,}759\frac{1}{5}$ days.

place he had occupied when in opposition (his advancing arc afterwards lying to the north of his former retrograde arc). In fig. 4, Plate IV., let P, E be the positions of Saturn and the earth respectively, when Saturn is in opposition; P'', E'' their respective positions four months and a half afterwards; then, since Saturn is seen from the earth at E'' in the same direction as when he was in opposition to the sun (neglecting his northerly deviation, which corresponds to a very slight elevation above the plane of the paper on which the figure is drawn), E''P'' must be parallel to E P. Now, since E E'' is the arc passed over by the earth in four months and a half, E S E'' is an angle of about 135°; thus the point E' is known. We have then only to measure the arcs E E' E'' and P P'' passed over by the earth and Saturn, in the same time, to determine their relative rates of motion. The result confirms those already obtained.

It may be mentioned that this method is independent of the two first, for it is not necessary (in applying it to determine Saturn's velocity) that S P should be even approximately known. All that is required to be known is the fact that Saturn's orbit is large compared with the earth's; this being the case, it is easily seen that the arc P P' does not differ greatly from S A, the perpendicular on E''P''. Having determined Saturn's velocity, his distance may be determined, as before, from his rate of motion in opposition. The construction may then be repeated, using this result to represent S P more correctly. In this way the ratio of Saturn's velocity to the earth's may be obtained with greater exactness than by the two former methods; for, in the first, it is necessary that the angle between E'P' and E P should be very accurately measured, a slight error in this measurement having an important effect in vitiating the construction or calculation for determining Saturn's velocity. In the second method, Saturn's daily motion is too small to be accurately measured, except by very delicate and trustworthy instruments, very skillfully used. His rate of motion is also increasing each day, when he is in quartile after opposition, and the determination of the exact instant in which he assumes this aspect is not very easy. The third method, on the other hand, is founded on an observation of the simplest nature, and the arcs E E'' and P P'' may be easily measured or calculated.

The results, however, of the first few months' observations of the distant stranger could, of course, be viewed only as rough approximations, to be corrected as time enabled the astronomer to apply more exact and trustworthy methods of investigation.

When a year had passed from the time at which Saturn was in opposition, the celestial sphere had apparently made a complete revolution round the earth, so that each star rose, culminated, and set at the same hours at the end as at the beginning of that interval. But Saturn had been slowly advancing in his orbit during that period—that is, he had been moving from west to east; and since the apparent annual revolution of the celestial sphere (like its apparent daily motion) is from east to west, Saturn had not yet reached opposition when the year was completed, but was to be found at midnight somewhat to the east of the meridian. Twelve days and three quarters elapse before he is in opposition, or has completed a synodical revolution, as it is termed, about the earth.

If the exact moments at which Saturn was in opposition, or the beginning and end of a complete synodical revolution, had been accurately noted, his period could have been at once determined, on the supposition, at least, that both Saturn and the earth move in uniform circular orbits. It is not, however, probable that the ancient astronomers could accurately determine the moment at which a planet arrived at opposition—an operation of some difficulty. After a few years, however, Saturn's average synodical period was no doubt determined with considerable accuracy. As already mentioned, this period exceeds a year by twelve days and three quarters. Let us consider how this result may be applied to determine Saturn's period of revolution in his orbit, or his sidereal period.

Let S (fig. 1, Plate V.) be the sun, E, P the positions of the earth and Saturn in their orbits when Saturn is in opposition at the beginning of a synodical period, E′, P′ their respective positions when Saturn is next in opposition—that is, at the end of the synodical period. Thus SEP and S′E′P′ are straight lines; the arc PP′ is passed over by Saturn in a year, twelve days, and about eighteen hours, or in rather more than 378 days; and during this time the earth has performed a complete revolution, and in addition

PLATE V.

*Fig. 1.*

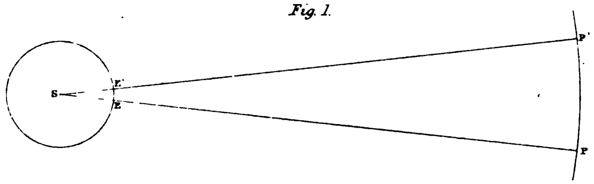



*Fig. 2.*

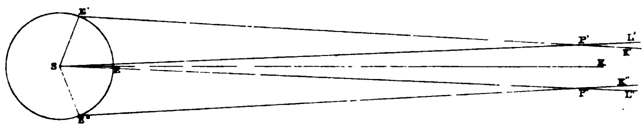



*Fig. 3.*

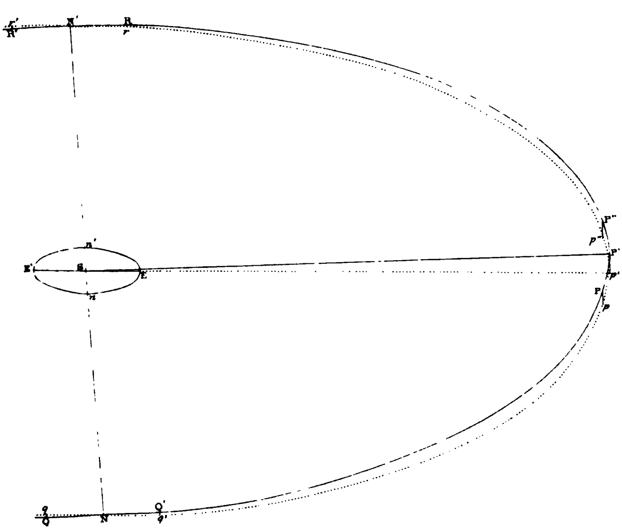


*Fig. 4.*

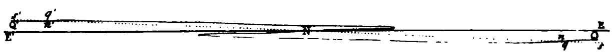
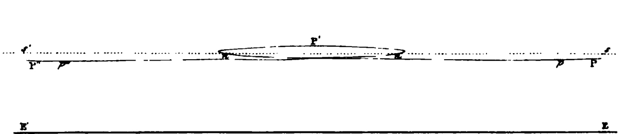


*Fig. 5.*



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the arc E E′. Thus the earth has passed over the arc E E′ in $12\frac{3}{4}$ days. Hence, since Saturn and the earth pass over the arcs P P′ and E E′ in 378 and $12\frac{3}{4}$ days respectively; and since, further, the arc P P′ bears the same proportion to Saturn's complete orbit that the arc E E′ does to the earth's orbit; the times in which Saturn and the earth perform their complete orbits are to each other in the proportion of 378 to $12\frac{3}{4}$—that is, of 1512 to 51. But the earth performs her complete orbit in a year; hence Saturn's period is $\frac{1512}{51}$ years, or rather more than $29\frac{1}{2}$ years.

From careful observations of each return of the planet to opposition, Saturn's synodical period became still more accurately ascertained, and thus his sidereal period was more correctly determined. This period is 10759·2197106 days. When this interval had elapsed from the time of his first discovery, Saturn had completed a revolution about the sun. It is not, however, so simple a matter as it might at first sight appear to determine from Saturn's position on the celestial sphere the exact instant at which a sidereal period, commencing at any given moment, is completed. In a sidereal period Saturn completes very nearly $28\frac{1}{2}$ synodical revolutions; * thus Saturn is in altogether different aspects with respect to the sun at the beginning and at the end of such a period. For instance, if Saturn is in opposition at the commencement, he is very near conjunction at the end of a sidereal revolution, and is therefore not visible. On the other hand, if he is in quadrature *preceding* opposition at the commencement, he is very near quadrature *following* opposition at the end of a sidereal period, and *vice versâ*. Now it follows, from what has been already shown, that from conjunction to opposition Saturn appears in advance of his true position † in his orbit, whereas from opposition to conjunction he is behind his true place. Hence, if Saturn is in quadrature preceding opposition, or apparently in advance of his true position, at the *beginning* of a sidereal period, then, at the *end* (when, of course, his true position is the same as at the beginning), he appears not to have

* The number of synodical revolutions in a sidereal period is obtained by dividing 10759·2197106 by 378·090: it is therefore 28·457 very nearly.

† That is, his position in his orbit as it would appear to an eye placed at the sun's centre; or, as it is termed, his heliocentric position.

reached his true place, still less the place he appeared to occupy at the commencement of the period. And in whatever aspect we suppose Saturn to be at the beginning of his sidereal period, the same difficulty presents itself. From the knowledge of Saturn's period and distance already obtained, the astronomer could correct the discrepancies due to this cause, and, if necessary, apply the period thus deduced (which would be more nearly correct than his former results) to obtain a more accurate approximation. These results could be still further corrected by comparing the results obtained when different epochs are assumed from which the sidereal periods are supposed to commence; and when Saturn had completed several complete revolutions about the sun from the time of his first discovery, there can be little doubt that the length of his sidereal period had been very accurately determined by astronomers.

Sir John Herschel has shown in his 'Outlines of Astronomy' that the distance of a superior planet whose period is known may be determined by observing its motion during a single day when in opposition. In the case of Saturn, however, this method would not be more exact than those already indicated, since Saturn's motion when in opposition is very slow,* and a very small error in the determination of the arc he daily traverses at this time would altogether vitiate the result of calculation or construction founded upon such determination. The following process is more trustworthy:—

Let the time in which Saturn passes from opposition to his stationary point, and the arc on the celestial sphere passed over by him in that time, be accurately noted. Then, knowing Saturn's

* When Saturn is near opposition he passes over a space on the celestial sphere equal to the mean diameter of the moon's disc in about 5½ days. It may be mentioned, however, that this is a much smaller space than might be supposed. The brilliancy of the moon deceives the unaided eye, and the impression is conveyed that the moon covers a larger space on the celestial sphere than it actually does. Thus the space between two neighbouring stars of the three (nearly equidistant) forming Orion's belt would be considered by the unaided eye as somewhat less than the moon's apparent diameter, which yet it exceeds in the proportion of three to one. The distance between the two stars Mizar and Alcor (the middle star in the tail of the greater Bear), which appear so close to the naked eye, is equal to the moon's apparent semi-diameter.

period, we know also the arc of his orbit he actually passes over in that time, on the supposition, at least, that his orbit is circular and his motion uniform; and we know, also, the arc of her orbit passed over by the earth in the same time. We can proceed, then, to the following construction:—

Draw the circle E″E E′ (fig. 2, Plate V.) to represent the earth's orbit about S, the Sun. Let S E K be the line on which the earth, Saturn, and the Sun are situated when Saturn is in opposition. Let the angles K S L′ and K S E′ be the angles swept out about the Sun by Saturn and the earth, respectively, during the time in which Saturn passes from opposition to his stationary point: thus, when Saturn is stationary, the earth is at E′, and Saturn *somewhere* in the line S L′. Again, through E′ draw E′K′ inclined to S K in an angle containing as many degrees, minutes, and seconds, as the arc on the celestial sphere passed over by Saturn in the interval of time we are considering: thus E′K′ is the line of sight from the earth to Saturn when he is stationary; so that at this time he must be *somewhere* in the line E′K′. But we have already seen that at the same moment he is somewhere in the line S L′. Hence the point P′, in which the two lines E′K′ and S L′ intersect, is Saturn's actual position at this moment. We have, then, only to compare the lengths of the lines S P′ and S E by simple measurement, to determine the relation between the distances of Saturn and the earth from the sun; or we can obtain the required relation by a very simple trigonometrical calculation.*

Instead of determining the interval of time and the arc passed over from the moment of opposition to the following station (a matter of some difficulty), the interval and arc between the station preceding and the station following opposition may be noted. If E″ and E′ be the positions of the earth at those epochs respectively, S P, bisecting the angle E′S E″, is the line of opposition. If, then, S P′L′ and S P″L″ are each inclined to S P in an angle equal to half that swept out by Saturn in his orbit, in the interval between

* Thus:—in the triangle S E′P′ the side S E′ is known; the angle E′S P′, being the difference of two known angles (E′S K and P′S K) is known; and so also is the angle E′P′S, since it is the sum of two known angles (P′S K and the angle between the lines E′K′ and S K); hence we can determine the remaining sides and angles of the triangle S E′P′, and S P′ becomes known.

the two stations, while E′P′K′ and E″P″K″ are each inclined to S P in an angle corresponding to half the arc passed over by Saturn on the celestial sphere in the same interval, P′ and P″ are both points on Saturn's orbit; and the measurement of S P′ or S P″, or the trigonometrical calculation of the length of either as compared to the length of S E, enables us to determine as before the relation between the distances of Saturn and the earth from the sun.

If the orbits of Saturn and the earth were circular and in one plane, this method and those before described would be strictly exact. The results obtained would be affected only by errors of observation, of construction, or of calculation. To such errors, at first, the astronomers of old must have been inclined to attribute the discrepancies which appeared, not only between the results obtained by different methods, but between results obtained by the same method applied at different times. If the first three methods described be so applied, Saturn appears to be traversing an orbit at one time larger, at another smaller, than the orbit resulting as the average of a large number of observations; while his velocity is less in the larger orbit, and greater in the smaller, than his mean velocity. If, on the other hand, Saturn's sidereal period be assumed as the basis of calculation, there still appears a discrepancy in the magnitudes of the orbits determined at different epochs, but the velocities thence determined appear greater in the larger, and less in the smaller orbits. The ellipticity of Saturn's orbit and the variations in his velocity, to which these discrepancies are due, will be considered further on. In the case of Saturn, the resulting irregularities, though not so marked as those of the Moon, Mars, and Mercury, must have been sufficiently obvious to the careful observer, even of old times, and their periodicity could hardly fail to attract his notice. Indeed, there are reasons for supposing that the early Chaldæan astronomers detected these irregularities in the planetary movements, and assigned them to their true cause.*

Another circumstance in which Saturn's orbit differs from the uniform orbit we have imagined—an irregularity undoubtedly detected in very early times—remains to be considered.

* See Note A, Appendix.

If the orbits of Saturn and the earth lay in one plane, it is evident that the line of sight from the earth to Saturn would always lie in this plane, and thus, whatever effect the motion of the earth might have on Saturn's apparent motions, he would always be seen on the circle in which this plane meets the celestial sphere; in other words, Saturn would always be seen on the ecliptic. Hence, in retrograding, he would appear to retrace part of his former progressive path and *vice versâ*. His actual apparent movements are not of this nature. He follows a looped and twisted course, as shown in Plates II. and III.; his retrogressive path lying sometimes above and sometimes below his progressive path, and *vice versâ*. It is clear, then, that Saturn's orbit cannot lie in the same plane as the earth's orbit.

The inclination of the plane of Saturn's orbit to the ecliptic is small. Since both planes pass through the Sun's centre their line of intersection passes also through that point. This line is called the line of Saturn's nodes; and when, in travelling along his orbit, he reaches this line he is said to be *in a node*. One half of his orbit lies to the north,* the other half to the south of the ecliptic. When he is passing from the southern to the northern side of the ecliptic he is said to be in his *ascending node*; and in his *descending node* when he is passing from the northern to the southern side of the ecliptic.

Thus in fig. 3, Plate V., let N P′P″N′ represent the northern half of Saturn's orbit (viewed in perspective), $n$ E $n'$E′ the earth's orbit, and N $p\,p'p''$N′ the projection of Saturn's orbit on the plane of the earth's orbit. Let N S N′ be the line of Saturn's nodes on this plane, and let S P′ be at right angles to N S N′, so that, when at P′, Saturn is at his greatest distance from the ecliptic on the northern side. Then the angle P′S$p'$ is the angle of inclination of the plane of Saturn's orbit to the ecliptic; N is Saturn's ascending node, N′ his descending node.

The ancient astronomers determined the positions of the nodes of the planets, and the inclinations of the planetary orbits to the ecliptic, with tolerable accuracy. The exact determination of these

* That is, on the same side of the ecliptic as the north pole of the earth. Strictly speaking, the terms north, south, east, and west refer to the equinoctial only.

elements is not easy. Let us consider the methods applicable in the case of Saturn.

When Saturn is at a node, at N or N′, it is clear that, wherever the earth may be, the line of sight to Saturn lies in the plane of the ecliptic. It is equally clear that when Saturn is at any other part of his orbit he is not seen on the ecliptic, for the line of sight from the earth no longer coincides with the plane of the ecliptic. Thus, if we can determine the exact moment at which Saturn appears to cross the ecliptic, we know that at that moment he is in a node. It does not, however, necessarily follow that the point at which Saturn appears to cross the ecliptic indicates the position of the node. Saturn, as we have already seen, may appear behind, or in advance of his true place, at the moment of passing his node. If the correction due to this cause were made, however, the position of Saturn's node would become known from such an observation.

If the plane of Saturn's orbit were inclined at a considerable angle to the plane of the ecliptic, this method would be as accurate as it is simple. But the angle is so small in the case of Saturn (as of nearly all the planets) that it is difficult to determine the exact point at which he passes the ecliptic. For several degrees on either side of this point his distance from the ecliptic is scarcely appreciable. It must further be remembered that the determination of the exact position of the ecliptic itself upon the celestial sphere is a problem of no inconsiderable difficulty, and a very slight error in its solution would introduce a very important error in the determination of the nodes of a planet whose orbital plane is inclined at a very small angle to the ecliptic.

If it were not for the difficulty of determining the exact moment at which Saturn crosses the ecliptic, his period could be determined with far greater accuracy by successive observations of his nodal passages than by any other method. For, in the first place, the interval between successive passages of his ascending node (or of his descending node) is constant, being in fact no other than his sidereal period.* In the second place, the observation to be made

* Strictly speaking, this interval, which may be called Saturn's nodical period, is neither constant nor equal to his sidereal period; but both errors must be measured, not by days and hours, but by minutes and seconds.

is simple, and the position of the earth in her orbit exercises no modifying influence on the result as in other methods.

Let us next consider how the angle in which the plane of Saturn's orbit is inclined to the plane of the ecliptic may be determined. If it were not very small, all that would be necessary would be to observe the angle between Saturn's path and the ecliptic at the time of either nodal passage; it is plain that this angle, Q′N *q′* or R N′ *r*, is the same as the angle P′S *p′*, whose value is required. This method is inapplicable in the actual case, but a very simple method may still be employed. After passing a node Saturn moves farther and farther from the ecliptic, through about 90° of his apparent path, and attaining here a maximum distance from the ecliptic, approaches nearer and nearer to it, till he is again upon the ecliptic, or at a node. Now, if the observer were placed at the sun's centre these motions of separation and of approach would plainly be continuous, since Saturn and the earth would each appear to describe a great circle on the celestial sphere. Further, it is perfectly clear that the arc measuring Saturn's distance from the ecliptic, when he is farthest from that great circle, contains as many degress, minutes, and seconds, as the angle between the planes in which Saturn and the earth are moving. If, then, the supposed spectator in the sun were to measure this arc on the celestial sphere, he would know the angle we are seeking. But to the actual observer on earth Saturn's apparent motions of separation from and approach towards the ecliptic are not continous; or rather, though continuous, we cannot separate them into two periods, one of separation, the other of approach. Although, on the whole, Saturn's distance from the ecliptic appears to be increasing, through about 90° of his path from a node, his apparent path on the celestial sphere is twisted into loops of varying shape, in his motion along which he moves alternately from and towards the ecliptic. His return to the ecliptic is effected in the same manner. The reason may easily be seen: if Saturn is in any other part of his orbit except either node, the line of sight from the observer on earth only lies in the plane of Saturn's orbit when the earth herself is in that plane; in other words, Saturn is only seen on his true or heliocentric path when the earth is on the line of nodes, either at $n$ or $n'$. In moving from $n$ through E to $n'$, the earth is south of the plane of Saturn's orbit, and Saturn therefore appears north of his true path; similarly, while the earth moves from $n'$ through E$'$ to $n$, Saturn appears south of his true place. Hence, if the earth is at any other part of her orbit but $n$ or $n'$ when Saturn attains his greatest distance from the ecliptic, a corresponding correction must be made on this account. A further slight correction is necessary on account of the difference between Saturn's distance from the earth at the moment of observation and his mean distance from the sun. The inclination of Saturn's orbit to the ecliptic may, however, be determined very approximately without attending to the first of these corrections. For when Saturn is describing the part P P$'$P$''$ of his orbit, his distance from the ecliptic varies very slowly. But during this time the earth describes rather more than one complete revolution, and therefore passes both the points $n$ and $n'$ of her orbit. If the distance of Saturn from the ecliptic be measured when the earth is at either of these points, and increased in the proportion of the distances of the earth and sun from Saturn at this time, then the required angle contains as many degrees, minutes, and seconds, as the arc thus determined, very approximately. The nearer Saturn is to the point P$'$, or to the opposite point of his orbit, when the earth is passing $n$ or $n'$, the more exact will be the determination of the angle required. In the course of two or three revolutions of Saturn, one observation at least that is perfectly trustworthy may be effected.

It is found in this manner that Saturn's orbit is inclined to that of the earth at an angle of about 2½°. Owing to the causes mentioned in the preceding paragraph, his greatest departure from the ecliptic exceeds this angle by about a quarter of a degree. The arc of the celestial sphere, then, that measures Saturn's greatest possible departure from the ecliptic is rather more than five times as great as the moon's mean apparent semi-diameter. The distance between the two stars commonly known as the Pointers* is almost exactly double the arc we are considering.

Owing to causes which will be mentioned further on, both

* That is, $\alpha$ (Dubhe) and $\beta$ Ursæ majoris.

the position of Saturn's line of nodes and the inclination of his orbit to the ecliptic are variable. The annual variation in the inclination is always very small; for long intervals it operates to increase, and for corresponding intervals to diminish, the angle of inclination; so that this angle varies in an oscillatory manner, the period of oscillation being very great, and the total amount of variation either way being very small. The line of nodes moves sometimes from east to west, sometimes from west to east, but the westerly motion prevails, so that on the whole the line of nodes revolves in a retrograde direction, but so slowly that a complete revolution is not effected in less than 66,000 years.*

The looped nature of Saturn's apparent path on the celestial sphere is due to the inclination of the plane of Saturn's orbit to the plane of the ecliptic. The varying forms assumed by the loop correspond to Saturn's varying positions in his orbit. When he is near a node his path is twisted, but without a loop: for instance, when he is near his ascending node his path is as shown in Plate II. (where the path crosses the ecliptic). As he moves on in his orbit his path becomes looped, the loop lying to the north of his mean path† in the case we are considering (that is, after the passage of the *ascending* node), or on the side farthest from the ecliptic. The loop gradually developes: at first, the progressive path intersects the former retrograde path; in each successive loop the point of intersection falls farther and farther from the stationary point following opposition, till it reaches the stationary point preceding opposition; after this, for several successive loops the point of intersection lies on the former progressive path, being halfway between the stationary points when Saturn reaches his greatest distance from the ecliptic. From this point to the

* In tables of the planetary elements the longitude of Saturn's ascending node is described as subject to an annual decrease of 19″·54. This is to be understood as referring to the retrograde motion in longitude of the ascending node. Since the precession of the equinoxes is 50″·1 yearly (in longitude), the longitude of Saturn's ascending node increases annually by more than half a minute of arc.

† That is, his *heliocentric* path; in the maps forming Plates II. and III., Saturn's heliocentric path would be represented by straight lines drawn in the direction indicated by the *general* direction of his geocentric path, in such a manner that Saturn's greatest departures on either side of such line may be about equal. See the dotted lines in figures 4 and 5, Plate V.

descending node the loops undergo similar changes in a reverse order; the point of intersection passes to the station following opposition; thence along the retrograde path to the station preceding opposition (so that the opening between the loop and path, which before was towards the east, now lies towards the west); and finally, near the descending node, the path, as at the ascending node, is twisted without a loop. In passing from his descending to his ascending node, Saturn's path is similarly varied, the loop being now south of the ecliptic, or still on the side of Saturn's mean path farthest from the ecliptic.

The causes of these phenomena will be made sufficiently apparent if we consider Saturn's motion during a synodical revolution in each of two extreme cases—viz., first, when he is at a node, and secondly, when he is at his greatest distance from the ecliptic.

Suppose, then, first, that during a synodical revolution Saturn passes from Q to Q′ (fig. 3, Plate V.), and is in opposition when at his ascending node N. During this time the earth moves from a point slightly to the west of *n′*, through rather more than one complete revolution, to a point slightly to the east of *n′*. As the earth passes the point *n′* Saturn passes from the northern to the southern side of his heliocentric path. He remains to the south of that path as the earth moves from *n′* through E′ to *n*. When the earth is at *n* Saturn (in opposition at his ascending node) again crosses his heliocentric path and also the ecliptic, passing to the north of both these great circles of the celestial sphere. While the earth moves from *n* through E to *n′* Saturn remains to the north of his heliocentric path, passing to the south as the earth passes the point *n′*.

If, then, we draw the line E N E′ (fig. 4, Plate V.) to represent part of the ecliptic, and the dotted line *s* N *s′*, inclined at an angle of $2\frac{1}{2}°$ to E E′, to represent part of Saturn's heliocentric path, and combine the results of the preceding paragraph with the knowledge already obtained of Saturn's progressions and retrogressions, it is easily seen that Saturn's apparent path on the celestial sphere, during the synodical revolution considered, is of the form Q $n$ N $n'$ Q'; $n$, N and $n'$ being the points at which he appears to cross his heliocentric path $s\,s'$*.

Next let us consider the nature of Saturn's apparent path when he is at his greatest distance from the ecliptic. Suppose that during a synodical revolution he passes from P to P'' (fig. 3, Plate V.), and is in opposition when at his greatest distance from the ecliptic at P'. During this time the earth moves from a point slightly to the west of E' through rather more than a complete revolution to a point slightly to the east of E'. While the earth is moving to $n$ she is on the northern side of the plane of Saturn's orbit, and Saturn is on the southern side of his heliocentric path. He passes to the northern side as the earth passes the point $n$; remains on the northern side of his heliocentric path as the earth moves from $n$ through E to $n'$ (attaining his greatest departure from that path when in opposition at P'); crosses to the southern side as the earth passes the point $n'$; and remains on that side throughout the remainder of the synodical revolution we are considering.

If, then, we draw E E', fig. 5, Plate V., to represent part of the ecliptic, and the dotted line $s\,n'\,n\,s'$ (parallel to E E' and at a distance from that line corresponding to an arc of $2\frac{1}{2}$ degrees on the celestial sphere) to represent Saturn's heliocentric path, it is plain that Saturn's path during the synodical period is of the form P $n$ P' $n'$P''; $n$ and $n'$ being the points at which he appears to cross his heliocentric path.†

There is no difficulty in applying similar methods to determine the form of Saturn's apparent path when he is in any other part of his orbit. It will be found to vary in the manner

* While traversing parts of this path near Q and Q', Saturn is not visible from the earth, being near conjunction. If he were visible in these parts of his orbit, it would be found that at $n$ and $n'$ his departure from the ecliptic is greater than at any other moment during the synodical revolution considered. These points are therefore marked $q$ and $q'$ to indicate their correspondence with the points $p$ and $p''$ in the synodical revolution next considered.

† While traversing parts of the path near P and P'', Saturn is not visible from the earth, being near conjunction; if he were visible in these parts of his orbit, it would be found that at $p$ and $p''$, the positions he occupies when the earth is at E', he attains his greatest southern departure from his heliocentric path—or approaches nearest to the ecliptic—in the synodical revolution considered.

described above. The following consideration may assist the student:—

Since Saturn is seen on his heliocentric path whenever the earth is at $n$ or $n'$, his geocentric path crosses his heliocentric path once in every six months; now, Saturn completes a synodical revolution in a period exceeding twelve months by twelve days and three quarters; thus the points of intersection of his geocentric and heliocentric paths fall successively farther and farther back, in each successive synodical loop, by the space Saturn traverses in $6\frac{3}{8}$ days; they therefore occupy, successively, every part of Saturn's synodical loops.

## CHAPTER II.

### FALSE SYSTEMS—MODERN ASTRONOMY—ELEMENTS OF SATURN'S ELLIPTIC ORBIT.

Before turning to the consideration of the methods and discoveries of modern astronomy, a few words on the system which explained Saturn's motions (in common with those of the other planets) on the supposition that the earth is the centre of the universe, will not be out of place. This system, and the fanciful and superstitious dreams of the middle ages, may be considered as occupying a place midway between the simple systems and intelligent inquiries of the Chaldæan astronomers, on the one hand, and the analyses and discoveries of modern times on the other.

The difficulties connected with the Ptolemaic system are not due so much to the inherent error of the system itself, as to the fanciful hypotheses with which the originators of the system perplexed themselves. All the varieties of the planetary motions, except a few irregularities only to be detected by the most exact instrumental observation, may be as exactly explained on the supposition that the earth is the centre of the system as on the true theory, and with almost equal simplicity. But the Epicyclians set themselves a problem of far greater complexity. They sought to explain the apparent motions of the heavenly bodies, not merely on the supposition that the earth is the centre of the system, but with the additional hypotheses that all the members of the system move in circular orbits and with uniform velocities. Bodies terrestrial, they argued, are gross, corrupt, and imperfect—therefore they move in imperfect orbits, with varying velocities; bodies celestial are sublime, incorrupt, and perfect—therefore they move in perfect orbits with uniform velocities; the circle is the only perfect figure—therefore the heavenly bodies move in circles; but the supposition of uniform motion in simple circular orbits is insufficient to account for the apparent motions of the heavenly bodies—therefore those motions must be explained by properly combining two or more sets of circular and uniform movements. Such was the problem they set themselves; in what manner they solved it will appear by an illustration drawn from the motions of Saturn.

Let E (fig. 1, Plate VI.) be the earth, C C′C″ a circle about E as centre. Then, clearly, Saturn's progressive and retrograde motions cannot possibly be explained by supposing him to move uniformly in the circle C C′C″. Suppose, however, that P *p* P′*p*′ is a smaller circle, whose centre C is on the circle C C′C″; and that while Saturn moves with uniform velocity round the circle P *p* P′*p*′, the centre of this circle moves uniformly round the circle C C′C″. Then it is clear that if Saturn's velocity in the smaller circle is greater than the velocity with which the centre of that circle moves round the larger circle, his apparent motion will be retrograde when he is at or near *p*′; and further, that by assigning suitable dimensions to the two circles, and a proper ratio between the velocities considered, Saturn's period of retrogression and the length of his retrograde arc may be readily explained.*

We have seen that Saturn's distance from the earth, at opposition, is variable. These variations may be explained with tolerable accuracy by supposing that the earth occupies an eccentric position within the circle C C′C″, as at E′.

Saturn's looped and twisted path may also be easily explained. We have only to suppose the plane of the circle P *p* P′ inclined at a small angle to that of the circle C C′C″; or, instead of this, we may suppose both circles to lie in one plane which oscillates through a small angle about a fixed line through the earth at E′.

Smaller irregularities may be accounted for by supposing that P *p* P′*p*′ is not *Saturn's* orbit, but the path of the centre of a smaller circle, S *s* S′*s*′, along whose circumference Saturn moves uniformly.

* For this purpose the radius of the smaller circle must bear to the radius of the larger circle the proportion that the radius of the earth's orbit bears to that of Saturn; again, Saturn must revolve once in a year round the smaller circle, whose centre must revolve once in a Saturnian year round the earth.

Again, we may suppose that the circle C C′C″ is not the path of the *centre* of the circle P *p* P′*p*′, but of a point near the centre; in other words, that the circle P *p* P′*p*′ is eccentric as well as the circle C C′C″. We may extend this eccentricity to the circle S *s* S′*s*′, or introduce additional variety by supposing any or all of the circles to lie in different or in oscillating planes; in fine, by a series of such suppositions, which may be carried on *ad infinitum*, we may account for nearly every irregularity in Saturn's motion with a very close degree of approximation.

To explain how these motions were supposed to be impressed and maintained by a system of celestial spheres, and through the complicated effects attributed to their rotations, would be out of place. The whole system, with its

> . . . centrics and eccentrics scribbled o'er,
> Cycle and epicycle, orb in orb,

has been long since swept away, and its records merely remain as illustrations of perverted ingenuity.

One point, however, connected with the Ptolemaic system of the universe remains to be noticed. If the earth really occupied the central place in our system, the actual, and even the relative distances of the various members of that system must have remained for ever unknown. Let us consider, for a moment, how the geometer ascertains the distance of an inaccessible object. To effect this, he observes the directions in which the object is seen from two convenient points, the distance between which he measures. Then, either by geometrical construction, in which these relations are represented on a convenient scale, or, more exactly, by trigonometrical calculation, he determines the distance of the inaccessible object from either point. That this determination may be depended upon, it is necessary, not only that the instruments with which the requisite data are obtained should be trustworthy, but that the distance between the two points should not bear too small a proportion to the distance of the inaccessible object. For instance, a base line of ten yards, with good instruments, would be sufficient for the determination of distances up to three or four hundred yards; but it would obviously be altogether useless to apply such a base to determine the exact distance of an object two or three miles off. The slightest error in the determination of either of the base angles would make a difference of a mile or two in the result deduced by construction or calculation. Now the length of the earth's diameter being about one-thirtieth part of the moon's distance from the earth, this distance can be determined with tolerable accuracy from a base line whose extreme points lie on the earth's surface.* But the distances of the other members of our system (including the sun) from the earth are so vast that it would be altogether impossible to determine their actual distances by using any base line on the earth. To obtain any notion of their relative distances would require the utmost perfection and power of modern instruments, and the highest skill of the modern astronomer. Even with these appliances, our ideas of the relative distances of the planets would be as vague and uncertain, if the earth were the centre of our system, as are our present ideas of the relative distances of the fixed stars from the earth.† Nor is there any point in the Epicyclic theory that would enable its supporters to form any conjectures regarding the relative distances of the planets. It is plain that to an observer placed at E (fig. 2, Plate VI.) the appearance of a planet revolving uniformly round the circle $PP'P''P'''$, while the centre of that circle moved uniformly round the circle $CC'C''$, would be precisely the same as that of a planet revolving uniformly round the circle $pp'p''p'''$, while

* Yet from the most trustworthy modern measurement it appears that the determination of the moon's distance hitherto adopted has been about twenty miles too great.

† In the case of the sun, as in that of the moon, our base line is limited by the earth's dimensions; and since the sun's distance is so vast compared with such a base line, we could expect to obtain no very close approximation to that distance. Accordingly, we find that before the discovery of the telescope the ideas of astronomers on the subject of the sun's distance were of the most vague and indefinite kind; and the discovery lately made, that the modern determination of the sun's distance is probably too great by three millions of miles or more, shows that even in the present advanced state of the science of astronomy the problem is no easy one. In the planets Mercury and Venus, however, we have two objects, which serve, so to speak, as celestial instruments; the sun's disc, at the times of their transits, serving as an index-plate. Observers at different parts of the earth's surface, marking the different indications of this celestial theodolite, calculate thence the solar distance. At favourable parts of his orbit, Mars, though a superior planet, serves the same purpose in a somewhat different manner, the celestial sphere serving as an index-plate.

the centre of that circle moved uniformly round the circle $c\,c'c''$, if the periods of revolution of the two planets and of their orbit-centres were respectively equal.

It appears, then, that if the Epicyclians merely trusted to the results of observation applied on the hypotheses which formed their system, they could have had no accurate notions, even of the relative distances of the sun and planets from the earth, far less of their actual distances. For anything they could perceive to the contrary, Saturn might (after the moon) be the nearest of the heavenly bodies—Mars, Venus, or Mercury the most distant. Yet we learn that the order of the planetary distances was known to the ancients at a very remote period. In the fanciful scheme ascribed by Philolaus to Pythagoras, in which musical tones were supposed to be produced by the revolution of the spheres bearing the planets, the note assigned to the Saturnian sphere was the *hypâte*, or deepest tone, the note assigned to the moon's sphere the *neate*, or highest tone of the celestial harmonies, the spheres of the other heavenly bodies being placed in their just order in the scale. It seems probable, therefore, that the Greek astronomers had derived part of their knowledge from nations to whom the true system of the universe was not unknown.

Before turning to the discoveries of modern astronomy, it may not be uninteresting to dwell for a moment on the superstitious fancies of the astrologer. The origin of the system which ascribed an influence on the fates of men and nations to the planetary phenomena is lost in the obscurity of a far antiquity. It was probably connected with the Sabæanism of the ancient Chaldæans and Arabians, a form of religious worship derived from a purer system, in which the stars and planets were not themselves the objects of adoration, but simply regarded as types of the divine attributes. Astrology was gradually formed into a system showing few traces of the religious source from which it had been derived. Its complex and mystical character marks it as framed rather to deceive and impress the ignorant, than as possessing the confidence of its professors. Thus it became a weapon in the hands of the priesthood of Nineveh and Babylon, a weapon which might serve good or evil purposes, according to the character of him who wielded it, but which was too often employed to subserve the evil designs of the despotic emperors under whose sway the priestly orders were subdued. It would be out of place to record here, at length, the details of the system itself, or to trace the gradual process by which astrology—deriving its origin from pure and lofty conceptions of the divine power, wisdom, and goodness—fell to the position it has now so long occupied, and became the tool of cheats and charlatans. It may be mentioned, however, that the idea of physical influences exerted by the planets in their varying positions, has been entertained by many who fully recognised the absurdity of the so-called astrological systems. Bacon (who was, however, but superficially acquainted with astronomy, and strongly prejudiced against the Copernican system) considered an inquiry into such influences likely to lead to valuable results. 'Astrology,' he wrote, 'is so full of superstition, that scarce anything sound can be discovered in it; though we judge it should rather be purged than absolutely rejected.' He then propounded his 'Astrologia Sana,' which should contain inquiries into—(i.) the commixture of planetary rays in the different positions of the planets with respect to one another and on the zodiac; (ii.) the zenith distances of the planets, or the planetary seasons; (iii.) the influences of the planets at their apogees and perigees; and (iv.) 'the other accidents of the planets' motions, their accelerations, retardations, courses, stations, retrogradations, distances from the sun, &c.; for all these things affect the rays of the planets, and cause them to act either weaker or stronger, or in a different manner.'*

The following lines of Chaucer present the gloomy and dismal ideas which astrologers naturally associated with Saturn's dull light and sluggish motions:—

My dere doughter Venus, quod Saturne,
My cours, that hath so wide for to turne,
Hath more power than wot any man.
Min is the drenching in the see so wan,
Min is the prison in the derke cote,
Min is the strangel and hanging by the throte,
The murmure, and the cherles rebelling,
The groyning, and the prive empoysoning.

* 'Advancement of Learning,' Book iii. Chap. 4.

I do vengeaunce, and pleine correction,
While I dwell in the signe of the leon.
Min is the ruine of the high halles,
The falling of the toures and of the walles
Upon the minour, or the carpenter:
I slew Sampson in shaking the piler.
Min ben also the maladies colde,
The derke tresons, and the castes olde:
My loking is the fader of pestilence."

Another superstition, whose origin is equally obscure with that of astrology—the idea, namely, that the planets exerted influences (each on its respective metal) over the labours of the alchemist—is mentioned by the same poet in the Chanones Yemannes tale. He thus succinctly states the distribution of the metals among the planets—

Sol gold is, and Luna silver we threpe;
Mars iren, Mercurie quicksilver we clepe:
Saturnus led, and Jupiter is tin,
And Venus coper, by my faderkin.*

* No satisfactory explanation has been given, so far as I know, of the distribution indicated above. That the two most valuable metals should be assigned to the sun and moon needs no explanation; the silvery light of the moon, and the yellow or red light of the sun whenever it can be viewed by the naked eye, make the distribution still more appropriate. On a different principle one can understand why quicksilver should be assigned to Mercury, which is so difficult to detect, and whose motions are so rapid. On other principles the association of Mars and iron may be explained: for some resemblance can be imagined between the colours of the ruddy planet and of the red oxide of iron, or Hæmatite; or the employment of iron in war might suggest the association; or, lastly, the invigorating and tonic properties ascribed to medicines containing iron correspond with the influences attributed to Mars by astrologers. The association of lead with Saturn may be explained on similar principles: the protoxide of lead (or Massicot) is of a pale yellow colour, somewhat resembling that of the planet; or one may imagine lead assumed as the representative of the dull, slow-moving Saturn, from some such fanciful association of ideas as that expressed by Armado in 'Love's Labour's Lost,'—'Is not lead a metal heavy, dull, and slow?'; or, lastly, the association might have been suggested by the chilling and deleterious effects peculiar to medicines containing lead—still called by doctors Saturnine medicines. Why tin and copper should be assigned respectively to Jupiter and Venus is not very obvious. The connection between the name of the latter metal and that of the island Cyprus sacred to Venus is noticeable. A singular coincidence may be mentioned here:—in the list of metals in Numbers, chapter xxxi, verse 22, we have the representatives of the sun, the moon, and the four planets probably known to the Jews at that time; and these four, 'the brass, the iron, the tin, and the lead,' are arranged in the order of the distances from the sun of the corresponding planets. That the word translated brass signifies copper is clear from the words of Job, chapter xxviii, verse 2, 'brass is molten out of the stone.'

Let us now turn from the false systems and idle fancies which throve with rankest luxuriance—like fungous growths in darkened nooks—amid the ignorance and superstition of priest-ridden ages, to the awakening of science at the dawn of a new era. The life of Nicolaus Koppernik, or Copernicus—the restorer if not the discoverer of the true system of the universe—belongs to the latter part of the fifteenth and the beginning of the sixteenth century; an age—as has been well remarked by Humboldt—'coinciding in a wonderful manner with the age of Columbus, Gama, Magellan; the age of great maritime enterprises; the awakening of a feeling of religious freedom; the development of nobler sentiments of art.'* During the first years of the sixteenth century Copernicus was engaged at Rome, at Padua, and at Bologna, in discussing with the astronomers of the day the various theories which had been invented to explain the planetary motions. Struck with the complexity of these theories he was led, after trying several hypotheses (probably including the system generally attributed to Tycho Brahe) to the conviction that the sun is the centre around which the planetary scheme revolves. 'We find in this arrangement,' he says, 'what can be discerned in no other scheme—an admirable symmetry of the universe, an harmonious disposition of the orbits. For who could assign to the lamp of this beautiful temple a better position than the centre, whence alone it can illuminate all parts at once? Here the sun, as from a kingly throne, sways the family of orbs that circle around him.'†

The new system met with fierce opposition; not, at first, from the priesthood, but from astronomers. It was not merely that the views put forward were opposed to opinions that had been held so long: this would in any case have been sufficient to rouse a strong feeling of opposition; but the system presented by Copernicus was wanting in simplicity. If he could have done away altogether with the old hypotheses of eccentrics and epicycles, the new system might have been more favourably received. This, however, he was unable to effect. His own observations had shown him that the apparent planetary motions were too complex to be satisfac-

* 'Cosmos,' vol. ii. part 2, § vii.

† 'De Revolutionibus Orbium Cœlestium,' lib. i. cap. 10.

torily explained by any hypothesis of simple circular orbits. He therefore retained in a modified form parts of the cumbrous systems of his predecessors.

Nearly three-quarters of a century after the publication of the celebrated work of Copernicus, Kepler, who had become in early youth an ardent convert to the new doctrines, was able to remove from the scheme of the universe the last traces of the Ptolemaic hypotheses. Tycho Brahe, strenuously opposed to the views of Copernicus, had erected an observatory at Uraniberg, where he had traced the paths of the planets on the celestial sphere with instruments more powerful and accurate than those employed by Copernicus. Kepler availed himself of a series of observations of the planet Mars made by Tycho Brahe with these instruments, and applied them to an investigation of the Copernican system. It was not his object to overthrow the doctrines of circular motions and uniform velocities, but to determine by what combination of eccentrics and epicycles the actual movements of the planets could be explained. Mars was in every respect the best selection he could have made. This planet is the nearest of the superior planets, and therefore its motions on the celestial sphere are swifter than those of Jupiter and Saturn; its orbit is also very eccentric;* on both accounts the true combination of epicyclic and eccentric motions should be more easily detected in the case of Mars than of any other planet.

Kepler calculated the motions that would result from such combinations with wonderful patience and accuracy, compared them with the actual motions of the planet, and was compelled to reject successively nineteen different hypotheses. Having exhausted the combinations of circular and uniform motion, he began at length to inquire whether the orbit of Mars, obviously oval, might not be an ellipse; and whether his velocity, obviously variable, might not—on the supposition of an elliptic orbit—be found to vary by some simple law. At this new problem he worked with unflagging energy and patience, trying and rejecting

* Mars in aphelion is more than 152,500,000 miles, in perihelion little more than 126,500,000 miles from the sun; the difference of these distances is greater than one-fourth of the earth's mean distance from the sun. See fig. 3, Plate VI.

numerous hypotheses. Finally, his labours were rewarded by the discovery of the true laws of planetary motion, constituting the two first of the 'laws of Kepler.' They are these:—

1. Every planet moves in an elliptical orbit, in one focus of which the sun is situate.

2. The line drawn from the sun to a planet (or the radius-vector of the planet) sweeps over equal areas in equal times.

From Saturn's motions in his orbit we can draw an illustration of these two laws. Let S, fig. 3, Plate VI., be the sun, E E′E″E‴ the orbit of the earth, *s s′s″s‴* Saturn's orbit. These orbits are both ellipses, but in the figure they are represented by circles, because (on the scale of the figure) the difference of the axes of Saturn's ellipse would be very nearly, the difference of the axes of the earth's orbit altogether imperceptible even on measurement. The eccentricity of the earth's orbit is also too small to be noted in the figure;* the eccentricity of Saturn's orbit will be at once observed. At *e* the earth is in perihelion; E, E′, E″, and E‴ are the positions of the earth at the winter solstice, at the vernal equinox, at the summer solstice, and at the autumnal equinox, respectively: at *s* Saturn is in perihelion, at *s″* he is in aphelion, and S *s″* bears to S *s* a proportion rather greater than that of ten to nine. More exactly—the radius of the circle E E′E″E‴ being taken as 1, the radius of the circle *s* N *s′*N′ is 9·538850; S *s″* is 10·072533; and S *s* is 9·005167. The two orbits, as already stated, lie in different planes, the line of whose intersection passes through the sun: in our figure N S N′ is this line, N being Saturn's ascending node; thus if the earth's orbit be supposed to lie in the plane of the paper, the part N *s′*N′ of Saturn's orbit lies above the paper, and the part N′ *s‴*N below. The lines *k k′* and *l l′* indicate the distances from the plane of the ecliptic of the points *s′* and *s‴*, at which Saturn attains his greatest departure from that plane.†

* It is hardly necessary to remark that the eccentricity may be very observable in an ellipse, even when the outline differs inappreciably from a circle: the difference of the semi-axes of such an ellipse bears a very small ratio to the distance of either focus from the centre—the ratio, namely, of the versed sine to the sine of a very small angle. For instance, the distance of the sun from the centre of Saturn's orbit is no less than 48,917,000 miles, while the difference of the semi-axes of Saturn's orbit is only 137,000 miles, or less than $\frac{1}{357}$th part of the former difference.

† The orbits of the planets Mercury, Venus, Mars, and Jupiter, are respectively

By the first law of Kepler, then, we learn that Saturn's orbit s s′s″s‴ is an ellipse, and that the sun is situated at S, one of the

indicated by the circles *m m′*, V V′, M M′, and J J′, the points *m*, V, M, and J being the perihelia of those orbits. The line ☊ ☋ in each orbit is the line of nodes, ☊ being the rising node. In the case of Jupiter the greatest departures from the plane of the ecliptic are indicated by the lines *i i′* and *j j′*; in the other orbits the corresponding departures are too small to be thus represented. The angles of inclination of the orbits of Mars, Venus, and Mercury, to the ecliptic, are, respectively, 1° 51′ 5″·5, 3° 23′ 33″·2, and 7° 0′ 25″·0. The corresponding angle in the case of Jupiter is 1° 18′ 36″·7. It will be observed that the orbits of Mars and Mercury are more eccentric than those of the other members of the system. The dotted ring A A′A″ marks the probable extent of the zone of asteroids, the orbits of four of which—Harmonia, Nemausa, Polyhymnia, and Nysa—are indicated respectively by the curves *h h′*, *a a′*, *p p′*, and *n n′*, the perihelia of these orbits being at *h*, *a*, *p*, and *n*. The two first are the least eccentric of the asteroidal orbits, and differ little from the circular form. The orbits of Nysa and Polyhymnia are remarkably eccentric. Professor Nichol remarks that 'Nysa recedes farther from the sun than any of the others, and, with the exception of Hæstia, approaches him the nearest.' If, however, the elements of the asteroidal orbits are correctly given by him in his 'Cyclopædia of the Physical Sciences' (article Asteroids), Hæstia is by no means remarkable for its near approach to the sun either as respects mean or perihelion distance, while the perihelion distance of Nysa is *less than the mean distance of Mars*. As will be seen from the figure, part of the orbit of Nysa absolutely falls *within the orbit of Mars*, a circumstance that will seem still more remarkable when it is considered that the *centre* of the ellipse in which Nysa moves lies outside the orbit of the earth—falling, in fact, *very near the orbit of Mars*. The orbit of Polyhymnia is not so eccentric as that of Nysa; yet the centre falls only just within the earth's orbit. To avoid confusion, the nodal lines of the four asteroidal orbits are not drawn in the figure; the following table indicates their positions, and the angles at which the planes of the four orbits are inclined to the ecliptic:—

| | Longitude of the ascending Node. | | | Inclination of Orbit. | | |
|---|---|---|---|---|---|---|
| Harmonia . . . . | 93° | 32′ | 28″ | 4° | 15′ | 48″ |
| Nemausa . . . . | 175 | 39 | 8·2 | 9 | 36 | 37·9 |
| Polyhymnia . . . | 9 | 16 | 5·0 | 1 | 56 | 56·0 |
| Nysa . . . . | 127 | 6 | | 3 | 53 | |

It will be seen from this table that the path of Nysa does not actually intersect that of Mars.

The asteroid Melpomene is also remarkable for the close proximity of a part of its orbit to the aphelion of Mars.

It has been noticed by Mr. Cooper, of Markree Castle, that in the positions of the asteroidal orbits a speciality is observable which can hardly be the result of accident:—the perihelia and the ascending nodes are not distributed indifferently, but are found chiefly in the semicircle from 0° to 180°. The observation may be extended to the larger planets; all of those introduced in the figure have their perihelia and rising nodes within the semicircle from 330° to 150°, which—more nearly than the semicircle just indicated—corresponds to the region in which the asteroidal perihelia and rising nodes are most remarkably crowded. The planets Uranus and Neptune do not foci of this ellipse. The second law of Kepler indicates the law of Saturn's motion in this orbit, which may be illustrated as follows: — Suppose that P P′, Q Q′, and R R′ are arcs over which Saturn passes in equal intervals of time; then Kepler's second law asserts that if straight lines S P, S P′, S Q, S Q′, S R, and S R′ be drawn (to avoid confusion, these lines are omitted in the figure), the areas S P P′, S Q Q′ and S R R′, are equal. Since the sector S P P′ is plainly shorter than the sector S Q Q′, and S Q Q′ than S R R′, it follows from the equality of these areas that the arc P P′ is longer than the arc Q Q′, and Q Q′ than R R′—increase in the breadth of the sectorial area compensating deficiency in length. In other words Saturn's velocity in his orbit increases as he approaches perihelion, and diminishes as he approaches aphelion. Thus, when he is near perihelion, he appears to be describing an orbit smaller than his actual orbit, with a velocity greater than his mean velocity; when he is near aphelion, these relations are reversed. His period, therefore, would appear too small, if determined when he is near perihelion, and too great if determined when he is near aphelion.*

deviate from the same law: the longitudes of their rising nodes are respectively 73° 14′ 38″; and 130° 10′ 12″·3, the longitudes of their perihelia 168° 27′ 24″, and 47° 17′ 58″. The speciality as regards the perihelia is certainly remarkable, and its physical interpretation worth seeking. The congregation of the rising nodes in the region indicated is obviously due to the choice of the ecliptic as the plane to which we refer the positions of the other orbital planes. Convenient as this selection is in many respects, it has its disadvantages; in fact, with the single exception of Mercury, no planet could be selected the plane of whose orbit is less suitable as a plane of reference in viewing the grander relations of the planetary scheme.

The orbits of Uranus and Neptune have not been introduced into the figure on account of their dimensions. The mean distance of Uranus from the sun is about twice, the mean distance of Neptune more than three times, that of Saturn. The eccentricities of the orbits are respectively ·0466 and ·0087, their inclinations to the plane of the ecliptic 0° 46′ 29″·9 and 1° 46′ 59″.

* The absolute velocity of a planet at any point of its orbit varies inversely as the length of the perpendicular on the tangent at that point: the angular velocity of the planet about the sun's centre varies inversely as the square of the planet's distance from the sun. There is a slight error in Nichol's statement that 'by an appropriate choice of an eccentric circular orbit the sun's motion relative to the earth or to any planet,' (or, which is the same thing, any planet's motion relatively to the sun), 'may be very closely approximated to,' on the supposition of uniform velocities. See article 'Eccentric' in Nichol's 'Cyclopædia of the Physical Sciences.' On such a supposition the angular velocity of a planet about the sun's centre would appear to vary inversely as the distance, instead of as the square of the distance of the planet.

The absolute dimensions of the ellipse in which Saturn moves are as follows: his mean distance from the sun (or half the greater axis of his orbit) is no less than 874,321,000 miles, his least distance (or $ss$) is 825,404,000 miles, and his greatest distance (or $ss''$) is 923,238,000 miles. The eccentricity of the orbit is very nearly ·056. In this vast orbit he moves with a mean velocity of 21,160 miles an hour, sweeping out a mean hourly angle of 5″·025 about the sun. He occupies 10759·2197106 days in moving once round his orbit, or in completing a sidereal revolution.*

Kepler next inquired whether there existed any relation between the periods of the planets and the dimensions of the planetary orbits. He selected the mean distances (or the semi-major axes of the orbits) for the comparison, considering that some relation might probably be found between the powers of these distances and of the periodic times. It was, however, only after many years' inquiry, that he arrived at the conclusion that it was here, and thus, that some new harmony in the planetary scheme was to be sought. One would have thought the rest of the work was simple; yet even when the very law he was seeking had occurred to him, two months and a half elapsed before he was able to verify it. Let us consider how the law might have been determined from the orbits and periods of Saturn and the earth. Calling the mean distance of the earth 1, Saturn's mean distance is 9·53885; again, calling the earth's period 1, Saturn's period is 29·4566:—now what relation (if any) exists between these numbers, 9·53885 and 29·4566, or their powers? The first is less than the second, but the square of the first is plainly greater than the square of the second; we must therefore try higher powers of the second number. Trying the next power, that is, the square of the second number, we immediately find the relation we are seeking; thus:—The square of the first number is less than the square of the second; but the next power, or the cube, of the first number is almost exactly equal to the square of the second.†

* All the elements of Saturn's orbit are undergoing slow processes of change; the natures and causes of some of these are examined further on; the tables of Appendix II. indicate the amount of the annual variation of each element.

† The cube of 9·53885 is 867·9369; and the square of 29·4566 is 867·691, differing from the first by less than 0·246.

Here then is the required law, if, only, it shall appear that the relation is confirmed when we try it upon other pairs of planetary orbits. On trial it appears to be true for every such pair, and thus the third law of Kepler is established; viz., that,

3. The squares of the periodic times of the planets vary as the cubes of their mean distances.*

Such are the laws of Kepler—laws purely empirical as presented by him, but destined to prepare the way towards, if they did not directly lead up to, the grandest law of nature yet discovered by man—the law of universal gravitation. Strictly speaking, none of Kepler's laws are correct: the planets being of appreciable mass and exercising attractions upon each other and upon the sun, their motions deviate from the orbits they would follow if these conditions did not exist—orbits which would be strictly in accordance with the laws propounded by Kepler. The accuracy of the laws, however, corresponded with, if it did not surpass, the accuracy of instrumental observation in Kepler's time, and for many years following the announcement of his important discoveries.

In the latter half of the seventeenth century, Newton commenced the investigation of Kepler's laws. Kepler had sought to learn *what* are the paths of the planets, and *what* the laws they obey in pursuing those paths: Newton devoted the powers of his piercing intellect to inquire *why* the planets follow such paths and obey such laws. He sought, in fact, the physical interpretation of the observed phenomena.

Newton first proved that a body moving in such a manner with respect to any point that its radius vector describes equal areas about the point in equal times, is moving under the influence of forces constantly directed towards or from that point. According as the orbit thus described is concave or convex towards the point, the force acts towards or from the point. Since, then, each planet describes equal areas in equal times about the sun, and moves in an orbit whose convexity is towards him, the sun exerts an *attractive* force on each member of the system.

* The law may also be expressed as follows:—Fixed units of time and space being chosen, the square of the number expressing the periodic time of a planet bears a constant ratio to the cube of the number expressing the mean distance of the planet.

Secondly, Newton demonstrated that if a body revolves in an elliptical orbit (or in an orbit whose form is any of the conic sections) under a central attracting force residing in one of the foci, that force varies as the inverse square of the distance of the attracted body. He further showed that Kepler's third law was a necessary consequence of attraction so varying.

In obtaining these results, Newton may be considered to have empirically demonstrated the existence of an attractive force exerted by the sun's mass, and to have established the law under which that force acts. The reader must be careful, however, to distinguish such a result from the establishment of the great law of gravitation. The mere determination of the law of attraction exerted by the sun on the planets and by these on their satellites, however interesting, would have been neither particularly valuable nor—except in being demonstrated—novel. The idea of attractions so exerted, and the very law of such attractions, had occurred to many astronomers long before Newton's day; nor does it appear that Newton himself attached any great value to the result, thus far, of his inquiries into the planetary laws of Kepler.

The history of the process by which Newton arrived at the great discovery which has rendered his name famous has been repeated so often that it would be idle to give it here at length. The idea that the moon was retained in its orbit about the earth by the same attractive energy that causes unsupported bodies to fall to the earth,* appears to have occurred to Newton about the year

* The story of the apple, whose fall suggested the first idea of his great discovery to Newton, is probably apocryphal. Whether it is true or not, the manner in which it is usually related in works on popular science is calculated to lead to altogether erroneous ideas of the nature of Newton's discovery. It would not have been the question, 'Why does the apple fall?'—that Newton would have asked himself: the attraction of gravity had been known for many ages; the laws of its action on falling bodies had been discussed, however erroneously, by Aristotle, and had been correctly established by Galileo. The inquiry might have been suggested, 'What if this attraction of gravity, so familiar to philosophers, of whose operation I have just witnessed an effect, has a wider range of action? what if an attraction whose influence appears to be exerted alike on bodies of the most varying natures, and to be unaffected by differences of elementary conformation, of form, or of physical condition, in the bodies acted upon, is itself exerted equally by bodies so differing; is a property depending not upon the quality but simply on the quantity of matter;—is, in fact, a "primitive power of nature," exerted by every atom in immeasurable space, with a range altogether

1666. He was unable, however, at that time, to establish the identity of the attractive energies displayed by the earth upon the moon and at her own surface, owing to the erroneous measure of the earth's radius then accepted. At length, in 1684, making use of Picard's more correct determination of the earth's magnitude, he was able to remove the discrepancy which had till then baffled him. He had already proved that, so far as terrestrial bodies were concerned, the earth's attraction is not influenced by the nature of the attracted object—that all solids, liquids, and gases, elementary and compound, in whatever physical state, are in the same degree under the influence of this omnipresent agency; he had now shown that the only celestial object whose motions are guided chiefly by the earth's attraction, shows by its main movements that it is influenced in the same degree as any terrestrial object would be at the moon's distance, supposing the earth's attraction to diminish as the square of the distance; and, lastly, he had proved that this law of variation prevails in the attractions of the celestial bodies. The conclusion deduced was announced by Newton—the last to rush from particular phenomena to general theories—in the grand cosmical law:—'Every particle of matter in the universe attracts every other particle with a force varying directly as the product of the masses and inversely as the square of the distance.' Under this law the satellites sweep round their primaries, these round the sun, the sun on his course within the star cluster to which he belongs, that cluster amidst its companion nebulæ, and the whole system of nebulæ amongst other systems in immeasurable space—all in their movements acting on and reacted upon by each other. And through the same great principle of nature, the least movement of the smallest insect on our globe has its influence on the motions of the most important members and systems of the universal Cosmos.

It is interesting to notice how admirably the characters of the three men whose labours had led up to and culminated in this

unlimited, however it may be modified, by distance?' It is quite possible that some simple event of the nature described might have started such a train of ideas in a mind like Newton's; it is certain that the law he established after eighteen years of patient waiting, has no narrower significance.

magnificent discovery, were adapted to the parts each had to perform. To Copernicus was given the confidence without ostentation necessary to the philosopher who is to refute ideas long held unquestioned: '*Vir fuit*,' says Kepler on this point, '*maximo ingenio, et quod in hoc exercitio magni est, animo liber*.' * Kepler's mind was cast in a different mould. He was not one who could originate a system, but rather one who, receiving a system from the hands of another, could appreciate its value, investigate its relations, and trace in it laws and analogies hidden from its discoverer. Inquisitive, ingenious, and imaginative, he pursued his inquiries with singular energy and untiring patience. In the midst of poverty, and tried grievously by a series of the most distressing domestic afflictions, he pertinaciously pursued, during twenty-three years, the path he had adventured upon.† Newton was endowed with a more comprehensive genius than either of his predecessors: bold and original like Copernicus—as observant, inquisitive, and patient as Kepler—he added to these qualities a piercing insight into those hidden operations and laws of nature to which celestial and terrestrial phenomena are due, and a wonderful aptitude in inventing and conducting experiments to confirm or correct his views. He was, on the one hand, the true philosopher of the Baconian type, forcing nature to reveal her secrets by sedulous and reiterated inquiries; on the other hand, he afforded an early illustration of Bacon's error in supposing his system of philosophy would raise all its followers to one level, however various might be their talents or capacities:—As in genius, so in the work he accomplished, '*genus humanum superavit*.' ‡

* Preface to the 'Rudolphine Tables,' published by Kepler in the year 1628.

† We find him in the year 1595, at the age of twenty-three, seeking the laws of the planetary orbits in simple numerical relations, in 'the residua of sines and cosines,' and in the radii of circles inscribed in and circumscribed about triangles, squares, and polygons; he even adopted, temporarily, a rough approximation drawn from the relations among the radii of spheres inscribed in and circumscribing the regular polyhedra. The singular law called the law of Bode or Titius is due to the ingenuity of Kepler, who also preceded Olbers in the supposition that some invisible planet occupied the space between the orbits of Mars and Jupiter.

‡ It is a rather singular coincidence that Kepler and Newton, to whose labours, chiefly, the discovery of the system of the universe is due, were both prematurely born into the world:—Kepler four days before Christmas-day in the year 1571; Newton on

In succeeding chapters of this work, we shall see how the theory of gravitation enables us to determine Saturn's weight and density. It has been applied also to determine the weight, and thence the probable thickness of Saturn's rings. In the sixth chapter, the great inequality of Saturn and Jupiter produced by the mutual attractions of these, the two most important members of the solar system, is examined and explained.

If any doubts could have remained of the truth of the Copernican theory after the revelations of the telescope and the investigations and discoveries of Kepler and Newton, Bradley's discovery of the aberration of light must have finally removed them. By this important discovery he proved that every star in the heavens, in tracing out its yearly aberration-ellipse, reflects the motion of our earth about the sun,* and becomes, in fact, a shining record of the ceaseless movements of that world, which seems to the untutored mind the aptest type of immobility.

Christmas-day 1642, the year in which Galileo died. We read of Kepler that 'he was a seven-months' child, very sickly during early life, and at the age of fourteen he was forbidden all mental application;'—of Newton, that 'he was so small at birth, that he might have been put into a quart pot,' and 'that the attendants successively despatched for medical aid were astonished to find him alive on their return.'

* It is plain that the stars are not the only objects whose positions on the celestial sphere are affected by the aberration of light; the planets, asteroids, and satellites are similarly affected in different degrees according to the directions of their motions and those of the earth; the sun's position is also affected by aberration, but with less variation in the amount of such affection. The moon is the only celestial body whose motions are not affected by aberration due to the earth's motion: the aberration due to her own motion is very small. The planetary and solar aberrations have been exactly computed, and are duly taken into account in determining the daily motions and positions of the sun and planets.

PLATE VII.

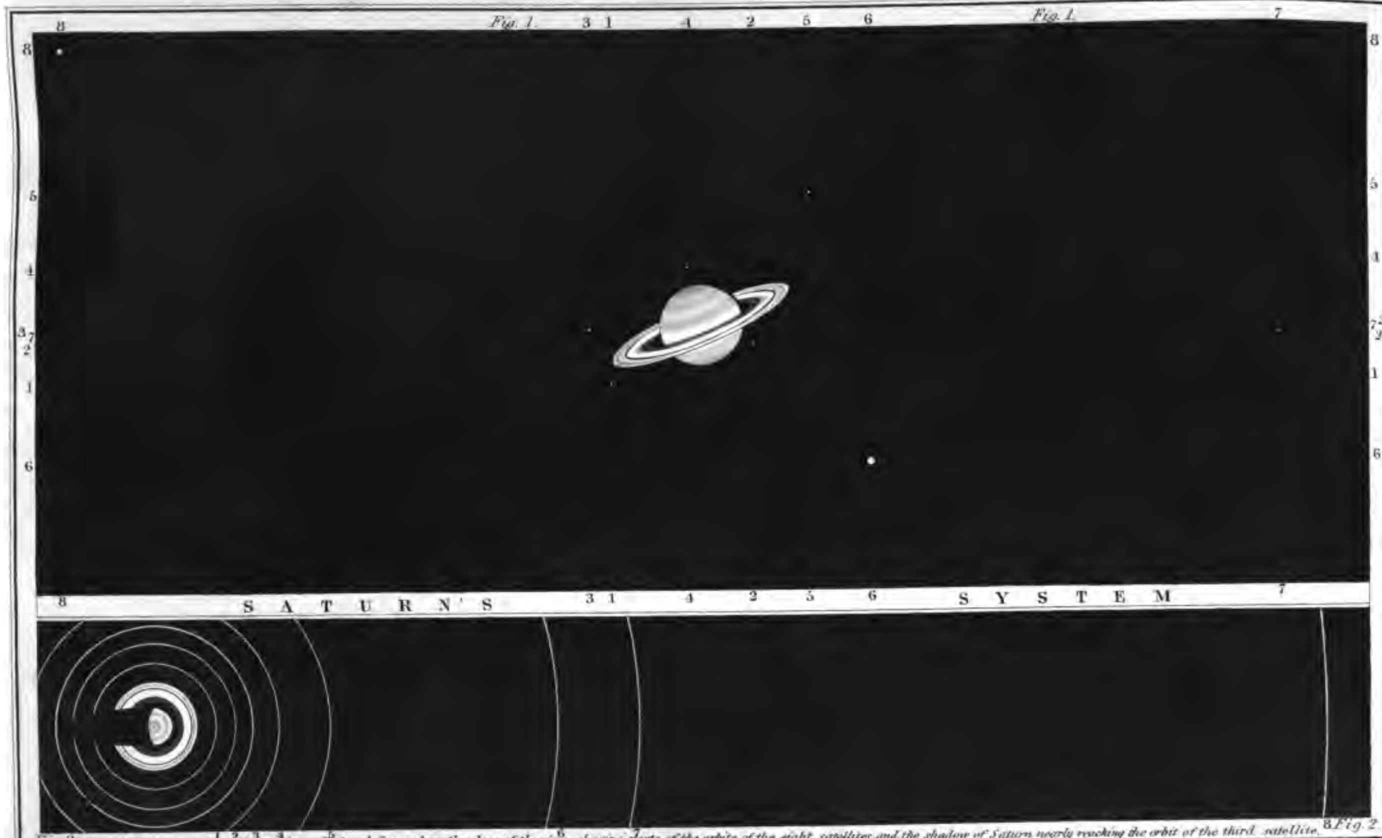


Fig. 2. Saturn's System on a smaller scale, and viewed from above the plane of the ring, showing parts of the orbits of the eight satellites, and the shadow of Saturn nearly reaching the orbit of the third satellite.

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# CHAPTER III.

## TELESCOPIC DISCOVERIES.

In the beginning of July 1610, at Padua, Galileo first examined Saturn with his largest telescope. Poor as this instrument would now be considered,* utterly insignificant as it would appear beside the gigantic tubes with which the Herschels, Rosse, Lassell, and Bond, have scanned the celestial depths, he had already effected with it a series of the most important discoveries. He had completed its construction in the preceding year—the year in which Kepler announced his first and second laws; and on the 7th of January, 1610, he had discovered by its means four new members of the solar system circulating around Jupiter, the least of which is nearly as large as our moon, while the greatest is equal in magnitude to the planet Mercury. We can imagine with what emotions of interest and expectation he applied his telescope to the examination of the more distant planet.

In July 1610, Saturn was approaching opposition, and very favourably situated for observation. Yet the result of Galileo's inspection was not satisfactory. He could detect a peculiarity in Saturn's appearance, but he was unable to determine the cause of that peculiarity. It appeared to him that on each side of Saturn's disc there was a minor disc. The two lesser discs seemed to be

* Galileo's largest telescope had a magnifying power of thirty-two diameters; two others he employed had powers of four and seven diameters only; the fields of view in all of them were very small. We learn from Brewster, who examined the largest a few years ago at Florence, that 'the object-glass was reduced to one-third of its area by a diaphragm of card, and the field was like a small hole.' A more powerful and far handier instrument may now be obtained in any optician's shop for a few shillings; yet if we regard the absolute importance of the discoveries effected by different telescopes, few, perhaps, will rank higher than the little tube now lying in the 'Tribune of Galileo' at Florence.

perfectly equal and symmetrically placed on opposite sides of Saturn, whose disc they appeared to overlap. Continuing his observations for several months, Galileo found that the two smaller discs retained the same position and were apparently unaltered in magnitude. These appearances were altogether perplexing to him; no phenomenon with which his telescope had hitherto made him acquainted, had prepared him to anticipate or understand a conformation so remarkable. The minor discs were evidently different from Jupiter's satellites: and, even if they were orbs attending on the central globe, it remained inexplicable that they should be always seen in the same position with respect to it, for this required that they should always be in the same position with respect to the line of sight from the observer on earth, a line whose motions partly depend, as we have seen, on the motions of the earth; so that it would appear as if these singular attendant orbs were partly guided by the earth in their movements about Saturn. Notwithstanding this apparently inexplicable circumstance, Galileo accepted the triplicity of Saturn as the only possible explanation of the phenomena, and in November 1610, he told Kepler that 'Saturn consists of three stars in contact with one another.' He announced the supposed discovery to the world of science, in the form of an anagram produced by transposing the letters of the sentence:—'*altissimum planetam tergeminum observavi*,' 'I have observed that the most distant planet is triform;' adopting this fanciful plan to prevent other astronomers from claiming the honour of the discovery.

After an interval of a year and a half, Galileo again examined Saturn. To his infinite amazement not a trace was visible of the appearances that had perplexed him before; there in the field of view of his telescope was the golden-tinted disc of the planet as smoothly rounded as the disc of Mars or Jupiter.* We can imagine how in his perplexity, he must have thought his telescope in fault, and how, adjusting the instrument, and cleaning the

* From Table X. it will be seen that the ring disappeared on December 28th, 1612, its plane passing through the sun; in the spring of 1613 the ring reappeared, its plane passing through the earth. There was no other disappearance at this passage of the ring's plane across the orbit of the earth. See Chapter IV.

glasses, he must again and again have brought the planet into the field of view,—still to see a single disc, where he had expected to see his triform planet. Finally, confused and amazed by a change so startling, he seems to have been inclined to put faith for the moment in the assertions of his enemies, that the discoveries he had reported had been mere illusions, justly sent to punish a spirit too prying and inquisitive:—'Is it possible,' he exclaimed, 'that some mocking demon has deluded me?'

The changes that Galileo afterwards detected in Saturn's appearance were still more perplexing. The minor orbs reappeared, and waxed larger and larger, varying strangely in form: finally, they lost their globular appearance altogether, and seemed each to have two mighty arms stretched towards and encompassing the planet.

From a drawing in one of his manuscripts it has been supposed that Galileo suspected the true cause of these startling changes. In this drawing Saturn is represented as a globe resting upon a ring. It seems more probable, however, that this drawing is a modern addition to the manuscript, and that Galileo was never able to explain the phenomena whose succession he had observed and recorded.*

Hevelius, with more powerful instruments, but in a climate less favourable to the astronomical observer, was not more successful than Galileo in explaining Saturn's mysterious changes of form. In the year 1656 he published his treatise '*de nativâ Saturni facie*,' in which he announced the result of his observations, concealing his real perplexity under a flight of sesquipedal words. 'Saturn,' he informed his contemporaries, with an amusing attempt at accuracy, 'presents five various figures to the observer—to wit: first, the mono-spherical; secondly, the tri-spherical; thirdly, the spherico-ansated; fourthly, the elliptico-ansated; fifthly, and finally, the spherico-cuspidated.'

A year or two before, Huygens, with a telescope of 12 feet focal length, had detected dark spaces enclosed within the as yet unexplained appendages on each side of Saturn's disc. Thus

* It will be seen from Table X. that the ring disappeared again in the year 1626, the plane of the ring passing through the earth early in September, and through the sun on the 15th of September in that year. Galileo became blind in 1637.

Saturn appeared as a globe, with two handles symmetrically placed on either side; or, as Hevelius expressed it, as an ansated spheroid. Subsequently, with a telescope of 23 feet focal length, and magnifying 100 times, Huygens saw these dark spaces more distinctly: but the true figure and structure of Saturn remained still a mystery to him. Some of the changes observed in Saturn's appearance could be explained by supposing the two appendages to be actually ansæ, or handle-formed structures attached to Saturn's body, but others remained inexplicable. It was not credible that the motions of Saturn's globe should be so exactly adjusted to those of the earth in her orbit, that the diameter through the ansæ should be always at right angles to the line of sight from the observer on earth; yet, if this were not the case, it remained impossible to explain how it happened that—whatever variations might appear in the forms of the ansæ—they always seemed to stand out to the same distance from the disc of the planet.

In the spring of 1656 Saturn appeared without his ansæ,* though Huygens examined him with a telescope of 123 feet focal length—one of the aërial telescopes he had himself invented. After observing the circumstances attending the disappearance and reappearance of the ansæ, and carefully investigating the theories which appeared most plausibly to account for the phenomena, Huygens at length arrived at the true explanation. He announced to his contemporaries, in the year 1659, that Saturn is girdled about by a thin flat ring, inclined to the ecliptic, and not touching the body of the planet.† He showed that all the variations in the appearance of this ring are due to the inclination of its plane to the ecliptic, while the tenuity and flatness of the ring explain its disappearance when the edge is turned to the spectator or to the sun. He found that the diameter of the outer circumference of the ring exceeded

* The plane of the ring passed through the sun early in March 1656 (see Table X); it had passed through the earth in the autumn of 1655, but Saturn was not then favourably situated for observation. After its plane had passed through the sun the ring became visible, but disappeared a few weeks after, its plane passing through the earth. The ring reappeared, finally, in the summer of the same year.

† Huygens propounded this important discovery in the form of the following sentence, anagrammatically transposed, 'annulo cingitur tenui, plano, nusquam cohærente, ad eclipticam inclinato.'

the diameter of Saturn's globe in the proportion of about 9 to 4; and he considered the breadth of the ring about equal to the breadth of the space between its inner edge and Saturn's body.

Four years before, on March 25th, 1655, Huygens had made another important discovery: by aid of the 12-feet telescope already mentioned, he had detected a satellite attending on Saturn. Judging from the brightness of this satellite at so vast a distance, he considered that it must greatly exceed the largest of Jupiter's satellites in magnitude, and be little, if at all, inferior to the planet Mars. It revolves round Saturn in rather less than 16 days, at a distance of nearly 760,000 miles. In 1659, Huygens published a table of its mean motions. As this discovery raised the number of secondary planets to six (including our moon); and as but six primary planets (including the sun) were known to Huygens, he sought for no more satellites—sharing the idea, then commonly entertained, that the numbers of the primary and secondary members of the solar system must certainly be equal. Otherwise, with the powerful telescopes he subsequently constructed, he could not have failed to detect two (if not all) of the four satellites discovered by Cassini.

Huygens discovered that Saturn's globe, like Jupiter's, is marked by belts parallel to the equator, and on one occasion he observed as many as five; but he was unable to detect any other signs of Saturn's rotation.

In 1665, William Ball discovered a black stripe of considerable breadth, running quite round the northern surface of the ring, and having its outer and inner edges concentric with the edges of the ring. Ten years later, Dominic Cassini observed a corresponding stripe on the southern surface of the ring. He observed also that the part of the ring's surface outside this stripe is not so bright as the part within. He suggested, in explanation of these phenomena, that the ring is divided into two concentric rings, the inner ring being the brighter.

Four years before, in October, 1671, Cassini had discovered a second satellite, revolving at a mean distance of about 2,209,000 miles from Saturn, in rather more than 79 days. This satellite is not so bright, and is therefore probably smaller than the satellite first discovered, but is certainly not inferior in magnitude to the largest of Jupiter's moons. Cassini soon detected a singular phenomenon in this satellite; through nearly one half of its revolution about Saturn, it disappears regularly, even when sought with the same telescope in which, through the rest of its revolution, it is a conspicuous object. He concluded that one half of the surface of the satellite must be less capable of reflecting light than the other, and that, like our moon, it rotates once on its axis in each revolution about its primary.* He subsequently abandoned these views; but they were confirmed by Newton and Herschel, the former showing that no explanation can be given of the regular disappearance of the satellite but that suggested by Cassini; the latter by a series of careful observations with his powerful reflectors, establishing the correctness of Cassini's observations. These, and similar observations by M. Bernard at Marseilles in 1787, and by later astronomers, seem to leave no doubt on the subject.† We have here, then, a secondary planet rotating on its axis in $2\frac{1}{2}$ months, while (as will presently appear) its primary, whose volume is 15,000 times as great, rotates on its axis in less than $10\frac{1}{2}$ hours.

On December 23rd, 1672, Cassini discovered a third satellite

* The only satellites whose motions of rotation have been detected exhibit the same peculiar relation between rotation and revolution. They are six in number:—our moon, the four satellites of Jupiter, and the outer satellite of Saturn. Either the surface of the largest of Saturn's satellites is little marked with irregularities, or these are distributed with tolerable uniformity, since it presents no appreciable changes of brilliancy. Of the other six satellites of Saturn, the satellites (variously estimated at four, six, and eight) of Uranus, and Neptune's satellite, nothing is likely to be known till telescopes far more powerful than any now in use shall have been constructed.

† In the year 1705, it was observed that this satellite was visible through a complete revolution, and it was hence concluded that the irregularities upon its surface are variable. Far more probably, however, the phenomenon was due to the exceptional clearness and steadiness of the earth's atmosphere during the interval of two or three weeks occupied by the satellite in traversing the part of its orbit in which it usually disappears. Any one who is in the habit of using a telescope of even moderate power systematically, must soon become aware that there are occasionally brief intervals during which the power of the telescope seems increased, though the eye detects no corresponding change in the appearance of celestial objects. Unfortunately, such intervals occur but rarely in our latitudes, and seldom last more than two or three days. They generally occur in early spring and late autumn; winter and summer are seldom favourable seasons for astronomical observation, notwithstanding the brilliance of some of our winter nights, and the softer splendour of the nocturnal skies in summer.

whose orbit lies within those of the other two. He effected this discovery by means of a telescope of Campani's, 35 feet in focal length. This satellite revolves about Saturn in rather more than 4½ days, at a mean distance of about 328,000 miles. Judged by its brightness, it is probably much smaller than either of the two satellites first discovered. It exceeds the outer satellite in brightness, however, when the latter is at or near its easterly elongation.

In March 1684, Cassini discovered two more satellites by means of Campani's object-glasses of 100 and 136 feet focal length.* These satellites revolve within the orbits of the first three, their mean distances from Saturn's centre being about 224,700 and 180,000 miles. Thus both are nearer Saturn's surface than our moon to the surface of the earth. They occupy about 2¾ days, and 1¾ days, respectively, in completing their revolutions about Saturn. They are about equal in brightness, being each slightly inferior in this respect, and therefore probably in magnitude, to the third satellite discovered.

Cassini found that the orbits of the five satellites hitherto discovered correspond with the laws of Kepler (see Table V.). He found also that the four inner satellites move in planes very nearly coincident with the plane of the rings, while the fifth moves in a

* Cassini also used object-glasses of 200 and 300 feet focal length, and Auzout constructed glasses having focal lengths of 600 feet. Of course, glasses of such enormous focal length were not fixed in tubes. They were attached to frames constructed to slide up and down tall uprights. The eye-glasses of such telescopes were simply connected with the object-glasses by wires of the proper length. Observation with such telescopes must have been wearisome work, and we cannot wonder that the invention of reflecting telescopes was gladly hailed as offering a relief from the use of such cumbrous and imperfect instruments. The reflector presented by Hadley to the Royal Society, in 1723, though it had a focal length of only 10 feet 5¼ inches, was fully equal in power to the refractor of 123 feet focal length given by Huygens to the same Society. Yet the difficulty of grinding the specula accurately, and of preserving them when ground from changes of form and loss of reflecting power, must always prevent reflecting telescopes from replacing refractors, now that the construction of achromatic object-glasses has attained such perfection. That absolute truth of form has been obtained in reflecting specula by the most ingenious systems of grinding may be doubted, when we remember that the Harvard refractor, with an object-glass of fifteen inches diameter, has clearly resolved nebulæ in which but doubtful indications of resolvability are afforded by the splendid 6-feet speculum of Lord Rosse's reflector.

It may be questioned whether, in certain applications of the telescope, tubeless telescopes might not be occasionally used with advantage, diminution of weight and consequent cheapness of construction compensating a slight loss of illuminating power.

plane inclined at an angle of about 15° to the plane of the ring. The younger Cassini investigated these relations more closely, and in 1717, published a table of the distances, mean motions, and inclinations of the orbits of these satellites. He determined also with considerable accuracy, the position of the ascending node of the rings' plane on the ecliptic, and on Saturn's orbit, and the position of the ascending node of the fifth satellite on the same circles. Halley corrected the results obtained by Huygens and the elder Cassini; and later, in 1720, published the elements of the orbits of the five satellites, corrected from a series of observations made by Pound. Halley also detected an eccentricity in the orbit of the largest satellite, and roughly determined its amount, and the position of the line of apsides.

Cassini called the four satellites he had discovered 'Sidera Lodoicea,' in honour of Louis XIV., under whose patronage his labours had been conducted.* This name has, however, long since been disused. The satellite discovered by Huygens has received the name of Titan; and the four discovered by Cassini have been called (in the order of their distances from Saturn), Tethys, Dione, Rhea, and Japetus. But the most convenient method of indicating these and the satellites since discovered, is by numbering them in the order of their distances from Saturn: thus the satellite discovered by Huygens is now known as the sixth satellite, while the satellites discovered by Cassini are known as the third, fourth, fifth, and eighth satellites. By referring to Plate I. the reader will be able to form an idea of the relative brightness of these bodies, and of the probable proportions they bear to each other, to the globe of Saturn, and to the other bodies represented in that engraving, all of which are on the same scale. In fig. 1, Plate VII., they are represented at their proper relative distances from Saturn; while in fig. 2, the dimensions of their orbits are represented on a smaller scale. The elements of the eight satellites are given in Table V., Appendix II.

For nearly a century after the discovery of Tethys and Dione no new features of importance were revealed by the tele-

* Cassini was naturalized in France in 1673. His son Jean Jacques Cassini, and his grandson César Francois Cassini, were both born in France. The family, however, originally came from Italy. Cassini himself was born at Périnaldo, in Nice.

scope in the Saturnian system. Several phenomena already suspected were verified, however, and others—not wanting in interest—detected. Hadley discovered that the outer part of the ring is thinner than the inner; he observed also the shadow of the ring on Saturn,* and the shadow of Saturn on the ring. He confirmed Huygens' observation of belts on Saturn's disc, and found that, like the belts of Jupiter, they vary in form and number. Halley, also, observed Saturn's belts, and concluded from their changes of form as Saturn traverses different parts of his orbit, that Saturn rotates on an axis perpendicular (to the sense) to the plane of the rings; in other words, that the plane of Saturn's equator coincides (to the sense) with the plane of the rings. In October, 1714, a few days before the disappearance of the rings, the earth being nearly in their plane, Maradi observed a singular phenomenon:—the narrowing ansæ of the ring appeared to be unequal in size, the eastern being the larger; yet after an interval of two nights the eastern ansa had disappeared, while the western was visible, though reduced to a faint line of light. From these observations he concluded that the rings are not of uniform thickness, and that they revolve about Saturn in their own plane.† In this conclusion may be traced the germ of the important discovery of the rotation of the ring afterwards made by Herschel. It may be noticed, however, that in arriving at this conclusion Maradi made two assumptions, neither of which (as will presently appear) is correct. He assumed, first, that the ring is a solid formation; and secondly, that it is a *rigid* solid. The first assumption was justified by the appearance of the ring, and was maintained, or rather never disputed, till the discoveries of the last few years led to a

* Cassini, in 1675, observed a dark belt on Saturn's body, parallel to the greater axis of the rings. This was probably either the shadow of the ring on Saturn, or the first indication of the existence of the dark inner ring lately discovered. In 1675, the rings were well opened, but not to their full extent; at such a time the outlines of the belts are elliptical. The outlines of the dark ring and of the shadow are, it is true, also elliptical, but they form parts of larger ellipses, and appear nearly straight and parallel to the greater axis of the ring.

† It will be seen from Table X. that the plane of the ring passed through the sun in February 1715, reappearing. After this the plane of the ring passed twice through the earth, disappearing at the first passage and reappearing at the second. These passages occurred in the summer of 1715, and within a few weeks of each other.

different view. The second assumption, on the other hand, is altogether unreasonable. It was not to be expected but that so vast a formation, subject to so many disturbing attractions, and whose thickness is obviously disproportionate to its other dimensions, should be subject to vast undulations; and these, for anything known to the contrary in Maradi's day, might sweep round the ring, altogether independently of any absolute motion of rotation in the system, and would thus sufficiently account for the phenomena observed by Maradi.

The motions and distances of the satellites, and the dimensions of the ring, were determined with considerable accuracy, by several astronomers, during the interval above mentioned. Some of these measurements will be made use of in a future chapter, but most of them have given place to the more exact determinations of the present century.

One or two observations, rather curious than valuable, were also made in the interval named. Thus, Whiston records that his father had seen a star through one of the openings between the planet and the ring. Such an occurrence, though uncommon with the telescopes in use in his day, is not infrequent with modern telescopes, especially when Saturn is traversing the constellations Taurus and Gemini in one part, Scorpio and Sagittarius in the opposite part of the Zodiac. No star of the first four or five magnitudes has ever, I believe, been seen through these openings. Again, Cassini has recorded that in 1692 he saw a fixed star occulted by Saturn's largest satellite, an occurrence that must be exceedingly rare even with the most powerful telescopes, and when Saturn is traversing those parts of the Zodiac in which stars of all magnitudes are most profusely scattered.

During the last fifteen years of the eighteenth century, many important discoveries were made by the elder Herschel in the Saturnian system. When the northern side of the ring was visible before the disappearance of the ring in 1789, he carefully examined the black line discovered by Ball. He appears during this time to have been strongly opposed to the idea that the ring is divided, even where this line is seen; still less was he willing to accept the hypothesis of the multiple division of the ring. Four observations in 1780 had appeared to indicate the possibility that other divisions besides the great one exist in the ring; and Laplace—his inferior as a practical astronomer, but his superior as a mathematician—had asserted that such divisions are absolutely necessary to the stability of the formation. Herschel maintained, however, and with some reason, that observation afforded no support to the theories of the French mathematician. Confident that his 20-feet reflectors were equal, if not superior, in power to the best telescopes of his day, he refused to put faith in the records of observations which his own telescopes failed to verify; and the idea of the *temporary* existence of such lines either never occurred to him, or was rejected as improbable.* His observations of the 'broad black mark,' as he at first spoke of the great division, were conducted with his usual accuracy and clearsightedness. He found that the outer and inner boundaries of this mark are both ellipses, concentric with and similar to the boundaries of the rings. He argued that the black stripe could not be the shadow of hills on the surface of the ring, since such a shadow would vary with the position of Saturn in his orbit, and when Saturn is in opposition no shadow would be visible at the ends of the longer axis of the elliptical mark, whereas it is precisely at these points that the mark is broadest. For similar reasons he rejected the idea that the line indicates the existence of a vast cavernous groove on the northern surface of the ring.

On the reappearance of the ring in the winter months of 1789-90,† he examined its southern face with his 40-feet reflector, and after carefully measuring the stripe on this face, he found that it corresponds exactly in form and dimensions with the stripe on the northern face. Accordingly, in the year 1790, he announced his suspicion that the formation is divided into two rings by a vast circular gap of uniform width—at the same time recording his opinion that this is the only division existing in the system.

On August 19th, 1787, Herschel thought he could detect a sixth satellite attending on Saturn. He remained uncertain as

* He has, in fact, recorded his opinion that the rings are undoubtedly solid formations, 'since they cast a strong shadow on the body of the planet.'

† The disappearances and reappearances of the rings in the years 1789–1790 are considered in Appendix II. See explanation of Table X.

to the existence of this body until the completion of his 40-feet reflector.* On August 27th, 1789, the first evening after the completion of this powerful instrument, he directed it towards Saturn. No sooner had he brought the planet into the field of view than he plainly saw six stars shining round its disc. Five of these were the satellites already discovered; it remained to be seen whether the sixth were a satellite or a fixed star. Saturn was then not far from opposition, and retrograding at the rate of 4′ 30″ daily; thus the motion of his system was carrying him across the celestial sphere, slowly indeed, but with a motion readily detected, even in a short time, by a telescope of such power as Herschel's. Thus, 2½ hours after the first observation, Herschel found all the six stars had accompanied Saturn in his slow motion across the celestial sphere—all, therefore, were satellites.

Herschel found that the orbit of the newly-discovered satellite is within those of the other five. It is less conspicuous, and, therefore, probably smaller than any of the satellites that had hitherto been discovered. It revolves about Saturn in rather less than one day and nine hours, at a distance of about 148,000 miles from Saturn's centre, or about 112,000 miles from the surface of the planet.

While continuing his observations of this satellite, and within three weeks of its discovery, Herschel detected a seventh satellite, about as small—or, at least, as little conspicuous—as the other, and following a smaller orbit. This satellite moves at a distance of rather more than 115,000 miles from Saturn's centre, or about 79,000 miles from his surface. Its mean distance from the outer edge of the ring is less than 32,000 miles. It accomplishes a revolution around Saturn in about 22½ hours—a period of revolution shorter than that of any known satellite in our system. Herschel published tables of the motions of the two satellites he had discovered. He found that the planes in which they move are either absolutely coincident with the plane of the ring, or so nearly so that no difference can be detected. Owing to this coincidence, as well as to their minuteness at so vast a distance from the earth,

* This splendid telescope, only exceeded in size by the great Parsonstown reflector, had a speculum four feet in diameter. It will serve to give an idea of the patience and energy of Herschel to record that, between the years 1775 and 1781, he cast, ground, and polished 80 specula of 23 feet, 150 of 10 feet, and 200 of 7 feet focal length.

they are not favourably seen, even in the most powerful telescopes, except when the ring is very nearly closed, as was the case at the time of their discovery.

Herschel examined the belts on Saturn's surface with great care. He found that their outlines are straight lines when the rings are invisible, and change into ellipses of less and less eccentricity as the rings open more and more; that, in fact, these outlines are always similar to the outlines of the rings. From this observation it follows, assuming that the belts are due to the rotation of the planet on an axis, that the axis of rotation is perpendicular to the plane of the ring; or, in other words, that the plane of the planet's equator coincides with the plane of the ring, as had been already suggested by Dr. Halley. Herschel established beyond doubt the connection between the belts and the rotation of the planet, by the discovery of certain spots on Saturn's surface. Carefully observing the motions of these spots for some time, he found that Saturn rotates upon his axis in 10 hours, 29 minutes, 16·8 seconds.* This rotation, like that of the earth, is from west to east; so that to the Saturnians the sun appears to travel across the sky from east to west, as with us. Instead of 365 days, however, the Saturnian year contains no less than 24,618 Saturnian days.

The investigations of Laplace into the stability of a solid flat ring (such as Saturn's was supposed to be) about a central attracting body, had led that distinguished mathematician to the conclusion that Saturn's rings must rotate about the planet in their own plane. In July, 1789, when the edge of the ring was turned directly towards the earth,† Herschel observed that it continued visible as a broken line of light when viewed through one of his 20-feet reflectors, and that certain spots of light were carried along this line as if by the rotation of the ring in its own plane. Continuing his observations, he found that the spots of light travelled nearly to the ends of the ansæ, so that he concluded they belonged to the outer ring. He found that they occupied 5 hours, 16 minutes, 7·5 seconds in travelling from end to end of the fine line presented

* Herschel first gave for this period 10 hours, 16 minutes, 0·44 seconds.

† The ring at this time was reappearing; the earth which for a few weeks before had been on the unilluminated side of the ring passing to the illuminated side. See explanation of Table X, Appendix II.

by the ring, and he therefore announced that the outer ring rotates in its own plane in 10 hours, 32 minutes, 15 seconds. *

Herschel's measurements of the diameters of the planet and rings are somewhat in excess of the measurements now generally adopted as the most trustworthy. He considered that the breadth of the system of rings was about one-fourth greater than the breadth of the space between the inner edge of the inner ring and the planet's equator. It will be remembered that Huygens considered those breadths equal. Pound, with the same telescope as Huygens, and using an excellent micrometer, considered that the breadth of the ring-system was even somewhat less than the breadth of the space between the planet and the rings. So remarkable a discrepancy can hardly be ascribed to errors of observation, and it will presently be seen that the change which would thus appear to have taken place in the shape of the rings between the years 1659 and 1790, was part of a progressive increase of the breadth of the system, that has continued to our own time.

Herschel at first considered the form of the planet to be spheroidal, and the polar axis shorter than an equatorial diameter in about the proportion of 10 to 11. He subsequently changed his opinion as to the form of the planet, concluding, from observations taken in April, 1805, that the outline of the planet's disc is not a regular curve. He compared its form to that of a parallelogram with rounded corners, whose longest diagonal is inclined at an angle of 43° 20′ to the equatorial diameter, while its shortest diagonal is the polar axis of the planet. Subsequent observation has not confirmed this view, which probably arose from an optical illusion. At the time of observation the ring was not favourably situated for the measurement of the planet's disc. For this purpose the ring should be altogether, or very nearly closed. In April, 1805, the line of sight from the observer was inclined at an angle of about 13° to the plane of the ring, so that the ring was sufficiently open to

* Doubt has been thrown on this conclusion, since Schroeter, at the next disappearance of the ring in 1802–3, and Bond in 1848, observed that spots and irregularities along the thin line of light maintain their positions absolutely unchanged for hours. The positive evidence of the ring's rotation afforded by Herschel's observation, is not affected, however, by the observations of Schroeter and Bond. The spots seen by these astronomers have been satisfactorily explained by the latter as belonging to the general configuration of the rings, not to irregularities of form at particular parts of the rings' surface.

interfere with the measurement of the disc, an operation at all times sufficiently difficult.

From a series of observations made in the years 1789-1790, during which the earth passed three times through the plane of the ring, Herschel arrived at the conclusion that the ring must be very thin. When the edge was turned directly towards the earth, the ring continued visible in his 20-feet reflectors as a fine line of light; and in his great reflector the ring was visible even when the earth and sun were on opposite side of the ring's plane,—that is, when the unilluminated side of the ring was turned towards the earth. Along the fine line of light visible in the former case, the satellites appeared to move 'like golden beads upon a wire,' as Herschel has described the phenomenon. He did not, however, conclude from this circumstance alone that the ring's thickness is necessarily less than the diameter of the least of the satellites; for he considered that the disc of the satellite might be rendered visible on both sides of the ring by refraction through an atmosphere which he supposed might envelope the ring.* He was doubtful whether the fact that the ring is visible when its dark side is turned towards the earth is due to the partial illumination of that side of the ring by light reflected from Saturn and from his satellites, or whether he only saw the illuminated edge of the ring. He judged that the edge of the ring is not perpendicular to the faces of the ring (so as to be part of a cylinder of very short axis), but that it is rounded (so as to form part of the surface of an oblate spheroid, whose axis is very short compared with its other dimensions); or, as he expressed it, 'that the edge of the ring is not flat but spheridical.'

Herschel found that when the satellites are occulted by Saturn, they appear both at ingress and egress to cling to his disc for a longer time than would be due to the dimensions of their own

* The supposition has not been confirmed by observation, however. To produce the effects described, the atmosphere of the ring must cling round the edges of the ring, since the mere presence of an atmosphere on the flat surfaces of the rings could have no other effect than to dim the lustre of the satellites. Now, it is plain that as the satellites reached (apparently) the ends of the line presented by the ring, their motion would appear to be considerably modified by refraction *round the ring's edge*; they would, in fact, appear to cling to the extreme end of the line for an appreciable interval: this is not the case, however; their apparent motions along and beyond the line being exactly those due to their motions in their orbits.

discs. This phenomenon can only be ascribed to the presence of an atmosphere of considerable extent and density surrounding Saturn. The existence of such an atmosphere, supporting vast masses of aqueous or other vapours, is indicated by the belts that cover varying zones of Saturn's surface. In November, 1793, Herschel obtained a favourable view of the Saturnian belts with his 40-feet reflector. He observed a broad and brilliant white belt of nearly uniform width, covering the equatorial regions; next to it he observed a broad dark belt of a yellowish colour,* divided into three unequal bands by two narrow and somewhat irregular white streaks less brilliant than the equatorial belt. At first sight the natural assumption would seem to be that the dark belts are bands of clouds upon the surface of the planet. It must be remembered, however, that the clouds of our own skies appear dark to us only because they intercept part of the solar light. When they are so placed as to reflect the sun's light to the observer they appear brilliantly white. Now it is precisely such reflected light—the 'silver lining' of the proverb—that an observer on earth receives from cloud-belts encircling a planet. On the other hand, the surface of the planet's body, diversified probably, like that of the earth, with continents and oceans, would exhibit in different districts varying shades of light and colour, and these—blended by the effects of distance and of the atmospheric envelope through which we see them—would combine to present precisely such dusky regions, faintly tinged with the prevailing colours of the planet's surface, as are visible in the belts of Saturn. The equatorial bright belt, which Herschel found to be permanent, may be ascribed to the presence of a permanent zone of clouds covering this part of Saturn's surface. On our earth we have a corresponding equatorial zone of calms, in which the sky is never free from clouds and vapours, and in which rain is almost constantly falling.

The series of discoveries effected by the elder Herschel cannot be better closed than by his own account of the features of the Saturnian system:—'There is not, perhaps,' he says, 'another object in the heavens that presents us with such a variety of extra-

* The best modern telescopes, under favourable circumstances, exhibit the dark belts as of a faint greenish colour.

ordinary phenomena as the planet Saturn: a magnificent globe encompassed by a stupendous double ring; attended by seven satellites; ornamented with equatorial belts; compressed at the poles; turning on its axis; mutually eclipsing its rings and satellites and eclipsed by them; the most distant of the rings also turning on its axis, and the same taking place with the farthest of the satellites;* all the parts of the system of Saturn occasionally reflecting light to each other—the rings and moons illuminating the nights of the Saturnian, the globe and moons enlightening the dark parts of the rings, and the planet and rings throwing back the sun's beams upon the moons when they are deprived of them at the time of their conjunctions.'

During the present century many observers of the highest reputation for skill and accuracy have detected divisions in the rings concentric with the great one. One such division, separating the outer ring into two rings nearly equal in breadth (see figs. 2 and 3, Plate I.) appears to be permanent, though it is only visible through its entire circumference when the ring is open to its full extent. Even then it can only be seen with telescopes of the first class for power and definition, and under the most favourable atmospheric conditions. The late General Mitchel, the American astronomer, has asserted that 'with the full power of the Cincinnati refractor†, defining in the most beautiful manner all the other delicate characteristics of Saturn and his rings,' he has never been able to perceive this division in the outer ring, or a trace of 'any other than the principal division.' In the spring of 1856, however, when the

* The care with which Herschel here confines himself to what is proved and established is in singular contrast with the magnificent audacity of his conceptions when he advanced, with no guide but his own genius, into unfamiliar and awe-inspiring regions of speculation. It is this combination of boldness and accuracy, enthusiasm and caution, which constitutes Herschel's claim to be classed in the very first rank among astronomers. The patient care with which he examined Saturn's ring for ten years before he would accept the theory of its division, and watched a satellite for two years before he would pronounce an opinion on its rotation, was as important a part of his character as the brilliant imagination that enabled him (as is well expressed on his monument at Upton) to 'break through the enclosures of Heaven,'—'*cœlorum perrumpere claustra.*'

† This refractor, made by Messrs. Merz and Mahler of Munich, has an aperture of twelve inches, its focal length being 17½ feet. Under favourable atmospheric conditions it will bear a magnifying power of 1200.

ring was open to its full extent, the division in the outer ring was seen by many observers; and it is therefore probable, though not absolutely certain, that it is a permanent division in the rings. The other traces of division that have been seen from time to time, have only been traceable through short arcs, and have not long continued visible. Those seen by different observers, or by the same observer at different times, have occupied different positions, and have belonged to different circles. If each division thus detected were considered as a satisfactory indication of a permanent division through a complete circumference, it would follow that the system consists, not of two or three, but rather of thirty or forty separate concentric rings. Strange as such a conclusion might appear, and manifold as are the conditions of instability the complexity of such a system would introduce, we should have no resource (on the assumption of the solidity of the rings) but either to accept this solution of the question, or else to reject the testimony of most accurate and skilful observers—of such men as Encke, the Struves, Captains Kater and Jacob, Mr. Dawes, and the astronomers of the Collegio Romano. The telescopes, also, through which these divisions have been repeatedly seen, have been among the most celebrated instruments of modern times. These appearances are examined and explained in Chapter V.

On September 19th, 1848, an eighth satellite was discovered by Bond, at Cambridge, America, with the Harvard refractor,* and by Lassell at Liverpool, with his great reflector. The orbit of this satellite lies between the orbits of Titan and of the outer satellite, so that it is the seventh in order of distance from the planet. It completes a revolution in rather more than 21 days—at a distance of about 919,000 miles from Saturn's centre.† Judged by its bril-

* This celebrated instrument has an aperture of 14½ inches, and a focal length of 21 feet. It will bear a magnifying power of 2000. The telescopes of Pulkova and Greenwich are of the same dimensions, and manufactured by the same opticians, Messrs. Merz and Mahler of Munich.

† A law similar to that known as 'Bode's Law of the Planetary Distances,' may be traced in the distances of Saturn's satellites from their primary. Thus, if to each of the series of numbers, 0, 1, 2, 4, 8, 16, 32, 64, we add 4, the resulting series corresponds in a remarkable manner with a series representing in order the distances of the eight satellites of Saturn from their primary (calling the distance of the first satellite 4). The following table presents these relations synoptically:—

liancy, it is probably the smallest of the system of satellites attending on Saturn, and presents in this respect a striking contrast to the two satellites between whose orbits it revolves. It has received the name of Hyperion.

Two years later a most remarkable discovery was made. A third ring, inside the two others, and of a singular appearance, was discovered on November 15th, 1850, by Bond, and a few days later (but independently) by Dawes and Lassell in England. This ring is not bright like the others, but exhibits a dusky, almost purple tinge; and *through it*, the undistorted outline of the planet's disc can be distinctly traced. The inner edge of this dark ring is concentric with the edges of the other rings, its outer edge appearing in general to coincide with the inner edge of the neighbouring bright ring. Mr. Dawes has remarked, however, that this is not always the case,—that the dark ring at times appears to be separated from the bright ring by a distinctly marked interval. This accurate observer considers, also, that the dark ring is occasionally divided into two or more concentric rings.

Perhaps the most remarkable circumstance connected with this mysterious formation is the fact that it was not discovered sooner. Had it existed in its present state in the time of the elder Herschel, it would have presented a marked appearance in his great reflector. For although the telescope with which Bond discovered the dark ring probably exceeds Herschel's reflector in power (though inferior in size), yet in this telescope it was so distinctly and easily visible, that its detection obviously taxed but lightly the powers of the instrument.* The reflecting telescope of Lassell is probably

| | I. | II. | III. | IV. | V. | VI. | VII. | VIII. |
|---|---|---|---|---|---|---|---|---|
| | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| | 0 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| | 4 | 5 | 6 | 8 | 12 | 20 | 36 | 68 |
| True distanc s | 4 | 5·13 | 6·36 | 8·14 | 11·37 | 26·36 | 31·88 | 76·60 |

In the three outer satellites there is some irregularity, corresponding with the breach of Bode's law in the case of Neptune.

* It is worthy of notice that Bond discovered this ring on a night, in other respects

little, if at all inferior in defining power to Herschel's great reflector; and Mr. Dawes' extraordinary vision supplements the powers of his telescope; but the same remark applies:—the dark ring is easily visible with both instruments. Further, soon after its discovery, this ring was found to be visible with telescopes far inferior in power to many that had been repeatedly directed towards it without discovering it; and, as the rings opened more and more, the dark ring became so conspicuous that it was visible, Professor Nichol asserts, in a good achromatic of *four inches aperture.* That the dark ring existed in Herschel's time is obvious from drawings of his, in which a belt is marked on the planet so exactly concentric with the edges of the bright rings, that it can be no other than this formation, mistaken by the astronomer for a belt on the body of the planet. That it was far less conspicuous in his day than at present is obvious from the fact that he so mistook its nature, though using so powerful an instrument as his great reflector.

Another remarkable circumstance connected with this ring is its increase in width since the time of its discovery. The measurements of the best observers of the day seem to leave no doubt of such an increase, the causes of which will presently be examined.

In the spring of 1856, when the ring was open to its greatest extent, Mr. Bond observed a singular darkening of those parts of the inner bright ring which lie nearest to the extremities of the apparent longer axis of the dark ring. These dark spaces, as represented by Mr. Bond, are bounded by well defined outlines, forming parts of an ellipse concentric with the other elliptical outlines of the rings, but of greater eccentricity. While the semi-major axis of this ellipse exceeds the semi-major axis of the outer boundary of the dark ring by about one third of the breadth of the inner bright ring, its minor axis is not greater than that of the inner boundary of the dark ring. Thus the outlines of these dark spaces meet the outer boundary of the dark ring in acute angles, at four different points. In pictures of the planet and rings taken at about the same time by other observers, correspond-

favourable for astronomical observation, but so hazy that only the brightest stars were visible to the naked eye.

ing dark spaces are exhibited, but the darkening is not bounded by a defined outline.* A similar darkening of the outer bright ring, near the extremities of the major axis of the great division, also appears in several pictures of the planet taken at this time. These dusky regions have been termed 'the shadows on the ring,' a term not very well chosen, as will presently appear. Figs. 2 and 3, Plate I., exhibit the general appearance of these dark spaces :† their nature is discussed in Chapter V.

In the years 1855, 1856, and 1857, Messrs. Bond and Dawes were occasionally able to trace dusky, ash-coloured, and mottled stripes, concentric with the outlines of the rings. These were not always visible, however; they reappeared along different circles on the rings' surface; and, in fact, were as variable and mysterious as the dark traces of division.

A singular discovery was made by Mr. Wray during the disappearance of the ring in the winter months of 1861–62. Observing Saturn on December 17th, 1861 (when the dark side of the ring was turned towards the earth), with an achromatic of only seven inches' clear aperture (with which he expected to be able to detect no trace whatever of the ring), Mr. Wray was surprised to find the illuminated edge of the ring distinctly visible, 'not only where it crossed the dark shade on the body,‡ but also extending on each side of the planet's margin.' Continuing his observations, he was led

* The same phenomenon is indicated in Plate I. of the first edition (1833) of Sir John Herschel's 'Outlines of Astronomy.'

† Mitchel, with a less powerful instrument than Bond used, saw these spaces less satisfactorily. He writes, 'I have sometimes been confident that the breadth of the dusky ring at the extremities of its longer axis was much greater than that which would be due to an elliptical figure concentric with the bright rings.'

‡ It seems clear, however, that what Mr. Wray has described as 'the edge of the ring crossing the dark shade on the body,' was a strip of the planet's surface. For, at the time of this observation, the earth was on the northern or dark side of the ring, and therefore, in an inverting telescope, the bright edge of the ring must have been the upper boundary of the dark surface; and the sun being on the southern side of the ring, the shadow of the ring was north of the ring, or, in an inverting telescope, below the ring. Both the dark stripes being below the bright edge of the ring, of course this edge could not be seen between them. But the sun being much more elevated above the plane of the ring on one side of it than the earth on the other, it is clear that a strip of the planet's surface must have been visible between the two dark stripes, and it is this strip (I imagine) which Mr. Wray mistook for the rim of the ring. Mr. Wray's sketches do not, it is true, accord very well with this explanation; but it is possible that, in drawing these figures (which he expressly describes

(Dec. 23rd) to suspect that the edge of the ring was 'thicker and somewhat nebulous about the region on either side, where it joins the planet's limb.' Finally, on Dec. 26th, he completed the discovery of certainly the most singular phenomenon detected in the appearance of the rings, since the discovery of the dark ring. The atmosphere being fine, and 'the image of Saturn exquisitely steady and well defined,' he observed 'a prolongation of very faint light stretched on either side from the dark shade on the ball, overlapping the fine line of light formed by the edge of the ring, to the extent of about one-third of its length, and so as to give the impression that it was the dusky ring, very much thicker than the bright rings, and seen edgewise, projected on the sky.' He saw this faint overlapping light on four other occasions, in January, 1862. M. Otto Struve, using the magnificent refractor of Pulkowa, rediscovered these singular appendages when the plane of the ring was passing through the sun, in May, 1862. M. Struve observed that 5½ hours before the computed time of this passage (which took place on the 18th of May, at 8h. 30m. A.M.), one ansa of the ring was plainly visible; and on the 19th of May he was able to trace the luminous appendages along the ansæ. He had seen them less distinctly on May 15th, when they appeared only on the southern (or, in an inverting telescope, the upper) side of the ansæ. He describes them as resembling 'clouds of a less intense light lying on the ansæ.' On May 19th, they appeared to him to 'differ much in colour from the ordinary colour of the ring;' to be, 'not yellow, but more of a livid colour, brown, and blue.' Later, he saw them still more favourably; they appeared unequal in length along the two ansæ; extending on one side to a distance equal to about two-fifths of the planet's diameter, on the other half as far again; the breadth 'of these appendages increased in the neighbourhood of the planet, giving them the form of sharp wedges.'

as rough), he was more careful in giving the details of the appearances which the sketches were meant to illustrate, than the outlines of the thin lines of light visible across the disc, and on either side of it.

In Chapter IV. the reader will find a complete account of the disappearances and reappearances of the rings in the years 1861–1862.

* The observations of Wray and Struve are recorded in the 'Reports of the Astronomical Society,' for January, 1863, from which the above extracts are taken.

Mr. Carpenter, using the Greenwich equatorial, observed Saturn on the same days as M. Otto Struve, without detecting these appearances. It may therefore be concluded that they are so little conspicuous, that a slight difference in atmospheric conditions affects their visibility. The earth was on the southern side of the rings throughout the observations of M. Struve, and raised more than five times as high above the plane of the rings as it had been during the observations of Mr. Wray; it was also nearer to the rings. On the other hand, the sun, which throughout the observations of M. Struve was nearly in the plane of the rings, was raised from 1° 50′ to 2° 16′ above that plane during Mr. Wray's observations. The discrepancies between the two accounts are not greater than we might expect from this difference in the circumstances under which Saturn was observed. The probable nature of these appendages will be considered in Chapter V.

On March 26th, 1863, Mr. Carpenter made an observation of some interest. When Saturn was passing across the field of view of the transit-circle of the Greenwich Observatory, it appeared to him 'that the dark space between the ring and the ball was much contracted.' Upon looking at Saturn with the equatorial, he 'found that this arose from a great increase in the brightness of the dusky ring, which appeared nearly as bright as the illuminated ring, and might easily have been mistaken for a part of it.'* At the time of this observation, however, the earth was raised about 4° 23′, the sun less raised above the plane of the ring; thus it is clear from the laws of refraction and reflection of light at the surfaces of transparent media, that the dark ring, whether we consider it to be a semi-transparent solid or fluid, would not, at the time of this observation, differ greatly in brightness from the outer rings. The results of Chapter V., however, will be found to afford a more probable explanation, both of the nature of this ring, and of the cause of its brightness when viewed at small angles.

Lastly, two observations by Mr. Dawes, rather singular than particularly valuable, are recorded in the February number of the 'Astronomical Reports' of the year 1863. He observed the transit of the shadow of Titan (the largest and brightest satellite) across

* 'Reports of the Astronomical Society,' April, 1863.

the disc of Saturn; and 'an eclipse of Titan itself in Saturn's shadow:' the former a rare, the latter, in Mr. Dawes' opinion, an unique phenomenon.

Among telescopic discoveries may be classed the measures that have been taken of the planet's polar and equatorial diameters, of the diameters of the various rings, and of the orbits of the eight satellites. The absolute measures, and even the proportions between the several dimensions obtained by different observers, vary considerably. The following are the dimensions of the rings as given respectively by Hind* and Struve:—

| | Hind. miles | Struve. miles |
|---|---|---|
| Exterior diameter of outer ring | 170,000 | 176,418 |
| Interior diameter of outer ring | 150,000 | 155,272 |
| Exterior diameter of inner ring | 147,000 | 151,690 |
| Interior diameter of inner ring | 114,000 | 117,339 |
| Breadth of outer ring | 10,000 | 10,573 |
| Breadth of inner ring | 16,000 | 17,175 |
| Breadth of division between the rings | 1,500 | 1,791 |
| Breadth of the system of bright rings | 27,500 | 29,539 |
| Space between planet and rings | 19,250 | 19,090 |
| Equatorial diameter of planet | 75,500 | 79,159 |

Struve's measures were probably taken at a later period than those adopted by Hind (? Bessel's).

The most remarkable feature, at first sight, in the comparison of the two tables, is the excess of every measure but one in Struve's table over the corresponding measure in Hind's. This excess will not appear so remarkable, however, when it is considered that, at the immense distance to which Saturn is removed from us, a space of about 4,240 miles corresponds to an angle of one second of arc.† The single measure of Hind's table which exceeds the corresponding measure of Struve's, marks a more significant discrepancy. Thus the breadth of the system, which is given by Hind as only 27,500 miles, Struve estimates at 29,539 miles; yet the space between the planet and the inner edge of the inner bright ring is given as 19,250 miles by Hind, or 160 miles greater than

* Superintendent of the 'Nautical Almanac.'

† That is, about $\frac{1}{1867}$th part of the angle subtended by the moon's apparent diameter (mean). Even in the most powerful telescopes an arc of 1″ of a great circle of the celestial sphere appears a very small space; so that a double star whose components are $\frac{1}{4}$th or $\frac{1}{3}$rd of a second apart, severely tests their defining powers.

the corresponding measure of Struve. In other words, while Hind gives the ratio between the breadth of the system of rings and the breadth of the space between the rings and planets as 10 : 7 exactly, Struve determines the same ratio as somewhat less than 10 : 6½. Such a discrepancy is not likely to be accidental; and it becomes still more significant when we compare the ratios just given with the corresponding ratios obtained by Huygens in the 17th, and by Herschel near the end of the 18th century. As already mentioned, these were respectively 10 : 10 and 10 : 8. Thus it would appear that from the first discovery of the rings to the present time, the ratio of the breadth of the rings to the space between the rings and planet has been continually increasing. As it does not appear that any perceptible change has taken place in the exterior diameter of the outer ring, it would follow that the rings have been continually spreading inwards. The absolute increase in the breadth of the rings, and the probable cause of this singular phenomenon, will be discussed in Chapter V.

At the time of the first discovery of the dark ring, its breadth was variously estimated—the lowest estimate being 6,000 miles, the highest not exceeding 8,000 miles; later, this ring appears to have grown broader, and the latest estimates of its breadth vary from 8,000 to 10,000 miles.

The compression of Saturn's globe has not been satisfactorily determined. It is usually given as $\frac{1}{10}$th; that is, the polar diameter is considered to be less than an equatorial diameter by about $\frac{1}{10}$th of such diameter. Herschel, on the other hand, considered the compression less than $\frac{1}{11}$th; Hind gives it as about $\frac{1}{15}$th; and in the 'Nautical Almanac' it is assumed to be the same as the compression of Jupiter's globe, or about $\frac{1}{14}$th. It is obvious that if the compression of Saturn's globe were accurately known, as well as the proportions of his equatorial diameter to the internal and external diameters of his rings, then the exact appearance of the system at any moment could be readily determined. We should only have to calculate (from the known orbits of Saturn and the earth about the sun) the elevations of the sun and the earth above the plane of the ring: these being known, the figure of the rings, the position of their shadow on the planet, and the position of the planet's shadow on the rings, are simply matters of calculation. Now it happens that if either of the tables given above be applied to such a calculation, the moment chosen being that at which the earth attains its greatest elevation above the plane of the ring (as in March, 1856), the calculated appearance of the system does not correspond with pictures taken at that time with the most powerful telescopes, unless we suppose Saturn's globe to be much more compressed than the best observers have considered it. For in these pictures the dark division on the ring is visible above the edge of the planet's disc (in a picture taken by Bond with the Harvard refractor nearly the whole breath of the division is thus visible), and it can readily be shown that, for the outer boundary only of this division to appear just touching the disc at its highest point, the compression of Saturn's globe should be nearly $\frac{1}{8}$th if Hind's measures are correct, and nearly $\frac{1}{7}$th if Struve's measures are correct. It is impossible to suppose so many observers deceived in a matter of such simplicity as the visibility or non-visibility of the great division above the disc of the planet; on the other hand, it is equally improbable that the compression of Saturn's globe should be so great as $\frac{1}{7}$th or $\frac{1}{8}$th. It seems more likely that some of the measures given above are erroneous. The measures in Tables III. and IV. (Appendix II.) have been adopted as the best average dimensions of the rings and planet. Assuming these measures to be correct, and that the compression of Saturn's globe is about $\frac{1}{11}$th, the system of rings, when open to their greatest extent, would present the appearance shown in fig. 3, Plate I.; and this appearance corresponds very well with pictures taken by the best observers.*

The thickness of the ring is a quantity too small to be made the subject of measurement, at the immense distance to which Saturn is removed. When the edge of the ring is turned to the observer, the ring appears, in the most powerful telescopes,

* The measures of the planet's equatorial diameter, of the exterior diameter of the outer ring, and of the inner diameter of the inner ring, adopted in the 'Nautical Almanac,' are respectively 75,000 miles, 173,430 miles, and 115,330 miles. The two first dimensions correspond closely with those I have adopted; the last belongs, as we have seen, to a variable quantity. The compression of Saturn's globe is, however, certainly greater than $\frac{1}{14}$th, the amount adopted in the 'Nautical Almanac.'

as an inconceivably delicate line of light. Beside it, the filaments of a spider's web across the field of view of the telescope look like cables. Sir William Herschel considered that the thickness of the rings certainly does not exceed 250 miles, but of their actual thickness even his great reflector gave no indication. Sir John Herschel considers that the rings are certainly not more than 100 miles thick. Bessel, of Konigsberg, calculated the mass of the rings, from their effect in disturbing the motion of Titan's line of apsides. He found that their mass must be about $\frac{1}{118}$th part of the mass of the planet. As we have no means of determining the ratio which the mean density of the ring bears to the density of Saturn's globe, Bessel's calculation does not enable us to determine certainly the thickness of the system. If we suppose the mean density of the rings to be about equal to the mean density of Saturn's globe, then it follows, from Bessel's determination of the mass of the rings, that their thickness does not greatly exceed 100 miles—about $\frac{1}{376}$th part of the breadth of the system. An idea of the proportions of the system may be readily obtained by cutting a ring of stout writing paper, whose exterior diameter shall be 2 inches, and its breadth half an inch. If such a ring be shaded on both surfaces across a breadth of nearly $\frac{1}{8}$th of an inch from its inner edge, and a dark circle about $\frac{1}{50}$th of an inch in breadth be described on both sides of the paper at rather more than $\frac{1}{8}$th of an inch from the outer edge, a tolerably exact conception may be formed of the dimensions of Saturn's ring-system.*

The distances at which the satellites of Saturn revolve have already been mentioned. I shall close the series of telescopic discoveries in the Saturnian system by showing how Saturn's mass and density may be determined from the observed distance of one of his satellites—selecting for this purpose the satellite Titan, the first discovered, and largest of the system.

We have the following data for the solution of our problem:—

* The dimensions of the rings in Plate XII. correspond to a thickness far less than that of the paper on which they are engraved, even if we suppose the thickness of the rings to be 250 miles. On the scale of Plate I. the thickness of the rings would probably be represented pretty exactly by the thickness of the paper.

The moon revolves round the earth in 27·32166 days, at a distance of 238,767 miles; Titan revolves round Saturn in 15·94543 days, at a distance of 759,990 miles. We may, for our present purpose, consider both orbits as circles described with uniform velocity.

First, let us consider what proportion Saturn's mass should bear to that of the earth, that a satellite at Titan's distance should move with the same velocity as the moon. This imaginary Titan would, of course, be longer than the moon in completing a revolution, in the proportion that the radius of its orbit bears to the radius of the moon's orbit. Hence it follows that the attractive force by which our pseudo-Titan would be retained in its orbit is less than the earth's attractive force on the moon, in the proportion of the moon's distance from the earth to Titan's distance from Saturn. For the direction of the moon's motion is altered through four right angles while she completes one revolution, and so of Titan; and the former period is less than the latter, and therefore (since the velocities are equal) the deflecting force in the former case greater than in the latter, in the above-named proportion. Now, if Saturn's mass were equal to that of the earth, his attractive force at Titan's distance would be less than the attractive force of the earth at the moon's distance, in the inverse proportion of the *squares* of those distances; whereas we have seen that for the false Titan to move as supposed, the former force should be less than the latter in the inverse proportions of the *simple* distances. Thus, if Saturn's mass were equal to that of the earth, his attractive force would be too small; and, it is perfectly clear that for Titan to move in the manner imagined, Saturn's mass must exceed that of the earth in the *direct* proportion of the distances of Titan and the moon from their respective primaries; the diminution of attraction in the proportion of the *squares* of the distances thus leaves Saturn's attractive force less at Titan's distance, than that of the earth at the moon's distance, in the required proportion.

Now let us take into account the difference in the orbital velocities of the real and false Titans. In the first place, it is clear that since the actual Titan moves more rapidly than the imaginary Titan, a greater deflective power is necessary to alter the direction of Titan's motion through a given angle in a given time, than would be requisite in the case of the imaginary Titan. And in the case of circular orbits uniformly described, it is easily seen that the former force should exceed the latter in the direct proportion of the velocities of the real and false Titans respectively. But the direction of the motion of Titan is moved through a given angle in *less* time than that in which the direction of motion of the false Titan is so moved; for it is deflected through the four right angles while Titan is completing one revolution, and he accomplishes this in less time than the imaginary Titan in the inverse proportion of their respective velocities (since their orbits are equal). The attractive force, then, which would have had to be greater on the real than on the imaginary Titan in the proportion of their velocities, if their deflections were equal in equal times, must, in the actual case, be greater in the proportion of the *squares* of their velocities. Thus the mass obtained, on our first hypothesis, by increasing the mass of the earth in the proportion of the distances of Titan and the moon from their respective primaries, must be still further increased in the proportion of the square of Titan's velocity to the square of the moon's velocity. This could be readily effected from the data given above; but the calculation will be simplified if we consider that the velocity of a body revolving in a circle varies directly as the radius of the circle, and inversely as the period of revolution. Thus, to obtain Saturn's mass, the earth's mass must be increased as the cubes of the distances of Titan and the moon, and the result increased as the squares of the periods of revolution of the moon and Titan.* Hence,

$$\text{Saturn's mass} = \text{the earth's mass} \times \left(\frac{759{,}990}{238{,}767}\right)^3 \times \left(\frac{27{\cdot}32166}{15{\cdot}94543}\right)^2$$

= the earth's mass × 94·6766. On several accounts this result requires modifying, however. The moon's mass bears an appreciable (though small) proportion to the mass of the earth, and, strictly speaking, it is not about the earth that the moon revolves, but about

* Thus,—let $d$, D be the respective distances of the moon and Titan from their primaries; $v$, V their respective orbital velocities; $p$, P their respective periods: then the mass of Saturn = the mass of the earth × $\frac{\text{D}\,\text{V}^2}{d\,v^2}$; but $\frac{\text{V}}{v} = \frac{\text{D}}{\text{P}} \div \frac{d}{p} = \frac{\text{D}\,p}{d\,\text{P}}$; thus the mass of Saturn = the mass of the earth × $\frac{\text{D}^3 p^2}{d^3 \text{P}^2}$.

the centre of gravity of the earth and moon. Again, Titan's mass bears an appreciable (though very small) proportion to that of Saturn; the masses of the rings and of the seven other satellites disturb Titan's motion; and other such considerations have to be taken into account. But a more important circumstance than any of these is, that the attraction of the sun operates, on the whole, to diminish the earth's attraction on the moon; and thus to increase the moon's mean period of revolution about the earth. Taking these considerations into account, it appears that Saturn's mass is about 91·433 times as great as that of the earth.

Saturn's volume exceeds that of the earth in the proportion of the squares of their respective equatorial diameters, multiplied by their respective polar diameters. Thus, Saturn's volume = the earth's volume $\times \left(\frac{72250}{7924}\right)^2 \left(\frac{65680}{7898}\right)$ = the earth's volume $\times$ 691·362. Now we have seen that Saturn's mass is only 91·433 times as great as that of the earth. Hence Saturn's density is less than that of the earth in the proportion of 91·433 to 691·362. If we call the earth's mean density 1, then the mean density of Saturn is ·1323 The mean density of the earth is about $5\frac{1}{2}$ times as great as the density of water. Or, more exactly, if the density of water be called 1, the mean density of the earth is 5·6747. Thus Saturn's mean density (if the density of water be called 1) is only ·7505; or about equal to the density of oak (·75), and very little greater than the density of sulphuric ether (·72).* It will be shown, however, in Chapter VII., that the materials of which Saturn is composed are not necessarily different from those constituting our earth.

The relative proportions of Saturn and his rings, and of the smaller members of the solar system, are exhibited in Plate I. The dimensions of the satellites must be considered merely as guesses, founded on their relative brightness; no actual measurements have yet been made of these bodies. On the same scale the distances of the satellites would be respectively $2\frac{1}{5}$in., $4\frac{1}{5}$in., $5\frac{1}{12}$in., $6\frac{1}{2}$in., $9\frac{1}{12}$in., 1ft. 9in., 2ft. $1\frac{1}{2}$in., and 5ft. $1\frac{1}{8}$in. The relations of

* Sir John Herschel, in his 'Outlines of Astronomy,' remarks that Saturn 'must be composed of materials not much heavier than cork.' The density of cork, however (water as 1), is only ·24; Saturn's mean density is more than three times as great.

these orbits are exhibited on smaller scales in Plate VII. The distance of the moon from the earth, on the scale of Plate I., is 6·6 inches, corresponding very nearly to the distance of the figure representing Saturn's eighth satellite from the figure of the earth. On the same scale the diameters of Uranus and Neptune would each be rather less than 1 inch, and the diameter of Jupiter rather less than 2½ inches, while the diameter of the sun would fall short of 2ft. by less than half an inch. Again, on the same scale the mean distance of the earth from the sun would be nearly half a mile, and the mean distance of Saturn from the sun more than 4½ miles. From these relations the reader will see:—first, how insignificant are the dimensions of our earth compared with those of the larger members of our system; secondly, how small even these globes appear when compared with the sun; and lastly, how minute are the proportions even of that gigantic globe when compared with the distances at which his attendant orbs revolve around him.*

Only the most powerful telescopes exhibit (under favourable atmospheric conditions) the complete series of phenomena described in this chapter. The two inner and the seventh satellites of Saturn are especially difficult objects. Mr. Wray records, however, that Mimas and Enceladus were visible in December, 1861, (when the dark side of the ring was turned towards the earth,) with his achromatic of only 7 inches clear aperture. The third, fourth, and fifth satellites are not very difficult objects. The fifth is slightly brighter than the other two; but all three are visible with a good achromatic of 4 inches aperture, the atmosphere being clear and steady.† The sixth and eighth can be readily detected with telescopes of

* When we compare the dimensions and orbits of the satellites with the diameters of their primaries, we do not find the same uniformity of disproportion. Thus, while the satellites of Jupiter and Saturn are mere atoms compared with their primaries, the volume of the earth does not exceed that of the moon more than fifty-five times; on the other hand, while the three outer satellites of Saturn, the outer satellite of Jupiter, and our own moon, revolve at distances from their respective primaries compared with which the diameters of those primaries appear very small, the same is not true of the other satellites of Saturn and Jupiter. The three inner satellites of Saturn revolve in orbits of very moderate dimensions, the mean distances of all falling within five and a half semi-diameters of Saturn.

† Wargentin relates that he saw the five brightest satellites with an achromatic of 10 feet focal length. On December 10th, 1793, Sir Wm. Herschel saw them with a

moderate power, Japetus requiring a power of 100, and Titan being easily visible with a power of 80. In general, Japetus must be sought for at a considerable distance from the disc of his primary.

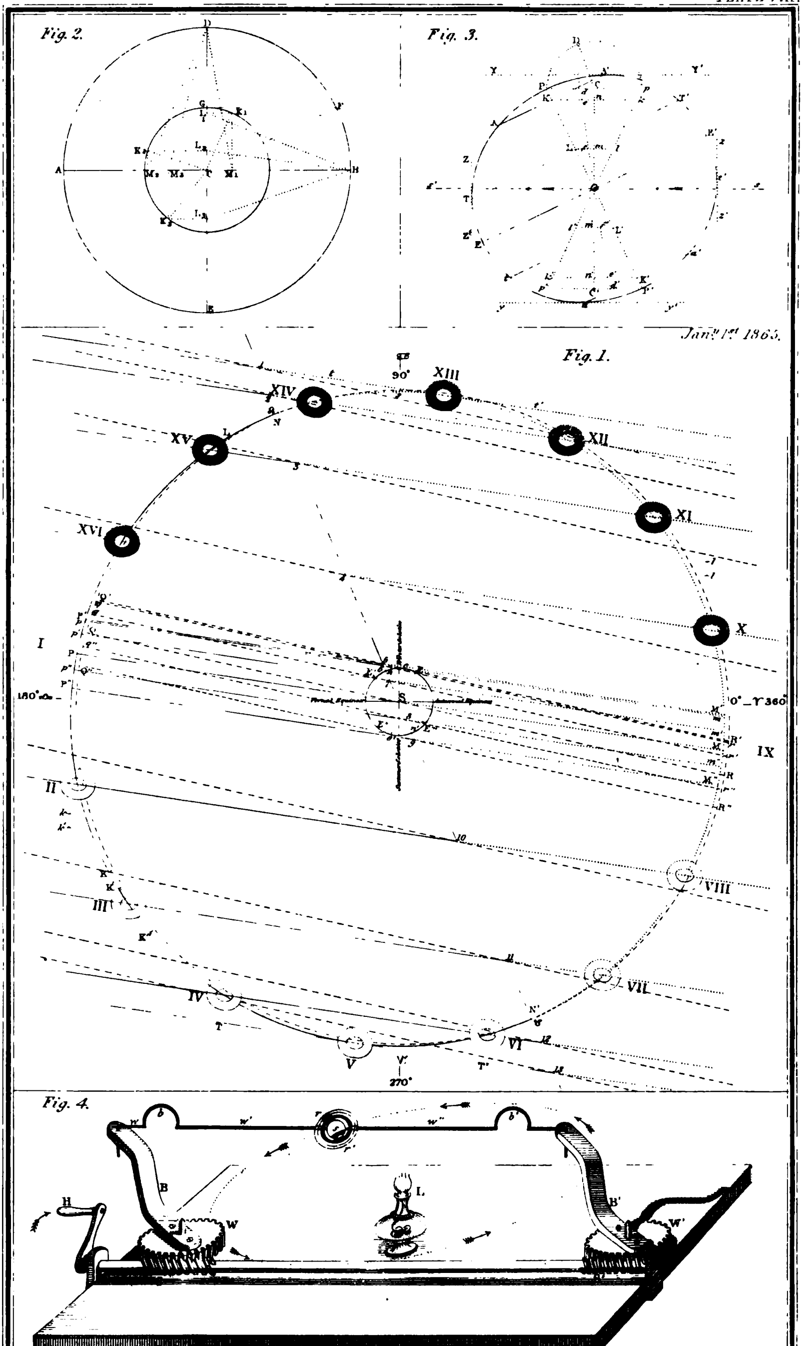
The lowest power with which the rings become visible (as such) is about 50.* A power of 150 is required to exhibit them with distinctness. The division between the rings can be seen with a power of 200, when the rings are open to nearly their full extent and viewed under favourable conditions. Under the same circumstances, the dark ring may be seen, as already mentioned, with a good achromatic of 4 inches aperture. The division in the outer ring and the variable divisions are only visible with a few of the finest reflectors and refractors in the world.

The belts on the surface of the planet are visible, under favourable atmospheric conditions, with a good achromatic of 4 inches aperture. In general, however, they require telescopes of greater power to reveal their outlines with distinctness.

No object in the heavens presents so beautiful an appearance as Saturn—viewed with an instrument of adequate power. The golden disc, faintly striped with silver-tinted belts; the circling rings, with their various shades of brilliancy and colour; and the perfect symmetry of the system as it sweeps across the dark background of the field of view, combine to form a picture as charming as it is sublime and impressive.

power of 60 applied to a reflector of 10 feet focal length. They can be detected, however, with smaller instruments. I have repeatedly seen all five with perfect distinctness through an achromatic of 4 inches aperture, and 5½ feet focal length.

* Since the diameter of Saturn when in opposition is about 19″·2, a power of 50 presents him to the eye with an apparent diameter of 16′,—rather greater than the mean apparent semi-diameter of the moon; and it might at first sight appear that when the planet has so great an apparent diameter, the rings must be conspicuously visible. Such is not the case, however; and, in fact, this method of estimating the appearance of the magnified disc of a planet is altogether deceptive. The disc really appears of the dimensions calculated; but, in the first place, the apparent size of the moon is over-estimated by the unaided eye; and, secondly, when we so view the moon, its disc is not perceptibly distorted by atmospheric undulations, whereas these undulations are all magnified fifty-fold when we use a telescopic power of fifty, and the disc of a planet so viewed is correspondingly distorted. If to these considerations be added the difference in the illumination of Saturn and of the moon, (see Chap. VII.), and the loss of light by reflection at the surfaces of the lenses and by absorption in passing through them, it will readily be seen that an observer who should found his expectations on such a calculation as that given above, would be altogether disappointed by the view he would actually obtain of the planet.




SATURNIAN ORRERY.

R. A. Proctor del.

H. Adlard sc.

London: Longmans & Co.

# CHAPTER IV.

## THE PERIODIC CHANGES IN THE APPEARANCE OF SATURN'S SYSTEM.

In describing his orbit about the sun, Saturn retains the direction of his polar axis (or axis of rotation) unaltered, or very nearly so. As in the case of the earth, there are small motions of this axis, owing to which, in the course of many thousands of years, the poles of the Saturnian heavens travel with an undulatory movement round two opposite small circles of the celestial sphere;* but so far as a single revolution about the sun is concerned, we may consider the polar axis of Saturn as retaining its direction absolutely unchanged. This axis is inclined at an angle of 63° 10′ 32″ to the plane of Saturn's orbit—in other words, the plane of Saturn's equator is inclined at an angle of 26° 49′ 28″ to the plane in which Saturn moves. I propose, in this chapter, to consider the effect of this inclination in producing changes of appearance in Saturn's rings and disc, and in modifying the apparent orbits of his satellites; leaving to a future chapter the consideration of the variations—due to the same cause—in the Saturnian seasons, and in the appearance of the rings to the Saturnians.

* The complete revolution of Saturn's vernal equinox occupies upwards of 412,080 years, the annual precession of his equinoxes being 3″·145. The right ascension of the north pole of the Saturnian ecliptic is 18h. 23m. 31·7s.; and the declination 67° 22′ 20″N. The poles of the Saturnian heavens revolve in two small circles, having this point and the opposite point on the celestial sphere for their respective poles, the angular radius of each small circle being 26° 49′ 28″.

At present no conspicuous star lies near either pole of the Saturnian heavens. The right ascension of the northern pole is 2h. 23m. 1·7s., the declination 82° 52′ 36″; this pole, therefore, lies near the northern foot of Cepheus, and nearly 6° from the polar star; the nearest visible star is 2 Ursæ minoris of the fifth magnitude—about 3° from Saturn's north pole. Saturn's south pole lies in the constellation Octans, and less than 1° from the star δ Octantis of the fifth magnitude. As with the earth, the conspicuous stars, α Draconis and α Lyræ (the brilliant Vega) are possible north-polar stars for Saturn, and ι Argûs is a possible south-polar star; but many centuries must elapse before any of these stars will occupy such positions.

The plane of the rings coincides in general with the plane of Saturn's equator, but is subject to oscillatory movements, whose extent and period have not yet been determined. Such oscillations will not be taken into account in considering the general changes of appearance presented by the ring, as, following the planet, it sweeps on its path round the sun.

In fig. 1, Plate VIII., let N P N′M be Saturn's orbit, E E′E″E‴ the orbit of the earth about S, the sun; let N S N′ be the line of nodes of Saturn's orbit on the ecliptic, ☊ being the ascending, and ☋ the descending node; suppose, also, that N Q N′R represents the projection of Saturn's orbit on the plane of the ecliptic. The longitude of the ascending node of Saturn's equator—and therefore the mean longitude of the ascending node of his ring—on the ecliptic, is at present 167° 43′ 29″, while the longitude of the point N is 112° 29′ 18″: so that if we draw the line Q S R inclined to S N at an angle of 55° 14′ 11″, Q S R is the *direction* of the line of nodes of the ring's plane on the ecliptic. It is not *actually* the line of nodes, since that moves with the ring; but wherever the ring may be on N P N′M, the line in which its plane intersects the ecliptic is parallel to Q S R; and twice in each revolution of Saturn this line coincides with Q S R. Now the plane of the ring is inclined at an angle of 28° 10′ 22″ to the plane of the ecliptic; hence, when the line of nodes of the ring's plane on the ecliptic coincides with the line Q S R, the plane of the rings intersects the plane of Saturn's orbit in the straight line P S M, such that the plane P S Q is inclined at an angle of 28° 10′ 22″ to the plane of Saturn's orbit. The heliocentric longitude of the point P is 171° 43′ 35″; the latitude of P is 2° 8′ 26″ N; and the arc N′P is 59° 15′ 42″.* The line P S M gives the *direction* of the line of nodes of the ring's plane on Saturn's orbit; wherever the ring may be, the line of nodes is parallel to P S M, and when Saturn is at P or M, the line of nodes coincides with the line P S M.

Let us now trace the motion of the ring through a revolution about the sun, starting from the point P. In the first place, let us neglect all consideration of the earth's motion in her orbit, and suppose the observer to be placed at the sun's centre.

* The arc from P to Q is 4° 32′ 15″.

When Saturn is at P, P S R is the line of nodes of the ring's plane on his orbit, and therefore the line of sight, S P, coincides with the plane of the ring.* Thus, when Saturn is at P, the spectator at S is looking at the edge of the ring, and it appears to him as a fine line of light inclined at an angle of 26° 49′ 28″ to the line of Saturn's motion—the eastern extremity of the ring being elevated —as shown at I, fig. 1, Plate IX. As the ring passes on to position II (fig. 1, Plate VIII.), the line of sight from S becomes more and more inclined to the line of nodes of the ring's plane on the plane of Saturn's orbit. When the ring is at II, the line of sight from S, inclined at an angle of about 23° to the line II–VIII, passes above the nearer, and below the farther half of the ring, and is inclined at an angle of 10° 10′ to the plane of the ring:—thus the appearance of Saturn and his rings is as shown at II, fig. 1, Plate IX., the northern side being visible. As Saturn passes on through the positions III and IV to V, the angle between the line of sight from S and the line of nodes of the ring's plane on Saturn's orbit, increases continually. When Saturn is at III this angle is about 46°, when he is at IV it has increased to about 68°; and, finally, when Saturn is at V it is a right angle, the corresponding angles at which the line of sight is inclined to the plane of the ring being respectively 18° 57′, 24° 44′, and 26° 49′ 28″. Thus the planet and rings appear as shown at III, IV, and V, fig. 1, Plate IX., the northern side being still visible. At V the ring is seen opened to its greatest extent, the division in the ring just appear-

* It must be understood that in fig. 1, Plate VIII., the curve N Q N′R is supposed to lie in the plane of the paper; thus the half N P N′ of Saturn's orbit lies above, and the dotted half N′M N below the plane of the paper. The lower half of each figure of the ring is supposed to lie above the plane N P N′M, the upper half lying below that plane; to avoid confusion the figure of the ring is omitted at the points P and M. The lines $t$ XIII $t'$, XIV–XII, XV–XI, . . . T V T′, represent the line of nodes of the ring's plane on Saturn's orbit in different positions; these lines are parallel to one another, and the dotted part of each is supposed to lie below the plane of the paper. The other lines through the points 1, 2, 3, . . . 13 (in which the former set of lines meet the line of nodes of Saturn's orbit on the ecliptic), represent the corresponding positions of the line of nodes of the ring's plane on the plane of the ecliptic; these lines also are parallel to one another. The proportions of the rings are greatly exaggerated: on the scale of the rings the diameter of the orbit should be more than two miles long; on the present scale of the orbit the sun's diameter would be less than $\frac{1}{740}$th part of an inch, the outer diameter of the rings less than $\frac{1}{3700}$th part of an inch.

ing above the edge of Saturn's disc. Throughout these changes the apparent major axis of the ring has become less and less inclined to the line of Saturn's motion, until at V it coincides with that line.

As Saturn passes on to the position IX (fig. 1, Plate VIII.) the line of sight from S becomes less and less inclined to the line of nodes, and the planet and ring pass through the same changes as before, but in a reverse order: the northern side of the ring still continues visible, but the apparent major axis of the ring now has its western extremity elevated above the line of Saturn's motion. Thus the planet and rings appear as shown at VI, VII, and VIII, fig. 1, Plate IX. Finally, when Saturn is at IX, fig. 1, Plate VIII., the line of sight, S M, again coincides with the line of nodes, and the ring appears as a fine line of light inclined at an angle of 26° 49′ 28″ to the line of motion of Saturn's centre (or as shown at IX, fig. 1, Plate IX.).

As Saturn passes from IX to XIII, he goes through the same changes, in the same order, as in moving from I to V. But it is plain that the line of sight from S now passes below the nearer and above the farther part of the ring; thus the planet and rings appear as shown at X, XI, XII and XIII, fig. 1, Plate IX., the southern side being visible. The western extremity of the apparent major axis of the rings still continues elevated above the line of Saturn's motion until it coincides with that line when Saturn is at XIII.

Finally, as Saturn moves up to I, the system presents successively the appearances shown at XIV, XV, and XVI, fig. 1, Plate IX., the southern side continuing visible, but the eastern extremity of the apparent major axis of the ring elevated above the line of Saturn's motion.*

* The mathematical reader will find no difficulty in verifying the following simple construction for determining the appearance of the ring—supposed to be viewed from the sun. With centre C, fig. 2, plate VIII., describe a circle A E B D, whose diameter shall represent the apparent major axis of the ring's outer boundary; draw the diameters A B and E D at right angles to each other; through C draw C F, so that F C B is an angle of 26° 49′ 28″; draw F G perpendicular to C D; with centre C describe the circle G $K_2$ $K_3$ $K_1$; and from C draw lines C $K_1$, C $K_2$, and C $K_3$. First let $K_1$C B be the angle that, at the moment considered, Saturn has swept out around the sun from the point P (fig. 1) of his orbit; draw $K_1$ $L_1$ and $K_1$ $M_1$ perpendicular to C D and C B respectively, and join B $L_1$ and D $M_1$. Then C $L_1$ is the apparent semi-minor axis of the outer boundary of the ring. Further, C B $L_1$ approximately represents the angle at which the line of sight is

The variations in the appearance of the system, viewed from the earth, are not different from those just described; but they are presented in a less simple succession. Besides the continuous increase and decrease in the minor axis of the ring, corresponding to the motion of Saturn in his orbit, there are changes corresponding to the motion of the earth in hers. When the line of nodes of the ring's plane on the ecliptic is passing across the limits of the earth's orbit, the variations arising from the motion of the earth are still more marked, as will presently appear.

To illustrate the general effects of the earth's motion in modifying the appearance of Saturn's system, let us trace the motions of Saturn and the earth from the conjunction which took place in the autumn of 1864, to the conjunction which will take place in the autumn of the current year (1865)—neglecting, for the present, the departure of the earth from the plane of Saturn's orbit; that is, supposing both orbits to lie in the same plane. The first-mentioned conjunction took place on October 14th, 1864, at 3h. 9m. A.M., Saturn being at K and the earth at the point in which the line from K, through S, meets the orbit E E′E″ (beyond S). If, then, Saturn had not been at this time hidden from view by the superior effulgence of the sun, he would have presented the same appearance (on a slightly diminished scale owiugto increased distance) to the observer on earth as to the supposed observer at the sun's centre;

inclined to the plane of the ring, and C D $M_1$ approximately represents the angle at which the apparent major axis of the ring's outline is inclined to the apparent path of Saturn's centre. [The true values of these angles are somewhat greater, and are obtained by describing circles about C as centre, with radii C $L_1$ and C $M_1$ respectively, and drawing tangents to these two circles from the points B and D respectively; these will be inclined to the lines C B and C D, respectively, at the required angles.] Similarly, if the angle $K_2$C B, or $K_3$C B (greater than two right angles), represents the angle swept out by Saturn about the sun (from P), we can obtain (i.) C $L_2$ or C $L_3$, the semi-minor axis of the apparent outer boundary of the ring; (ii.) the angle C B $L_2$ or C B $L_3$, at which (approximately) the line of sight is inclined to the plane of the ring; and (iii.) the angle C D $M_2$ or C D $M_3$, at which (approximately) the apparent major axis of the ring's outline is inclined to the line of motion of Saturn's centre. According as the point corresponding to $L_1$ lies above or below A B, the northern or southern face of the ring is visible, and according as the point corresponding to $M_1$ lies to the right or to the left of D E, the eastern or western extremity of the ring's major axis is elevated above the line of motion of Saturn's centre. If C B represents the semi-major axis of any other outline of the ring than the outer boundary, then the line corresponding to C $L_1$ represents the semi-minor axis of the *corresponding* outline.

for the line from the earth to Saturn is inclined at the same angle to P M (or, which is the same thing, to the line through K parallel to P M) as the line from the sun. But as the earth moved on in her orbit, Saturn moving on slowly in his, the former angle obviously increased more rapidly than the latter, until Saturn was in quadrature preceding opposition (which happened on January 20th, 1865, at 0h. 5m. A.M.). At this time the earth was between $n$ and E′, Saturn at K′, and the former angle exceeded the latter by about 6°. Hence, since the extent to which the rings appear opened obviously depends altogether on the angle at which the line from the spectator is inclined to their plane,* the rings were opening out more rapidly to the observer on earth, during the interval from conjunction to quadrature, than they would have been to an observer placed at the sun's centre. On the other hand, as the earth and Saturn move on in their respective orbits to opposition (which takes place on April 17th, at 0h. 32m. A.M.), it is plain that the line of sight from the earth moves up to coincidence with the line from the sun's centre. During the first part of this interval, the rings appear to the observer on earth to be opening out, but more and more slowly, till in the beginning of February they attain their greatest expansion (the line of sight from the earth being then inclined at an angle of about 16° 15′ to the plane of the rings). After this the rings appear to close, more and more rapidly, till finally, when Saturn is in opposition (at III, the earth being in the line from S to III, near E″), his system presents the same appearance (on a slightly increased scale owing to diminished distance) to the observer on earth as it would to an observer placed at the sun's centre. As Saturn and the earth move on to conjunction, the rings continue to close, but more and more slowly, till the last week of June (the line of sight from the observer on earth being then inclined at an angle of about 13° 40′ to the plane of the rings). After this they open out till Saturn is in quadrature, following opposition (which happens on July 16th, at

* If a figure be constructed as shown in the preceding note, to determine the appearance of the ring as seen from the earth and from the sun at this time, it will be found that the angle at which the line from the earth is inclined to the plane of the ring, exceeds the angle at which the line from the sun is inclined to that plane, by about 2° 15′.

1h. 2m. P.M., Saturn having moved through nearly 2° from III, and the earth being between $n'$ and E‴); and thence more rapidly till finally at conjunction (which happens on October 26th, at 1h. 16m. P.M., Saturn being at K″ and the earth not far from E), the line from the earth is again coincident with the line from the sun to Saturn. It appears, then, that although during nearly five months of the synodical revolution considered, the rings will have been closing up, yet on the whole they will have opened out by the same amount to the observer on earth as to an observer supposed to be placed at the sun's centre.

The departure of the earth from the plane of Saturn's orbit is so small compared with Saturn's distance from the earth, even when in opposition, that its effect in modifying the appearance of Saturn's rings (as viewed from the earth) is very slight. At opposition, when, in general, this effect is greatest, the opening of the rings may be diminished or increased according to the position of the line on which opposition occurs. If this line lies within the angles N S P or N′S M, the opening of the rings is slightly increased: for when within the first angle, the earth is below the plane of Saturn's orbit, and also below the visible face of the rings, so that the departure of the earth is *from* that face; and, within the second angle, the departure of the earth is still *from* the visible face of the rings, for the earth is above that face and also above the plane of Saturn's orbit. On the other hand, if opposition occurs within the angles P S N′ or M S N, the departure of the earth from the plane of Saturn's orbit plainly brings the earth *towards* the plane of the ring; for within the former angle the earth is below the plane of Saturn's orbit, and above the visible face of the rings; and within the latter angle the earth is above the plane of Saturn's orbit, and below the visible face of the rings: thus the opening of the rings is diminished at or near opposition occurring within these two angles. The increase or diminution is very small, however, in either case, and any change in the opening of the rings when Saturn and the sun are not in opposition, is in general still less important. There is one case, however, which requires to be noticed. If the earth moved in the plane of Saturn's orbit, it is clear that the rings could never appear more open to the observer on earth than they would appear to an observer placed at the sun's centre when Saturn is at V or XIII. The opening of the rings would, in fact, only actually attain such an extent when opposition happened to take place along one of the lines S V or S XIII,—though the difference would be inappreciable if Saturn were even in quadrature when at V or XIII.* But since the earth moves in a plane inclined to the plane of Saturn's orbit, the observer on earth is occasionally able to see Saturn's ring slightly more open than it would ever appear to an observer placed at the sun's centre. For instance, suppose Saturn at or near XIII, and the earth at *e* (as in the winter of 1855-1856); then we have just seen that the earth, being slightly above the plane of Saturn's orbit while moving from *e* to *n*, and below the rings' plane, the rings appear less open than they would if the earth were *in* the plane of Saturn's orbit, or than they would to an observer at the sun's centre; but when the earth has passed *n*, and begins to dip below the plane of Saturn's orbit—that is, away from the plane of the rings—these appear more open to the observer on earth than they would to an observer at S. In the meantime, Saturn is moving away from that part of his orbit at which his rings would appear most open to an observer at S, and so far as this motion of Saturn's is concerned, the rings would appear to be closing to the observer on earth. The departure of the earth from the plane of Saturn's orbit has at first the greater influence; but as it gradually becomes slower and slower the effect of Saturn's motion in his orbit begins to show itself,—the rings cease to open, and commence slowly to close. Thus, on January 1st, 1856, the proportion of the minor to the major axis of the ellipse presented by the outer circumference of Saturn's ring to the observer on earth was ·4491; on January 15th this proportion was ·4501 (or very nearly the same as the corresponding proportion—when the ring is most open—to an observer placed at the sun's centre); on March 21st, this proportion had increased to

* The proportion of the minor to the major axis of the ellipse presented by any outline of Saturn's ring (say the outer edge) to an observer at S, when Saturn is at V or XIII is ·45015; the corresponding proportion in the case of an observer on the earth (Saturn being supposed to be then in quadrature to the sun) is ·44765. If two ellipses, having equal major axes, and minor axes in these proportions respectively, were drawn, the minor axis of the first would exceed that of the second by only $\frac{1}{400}$th part of the major axis of either.

·4548;* on April 15th it had diminished to ·4516, though the earth had then attained its greatest distance from the plane of Saturn's orbit; and, finally, when the earth again reached the plane of Saturn's orbit at $n'$, this proportion had diminished to ·4349.

Let us next examine the phenomena presented when the line of nodes of the ring's plane on the ecliptic is travelling across the orbit of the earth (E E′E″E‴). It is clear that during this passage the plane of the ring must pass once through the sun; once, at least, through the earth; and (unless those two passages happen to be simultaneous) the plane of the ring must lie for a time between the earth and the sun. In any one of these cases the ring will be invisible except through telescopes of great power: in the first, because the sun is shining on the outer edge, and does not illuminate either face of the ring; in the second, because the edge of the ring is turned directly towards the observer on earth; and in the third, because the dark side of the ring is turned towards him.

It may happen, however, that the plane of the rings will pass more than once through the earth during the interval we are considering. Let us examine two or three ways in which the passage of the line of nodes across the orbit of the earth may take place. To avoid confusion the words *invisible, disappearance* and *reappearance*, are used without the addition of the words 'in ordinary telescopes;' it is to be understood, however, that in telescopes of great power the rings are probably never altogether invisible.

Draw the lines Q′$e$ R′ and Q″$e'$R″ parallel to Q S R, and touching the circle E E′E″E‴ at the points $e$ and $e'$, respectively: then, when the line of nodes of the rings' plane on the ecliptic is in any position between Q′R′ and Q″R″, it passes across the earth's orbit. Through the points 5 and 9, in which the lines Q′ R′ and Q″R″ meet the line N S N′, draw P′5 M′ and P″9 M″ parallel to P M; then, when the line of nodes is in the position Q′ $e$ R′, Saturn is either at P′ or M′; and when the line of nodes is in the position Q″$e'$R″, he is either at P″ or M″: thus, whenever Saturn is between the points P′ and P″, or M′ and M″ of his orbit, the plane of the ring intersects

* At this time the line of sight from the earth to Saturn was inclined at an angle of rather more than 27° 13′ to the plane of the rings. This is very nearly the greatest angle at which it can be so inclined. Saturn's appearance at this time is represented in fig. 3, Plate I.

the earth's orbit. Now each of the arcs P′P″ and M′M″ is slightly greater than the diameter of the earth's orbit; and therefore each is approximately equal to one third part of the circumference of that orbit. And since Saturn moves along the arcs P′P″ and M′M″ with a velocity rather less than one third of the earth's velocity in her orbit, it is clear that Saturn will occupy rather more time in traversing either of the arcs P′P″ or M′M″, than the earth in travelling once round her orbit; in other words, the line of nodes of the ring's plane on the ecliptic occupies rather more than a year in passing across the earth's orbit. The actual interval is about a year and a week for the arc P′P″, and about a year and three weeks for the arc M′ M″.*

Now let us suppose that when Saturn is at P′ (and the line of nodes in the position Q′*e* R′), the earth is at or near *n*. The earth moving on from *n*, continues in advance of the line of nodes while that line moves up to the position Q R. During this time the earth and sun are both on the same side (the southern side) of the plane of the ring, which therefore is visible during this interval. But when the line of nodes has arrived at the position Q S R (Saturn being at P) the plane of the ring passes through the sun, and the ring disappears. After Saturn has passed the point P the sun is on the northern side of the ring, and the earth (somewhere between *e*′ and *n*′ on its orbit) is on the southern side of the ring; the ring is therefore still invisible. When the earth, passing on towards E‴, meets the advancing line of nodes, the plane of the ring passes through the earth, and at this epoch the ring is invisible as before. But as the earth now passes to the northern side of the ring's plane—the sun being also on the northern side of that plane—the ring again becomes visible. The earth proceeds to traverse the part of E *n* E′ of her orbit; when the earth has nearly reached E′ Saturn is at P″, or the line of nodes is in the position Q″*e*′R″; thus the earth does not pass again through the plane of the ring, whose line of nodes on the ecliptic now passes beyond the range of the earth's orbit.

* The interval is different for the two arcs P′P″ and M′M″: in the former Saturn moves with a mean daily velocity of about 2′ 3″·4; in the latter with a mean daily velocity of about 1′ 59″·0. Thus the interval for the arc M′M″ is greater than the interval for the arc P′P″, in the proportion of about 28 to 27.

It appears, then, that in the case considered, the plane of the rings passes only once through the earth; there is one disappearance, and one reappearance of the ring; and for some weeks between these events the ring is invisible.

Again, if we suppose that when Saturn is at P′ the earth is at E″, it is clear that in this case also the ring's plane will only pass once through the earth: this will take place before Saturn reaches P. The plane of the ring will then disappear, and continue invisible until the planet passes the point P. After this, the sun being on the same side of the plane of the ring as the earth, the ring will be visible.

Further, if the earth is anywhere between *n* and E″* when Saturn is at P′, the plane of the ring will pass only once through the earth, and there will be one disappearance and one reappearance.

But now let us examine another case, and as it happens that in the years 1861–1862 the rings actually exhibited the phenomena corresponding to this case, let us take the actual dates of the disappearances and reappearances of the ring in those years. When Saturn was at the part P′ of his orbit, in the autumn of 1861, the earth was approaching the point E, and the southern side of the ring was visible. On Nov. 23rd, 1861, at 3 P.M., the earth was at E, Saturn at *p*, and the line of nodes of the ring's plane on the ecliptic was in the position *q* 6 E *r*, so that the plane of the ring passed through the earth: thus the ring disappeared at that instant, and the earth passing to the northern side of the ring's plane, the ring remained for a time invisible. On Feb. 1st, 1862, at 3 A.M., the earth was at E′, Saturn at *p*′, and the line of nodes in the position *q*′E′7 *r*′, so that the plane of the ring again passed through the earth: thus the ring was invisible at that instant; but the earth passing to the southern side of the ring's plane,† the ring reappeared. The ring continued visible until Saturn arrived at the point P, when the ring's plane passed through the sun. This took place on May 18th, 1862, at 8h. 30m. A.M., the earth being then at E″. The ring disappeared at this instant,

* And a few degrees beyond these points; in fact, over the whole of the arc *e*E′*e*′, except about 7° from the points *e* and *e*′.

† Fig. 1, Plate I., represents Saturn a day or two after this reappearance. The sun being raised about 1° 38′ above the plane of the ring, the shadow is visible as a dark line crossing Saturn's disc.

the sun illuminating only the edge of the ring. After this, the sun being on the northern, the earth on the southern side of the ring's plane, the ring continued for a time invisible. Finally, on August 13th, 1862, at 4 A.M., the earth was at E‴, Saturn at $p''$, and the line of nodes in the position $q''$8 E‴$r''$; thus the plane of the ring passed through the earth, and the earth passing to the northern side of this plane, the ring again became visible. When the earth had reached E (that is, in November, 1862), the line of nodes had passed to the position Q″$e'$R″; thus the northern side of the ring has since continued visible.

It appears, then, that while the line of nodes of the ring's plane on the ecliptic was crossing the earth's orbit in the years 1861–62, the plane of the ring passed three times through the earth, that the ring disappeared twice and reappeared twice, and continued invisible during two intervals, the first of nearly ten weeks, the second of twelve weeks.

If the earth had been at $e'$ when the line of nodes had reached the position Q′$e$ R′, it is clear that the plane of the ring would have passed three times through the earth in this case also. The first passage would have taken place before Saturn reached the point P, until which time the rings would have been invisible. They would have become visible when Saturn had passed the point P, and continued so until Saturn had very nearly reached the point P″, when —the earth being near $e'$—the plane of the rings would have passed twice through the earth, disappearing at the first passage and reappearing at the second.

Further, if the earth is anywhere on the arc $e$ E $e'$, and for a few degrees beyond the points $e$ and $e'$, when Saturn reaches the point P′, the rings' plane will pass three times through the earth, and there will be two disappearances of the rings, and two reappearances.

Similar remarks apply to the passage of Saturn through the arc M″M M′ of his orbit. Thus, if the earth is anywhere on the arc $e$ E′$e'$, or a few degrees beyond the points $e$ and $e'$, when Saturn reaches the point M″, the rings' plane will pass three times through the earth, and there will be two disappearances and two reappearances of the rings. But if the earth is anywhere on the remaining arc of its orbit when Saturn arrives at M″, the rings' plane will pass only once through the earth, and there will be one disappearance and one reappearance.

In general, when the plane of the rings has passed through either the earth or the sun, the rings disappear or reappear. For, it is clear, that, if before the passage of the rings' plane through the earth their illuminated side was turned to the observer, then after such passage the earth must be on the darkened side of the rings, and *vice versâ*. And again, if before the passage of the rings' plane through the sun, the earth and the sun are on the same side of the rings' plane—then, after such passage, the earth and the sun must be on opposite sides of the rings' plane, and *vice versâ*. It may happen, however, that the plane of the ring passes through the sun and the earth at the same or nearly the same instant of time; and it is perfectly clear that, in this case, if the ring is in-invisible before such passage it will be invisible after it, and *vice versâ*. Thus, if the ring's plane passes only once through the earth at this passage of the nodal line across the earth's orbit, there will be *no* interval during which the ring is invisible (except the brief interval of the passage of the ring's plane through the earth and the sun); and only *one* such interval if the ring's plane passes three times through the earth. It will readily be seen that in the former case Saturn and the sun would be in conjunction at the double passage, and Saturn therefore invisible;* in the latter case Saturn would be in opposition to the sun, and therefore favourably situated for observation.

Again, two passages of the earth through the plane of the ring may coincide. This will occur if, when Saturn is at P′, P″, M′, or M″, the earth happens to be on one of the points separating the two arcs of its orbit mentioned above. Thus, if the earth is a few degrees beyond the point *e* when Saturn arrives at P′, the line of nodes of the ring will overtake the earth (which is here moving in a path inclined at a very small angle to that line); but before the ring's plane has passed beyond the earth, the rapid motion of the latter

* To avoid confusion no notice has been taken of the invisibility of Saturn at and near conjunction, in the description of the passage of the nodal line of the ring's plane across the elliptic.

carries it (as the angle increases at which the direction of such motion is inclined to the line of nodes) again in front of the line of nodes, and the rings only disappear for the comparatively brief interval during which their plane passes through the earth. A similar double passage will happen if the earth is a few degrees from $e'$ when Saturn reaches P″; for, in this case, when Saturn is approaching P″, the earth will have moved round in its orbit (having passed once through the plane of the rings), and have reached the line of nodes; but before the earth has passed through and beyond that line, the direction of the earth's motion will have become inclined to the line of nodes at so small an angle, that that line will again pass in front of the earth. The earth will similarly hang for a short time in the plane of the rings, without passing through and beyond it, if, when Saturn reaches M′ or M″, the earth is a few degrees from $e$ or beyond $e'$. In these cases it will be seen that the rings are visible both before and after the double passage. It is plain, also, that at such a double passage the earth would be for a longer time in the plane of the rings than when merely passing through that plane, and thus the phenomena attending the disappearance of the ring would be very favourably seen. The coincidence described is, of course, very uncommon; if, however, the earth is *near* one of the points mentioned, she hangs longer in and near the plane of the rings than at an ordinary passage through that plane.* If the earth passes through the plane of the ring when Saturn is at or near opposition, an observer on earth will be between the planes bounding opposite faces of the ring for about 8 seconds—the thickness of the rings being as-

* It is stated in Sir John Herschel's 'Outlines of Astronomy' that the plane of Saturn's ring in crossing the earth's orbit generally passes twice through the earth; and in Hind's 'Introduction to Astronomy' that 'there are usually two, if not three, disappearances about the time of the planet's arrival at the nodes.' The above investigation shows that the earth can never pass *twice exactly* through the plane of the rings during the passage of this plane across the earth's orbit, but may pass either once or three times. And again, it is clear that there may be two disappearances of the ring, but never three; or, if reappearances and disappearances are both included under the term 'disappearances,' then there may be two or four, but can never be *three exactly*. The case supposed by Herschel would leave the earth on the same side of the ring's plane before as after the passage of that plane across the earth's orbit; the case supposed by Hind would leave the rings invisible after such passage.

sumed to be 100 miles—for he is carried from one plane to the other with a velocity equal to the difference of the velocities of the earth and Saturn, or at the rate of 44,000 miles an hour. If the earth passes through the plane of the rings when Saturn is in conjunction, and Saturn could be seen at such a time, the observer would be carried from plane to plane of the opposite faces of the ring in 4 seconds. On the other hand, when two passages of the earth through the ring's plane coincide, an observer might be nearly nine hours between the planes bounding opposite faces of the rings (supposing the thickness of the rings to be 100 miles),—not being carried through both planes, but twice through one plane. If two passages were very nearly coincident, an observer might be four or five hours between the two planes at one passage, and an hour or two at the other, passing each time through both planes. When the earth passed through the plane of the ring in November, 1861, the time of passage of each point on the earth from plane to plane of the bounding faces of the rings, was very short: the corresponding passage in February, 1862, occupied a longer interval.

Another consequence of the earth's motion in her orbit is that the shadow of the rings on the planet, and the shadow of the planet on the rings, become visible. It is clear that to an observer placed at the sun's centre these shadows would at all times be invisible,—since those parts only of the rings and planet are in shadow from which the sun is invisible (through the interposition of parts of the planet and rings respectively), and the sun being invisible from them, they would be invisible to an observer placed in the sun. And the more nearly the line of sight from the earth to Saturn approaches to coincidence with the line from the sun to Saturn, the less conspicuous will the shadows be. Thus, when Saturn is in quadrature to the sun, at which time the angle between these lines is greatest (having an average value of about 6° at that time*), both shadows are, in general, more conspicuous than at any other time; but there is a difference between the two shadows in this respect. When the rings are open to nearly their greatest extent, the shadow of the rings on the planet is not conspicuous, even

* When Saturn is in perihelion this angle is 6° 22′ 25″·6; when he is in aphelion it is 5° 41′ 50″·2.

when Saturn is in quadrature, for the motion of the earth in her orbit causes very little alteration in the apparent opening of the rings at this time; thus the earth being elevated at very nearly the same angle as the sun above the plane of the rings, those parts of the planet's disc which would be invisible from the sun (that is, in shadow) are also invisible—or only visible along very narrow strips of their surface—to the observer on earth, in whatever part of its orbit the earth may be. On the other hand, the shadow of the planet on the rings is very favourably seen when Saturn is in this part of his orbit, and in or near quadrature; for the portion of the rings concealed from the observer on earth, and the portion hidden from the sun (that is, in shadow), are shifted from coincidence with each other, through an angle of $5\frac{3}{4}°$ or $6\frac{1}{4}°$ (according as Saturn is near perihelion or aphelion)* about the centre of the rings, and a large part of the shadow of the globe thus becomes visible on one side or the other of Saturn's disc. Further, since the rings are open to their greatest extent, the shadow, which extends nearly across the width of the rings, is less foreshortened at this time than when the rings are less open. Thus the shadow appears as in fig. 3, Plate I., in the form of a broad curved black space, bounded by two elliptical outlines. This figure represents Saturn as he appeared when near the point XIII of his orbit (fig. 1, Plate VIII.), and in quadrature following opposition (that is, when the earth was between E′ and E″. The shadow lies to the right of the planet's disc: in reality the shadow lay at this time to the left of Saturn's disc, but in an inverting telescope the shadow appeared as represented. If there were no division in the rings the shadow would have been visible beyond the uppermost point of Saturn's disc, and for a short distance to the left of this point; for, as we have seen, the line of sight from the earth to Saturn was inclined at a slightly greater angle to the plane of the rings than the line from the sun. Since, however, the division in the rings becomes visible above the disc of Saturn, there is no visible shadow at this point or in its neighbourhood.† When Saturn

* The rings are open to their greatest extent when Saturn is near one or other of the apses of his orbit.

† In pictures of Saturn by Bond and other observers, the division is thus shown. Pos-

(near XIII) was in quadrature preceding opposition (in October, 1855), the shadow presented a similar appearance on the other side of Saturn's disc.

When Saturn is in other parts of his orbit, the shadow of the rings on the planet is more favourably seen. Thus, if Saturn is at L, and the earth near E—that is, Saturn in quadrature preceding opposition—it is clear that the earth is elevated at a smaller angle than the sun above the plane of the rings. Thus the ellipses presented by the rings to the observer on earth have smaller minor axes than the corresponding ellipses that would be presented by the rings to an observer at the sun's centre. The shadow of the rings is therefore seen outside the outer edge of the rings as a black stripe, whose outline forms part of an ellipse of larger minor axis than that of the ring's outline. Fig. 2, Plate I., represents the appearance of Saturn in an inverting telescope at such a time. He appeared thus in the earlier part of November, 1858. At this time the shadow of the planet on the rings appeared (in an inverting telescope) to the left of the planet's disc—more foreshortened than the corresponding shadow in fig. 3, but sufficiently conspicuous.

When Saturn had passed on to quadrature following opposition, or in the spring of 1859 (the earth being between E′ and E″), the shadow of the rings appeared within them* (since the earth was elevated at a greater angle than the sun above the plane of the rings), and the shadow of the planet to the right of the planet's disc—in an inverting telescope.

As Saturn's ring closes up, the portion of the shadow of the rings visible when Saturn is at or near quadrature increases, while the shadow of the planet on the rings becomes more and more foreshortened. When Saturn is near either of the positions I or IX, the whole of the shadow of the rings becomes visible, at and near either quadrature. As the sun is nearly in the plane of the rings the

sibly the shadow of the globe has been mistaken for the continuation of the division at this point.

* As the visible part of the shadow of the rings at such a time is plainly the shadow of the dark ring, it will appear probable, from the results of Chapter V., that the shadow is not so dark as in the former case, where the shadow is that of the outer bright ring. It would be very difficult to detect the difference, however, even with the most powerful telescopes.

shadow is very narrow. It appears within or without the outline of the rings, according as the earth or the sun is elevated at the greater angle above the rings' plane. Thus, in fig. 1, Plate I., the whole of the shadow is seen crossing the planet's disc below the outline of the rings. This figure represents Saturn as he appeared in an inverting telescope, a day or two after the reappearance of the rings in February, 1862, the earth being near E′, and Saturn near $p'$. Saturn was at this time between quadrature preceding opposition and opposition. The shadow of the planet on the rings was foreshortened almost to disappearance, and only traceable from its effect in taking off half the breadth of the fine and broken line presented by the ring.

In the other four quarters of Saturn's orbit, the shadows of the ring and planet present corresponding changes of appearance and position.*

The manner in which Saturn's rings move round the sun, their plane remaining always parallel to a fixed plane—or, as it is sometimes expressed, remaining always parallel to itself—may be conveniently illustrated by means of a parallel ruler. To one of the

* In Brewster's edition of 'Ferguson's Astronomy,' a table is given for the determining the proportion of the minor to the major axes of the ellipses presented by the outlines of the rings to the observer on earth. As this table has been calculated on the supposition that the plane of the rings is inclined at an angle of 31° 24′ to the plane of Saturn's orbit, whereas the true inclination is only 26° 49′ 28″; and as, further, the correction for Saturn's geocentric latitude is wrongly given, the table is not very valuable. In the 'Nautical Almanac' the elements for determining the appearance of the ring are given, at intervals of twenty days. The calculation of these elements for any intermediate day will be aided by Tables VII. and VIII. at the end of this work. (Appendix II.; see, also, explanation of the tables.)

I may notice here a slight error in Hind's valuable 'Introduction to Astronomy.' In describing the appearance of Saturn's rings when open to their greatest extent, he writes, 'The earth is then elevated 28° above the plane of the rings, and as that is the amount of inclination between the plane and the ecliptic, we view the ring as much open as it will ever be.' The earth, however, can never be elevated so much as 28° above the plane of the ring; it is clear that the angle of elevation depends mainly on the inclination of the plane of the rings—not to the ecliptic, but to the plane of Saturn's orbit. The inclination of the ecliptic to the plane of Saturn's orbit has an effect in slightly altering the angle of inclination, but not to the extent implied in the sentence quoted. The earth's orbit is so small compared with Saturn's, that, even if the plane of the ecliptic were inclined at an angle of 90° either to the plane of Saturn's orbit, or to the plane of the rings, these would never appear much more open than they do under the present arrangement.

movable rods of such an instrument let a ring of paper be fixed so that its plane is inclined at an angle of about 27° to the face of the rod; then (the other rod being held fixed) let the instrument be opened to its full extent, and closed again by carrying *on* the rod which bears the ring; then the ring will move through nearly a semicircle (a point near the centre of which will represent the sun) in the same way that Saturn's ring moves about the sun. The same motion may be more completely illustrated by such an instrument as that represented in fig. 4, plate VIII. If the handle, H, be turned uniformly in the direction indicated by the arrow, the two endless screws, S and S′, will communicate equal and uniform motions of rotation to the toothed wheels, W and W′; thus the extremities of the rigid curved bars, B and B′, will move uniformly and at equal rates round the circumferences of equal circles, bearing the bent wire, *w w′ w″*, with a uniform cranklike motion. If this wire bear a ball, *s*, circled about by a ring, *r r′*, to represent respectively Saturn's globe and ring, and a spirit lamp, L, be placed as shown in the figure, to represent the sun, then the motion of the wire will bear this miniature system about the lamp in a manner that will illustrate more clearly than any verbal description the motion of Saturn's system about the sun.* If all extraneous light be excluded, the instrument will illustrate the motion of the shadow of the rings on Saturn's globe, and the motion of the shadow of the globe on the rings. These phenomena are described in Chapter VII.

The changes in the appearance of Saturn's belts in the course of a complete revolution about the sun, correspond with the changes in the outlines of the rings. Since Saturn's equator is approximately concentric with the rings, and in the same plane, it would always appear (if it were actually a visible line traced on the surface of the planet) as part of an ellipse similar to and concentric with the outlines of the ring. Thus when the edge of the ring is turned to the observer on earth, the equator is coincident with the line pre-

* It will be seen that the lamp is placed somewhat eccentrically, so as to correspond with the eccentricity of Saturn's orbit about the sun. The wire is bent at *b* and *b′* (at unequal distances from its extremities) that it may pass freely over the wick and flame of the lamp. The wire can be readily removed, and the instrument used to illustrate the motion of the earth or of any other planet about the sun. The ball, *s*, may be made of pith; the ring, *r r′*, of card or paper.

sented by the ring. When the northern surface of the ring is visible, the equator appears as the half of an ellipse having its convexity turned southwards; and when the southern surface of the ring is visible, the convexity of the corresponding semi-ellipse presented by the equator is turned northwards. If the Saturnian latitude-parallels were actually lines on the surface of the planet, it is clear that they would appear as parts of ellipses similar to, but not concentric with, the ellipses presented by the outlines of the rings. Half of the equator is always visible, but of the parallels of latitude more or less than half will be visible according to their position on Saturn's globe, and the variations (according to Saturn's position in his orbit) in the forms of the ellipses of which they form part. Thus, when the convexities of these ellipses are turned southwards, parallels on the northern half of Saturn's globe are visible through more than half their circumferences, the visible portion of each increasing northwards, until of parallels near the north pole the whole circumferences are visible; but of parallels on the southern half of Saturn's globe less than the halves will be visible, the visible portion of each diminishing southwards, until of parallels near the south pole, the whole circumferences are invisible:* these relations are reversed when the convexities of the ellipses are turned northwards. Thus the Saturnian belts, whose outlines correspond as to their general contour with parallels of latitude on Saturn's globe, are presented with their convexities turned towards the concavities of the rings' outlines; and of the belts on the half of the disc farthest from the rings a greater part is visible than of the belts on the other half of the disc.

It is clear that when the edge of the ring is turned to us (and therefore the equator of the ring presented as a straight line), each of the poles of Saturn's globe lies on the edge of his disc. When the northern side of the ring is turned towards us, the north pole becomes visible within the edge of the disc, the south pole disappearing behind the disc; and when the southern side of the ring is turned towards us the south pole becomes visible, and the north pole disappears. To an observer placed at the centre of the

* Where the ring crosses the planet's disc, parts of parallels will of course be invisible; this effect is considered in Chapter VII.

sun, these changes would take place in a uniform and continuous manner, just as the opening and closing of the rings have been shown to do.* The north pole would appear to leave the edge of

* These changes of appearance, and therefore the changes of the planet's seasons which depend on them, may be illustrated as follows:—Let P E P′E′, fig. 3, Plate VIII., represent an oblate spheroid of crystal, or any transparent substance, P O P′ being the axis, O the centre; and suppose the spheroid to move in a circular orbit in the direction *s* O *s*′, its axis P O P′ retaining its direction unchanged throughout the motion. Then to an eye placed at the centre of motion, and referring the motions of the different parts of the spheroid to the disc presented by it, the points P and P′ will appear to move backwards and forwards along two equal lines, P C *p* and P′*c*′*p*′, both parallel to *s* O *s*′; not uniformly, but according to a simple law. Thus:—draw C O *c*′ through O, at right angles to *s s*′, to meet P *p* and P′*p*′ in C and *c*′; and with centre C draw the circle P D *p* (the lower semicircle is omitted in the figure to avoid confusion); then if P C D be taken to equal the angle that (at the time considered) has been swept out by the spheroid about the centre of motion, from the position represented in the figure, and D *d* be drawn perpendicular to P *p*, *d* will be the apparent position of the pole P at that time, and the line *d* O *d*′ through O will meet *p*′P′ in *d*′, the apparent position of the other pole P′; so that *d* O *d*′ is the apparent position of the axis at that time. Again, the outline presented by the spheroid will throughout the motion appear to touch two lines, Y A′Y′ and *y a y*′, parallel to and equidistant from *s* O *s*′. Thus the eccentricity of the disc's outline will appear continually to diminish as the pole moves to C, thence to increase as the pole moves to *p*, and to go through similar changes as the pole returns to P. The point O (referred to the disc's outline) remains fixed throughout. [In reality, of course, the motions of the points P and P′ (referred to the spheroid) are not in straight lines but circles, the motions of the axis P O P′ (referred to the spheroid) carrying that line over the surface of a double cone, whose semi-vertical angle is C O P. While the pole P is moving from P to *p* it is seen *through* the spheroid, and P′ is on the nearer hemispheroid; in the following half-revolution these conditions are reversed.] Again, let circles having P, and P′ as their poles (that is, latitude-circles), be traced on the surface of the spheroid. For instance, let circles A A′, *a a*′ corresponding to arctic circles (determined by the points A′, *a*′ in which Y Y′ and *y y*′, tangents parallel to *s* O *s*′, meet the ellipse P E P′E′) be so traced; and again, let circles T T′ and *t t*′ corresponding to the tropics (determined by the points T, *t*′, in which Z Z′ and *z z*′, tangents perpendicular to *s* O *s*′, meet the ellipse P E P′E′) be so traced; and, lastly, a circle E E′ corresponding to the equator of the spheroid. Then these circles, which, in the position of the spheroid represented by the figure, appear as straight lines, will in other positions of the spheroid appear as ellipses, always touching the outline of the spheroid's disc, one part of each such ellipse (up to the points of contact with the disc) being seen *through* the spheroid, the other upon the nearer hemispheroid; the centres K, L, L′ and K′ will appear to move backwards and forwards along the lines K *k*, L *l*, L′*l*′ and K′*k*′, (just as the points P and P′ move along P *p* and P′*p*′, so that when these two poles are respectively at *d* and *d*′, the four centres above-named will be found respectively at *e*, *f*, *f*′ and *e*′); and, lastly, if lines parallel to *s* O *s*′ be drawn through the points A, T′, E′, *t*′, T, E, *t*, and *a*′, the ellipse into which the circle A A′ is projected will throughout appear to touch the line Y Y′ and the parallel through A, the corresponding ellipse of the circle T T′ will appear to touch the parallels through T and T′, and so the ellipse corresponding to each of the circles will appear continually to touch the two parallel lines corresponding to it. From the

the disc as the northern face of the rings began to appear, to travel farther and farther from the edge as the rings became more and more open, and afterwards, as the rings closed, to approach the edge in the same continuous manner—the southern pole being throughout invisible. Then, as the southern side of the rings became visible the southern pole would appear, first on, and then within, the edge of the disc (the northern pole disappearing), and would make a similar advance and retreat. The ellipses that would be presented by the parallels of latitude on Saturn's disc, if these were visibly traced on his surface, would open and close in the same continuous manner. But to the observer on earth the poles would not appear to advance and retire continuously. Just as we have seen that, although, viewed from the earth, the rings, in any complete synodical revolution have been opening or closing, yet for an interval in each synodical revolution appear to reverse those movements, so the poles, while on the whole they present themselves within the disc of the planet with alternations corre-

last property it is clearly seen that the ellipses corresponding to the circles A A′ and *a a′*, always meet the outline of the spheroid's disc at the same points, respectively, as the lines Y Y′ and *y y′*.

The figures of Plate XIII. illustrate some of these properties; they indicate (among other matters) the changes of Saturn's appearance during one quarter of a revolution about the sun, and as seen by a spectator supposed to be placed at the sun's centre. Only the visible parts of the ellipses are introduced in these figures.

Fig. 2, Plate IX., represents the autumn (I.), winter (II.), spring (III.), and summer (IV.) phases for the northern (supposed the upper) hemisphere of a planet; or the spring, summer, autumn, and winter phases, respectively, for the southern hemisphere. They are placed in order, from right to left, that being the direction in which a planet would appear to move to an observer placed at the sun's centre.

All the propositions contained in the first paragraph of this note may be very easily proved. For instance, to show that the ellipse presented by a latitude-parallel would always appear to touch two lines parallel to *s* O *s′*, and drawn at fixed distances from O:—Since the spheroid's axis moves parallel to itself, the plane of each latitude-circle is carried parallel to itself, the centre moving parallel to the plane in which the centre of the spheroid moves; thus each latitude-circle may be conceived as sliding between two fixed planes, parallel to each other and to the plane of motion of the spheroid's centre; these planes meet the apparent disc of the spheroid (wherever it may be) in two parallel lines at fixed distances from the centre of the disc, and the circle touching those planes must therefore be projected into an ellipse touching those lines. And with similar simplicity the other propositions may be proved. They will appear identical propositions to the mathematician. It must be remarked that the distance of the spheroid from the observer is supposed to be very great compared with the dimensions of the spheroid.

sponding to the Saturnian year, yet have another set of movements corresponding to our own year. Thus at times they appear to advance from or retire towards the edge of the disc more rapidly than they would if the earth were the centre of Saturn's motion; at others they reverse their movements for intervals of several months, so that the advance or retreat (on the whole) of either pole takes place in an oscillatory manner. The belts on Saturn's surface appear to the observer on earth to open and close precisely as the rings have been shown to do.

Owing to the immensity of Saturn's orbit compared with the orbit of the earth, he never presents a gibbous appearance. It will be obvious from an inspection of fig. 3, Plate VI., that in the case of Mars (the only superior planet that ever presents a gibbous appearance), the orbits of the earth and planet are so related that the line of sight from the earth to Mars may be inclined at a large angle to the line from the sun to Mars,* and thus Mars may present to the observer on earth a considerable portion of his darkened hemisphere; when this is the case he appears gibbous. The corresponding angle, however, in the case of Saturn, is always small; it obviously attains its greatest value for each synodical revolution when Saturn is in quadrature, and for different synodical revolutions such maximum value will vary with Saturn's distance from perihelion; but even when Saturn is in perihelion at the moment of quadrature this angle is less than 6° 23′. At such a time a portion of Saturn's darkened hemisphere is actually turned to the observer on earth, and a portion very considerable so far as absolute extent of surface is concerned: † yet the alteration in the figure of Saturn's disc is altogether inappreciable, even on applying the most exact micrometrical measurement. That diameter which is most affected (that is, the diameter through the widest part of the darkened lune,) is not

* This angle may be as great as 46° 45′ if Mars is near perihelion at the time of either quadrature. In this case the breadth of the dark part of his disc (that is, the greatest width of the lune-shaped invisible portion of the disc) is about $\frac{3}{20}$ths of the diameter of the disc. In November, 1860, Mars presented this appearance, having passed his perihelion on September 16th, 1860, and being in quadrature following opposition on November 23rd.

† The extent of the darkened part thus turned towards us is considerably greater than the whole surface of our earth.

diminished by $\frac{1}{320}$th part of its length. Now this diameter subtends an angle of about 17″ when Saturn is in quadrature; thus the amount by which it is diminished corresponds to an angle of little more than $\frac{1}{20}$th of a second.

Since the seven interior satellites of Saturn move in orbits very nearly concentric with the rings, and in the rings' plane, it is clear that if those orbits were visible throughout their extent, they would appear as rings of light very nearly concentric with the rings, and similar to them in shape. These orbits would therefore appear to an observer at the sun's centre to open and close uniformly, while to the observer on earth they would appear to open and close in an oscillatory manner. And though the plane of the outer satellite is inclined to the plane of Saturn's orbit at a different angle than the ring's plane, and has its line of nodes in a different direction, yet the changes in the appearance of this orbit (supposed visible throughout its extent), would be similar to those of the other orbits. The investigation of the changes in the appearance of the rings, which is directly applicable to the orbits of the seven inner satellites, is applicable, *mutatis mutandis*, to the orbit of the outer. Like the ring this orbit opens out to the observer on earth in an oscillatory manner; but the extent to which it opens is different, and it does not attain its maximum opening in each successive synodical revolution, or its absolute maximum opening in each semi-sidereal revolution, at the same time as the ring.

Owing to the inclination of the planes of the eight orbits to the plane of Saturn's motion, eclipses, occultations, and transits are less frequent among the Saturnian satellites than among those of Jupiter. The latter revolve very nearly in the plane of Jupiter's orbit, and therefore always appear to lie very nearly in a straight line through the centre of Jupiter's disc: thus they are occulted, eclipsed, and transit his disc at nearly every revolution. On the other hand, Saturn's satellites move in orbits which, if visible throughout their extent, would in general appear as ellipses, whether viewed from the centre of the sun or from the earth: it is only when such ellipses, viewed from the sun's centre would be partly hidden by Saturn's disc, that eclipses of the corresponding satellites can take place; and only when such ellipses, viewed from the earth would be partly hidden, that occultations or transits of the corresponding satellites can take place. Now the mean distance from Saturn's centre at which the outer satellite revolves is no less than 2,208,720 miles, and on the scale of the figures in Plate I. this distance would be represented by a line more than five feet long: thus, if the orbit of this satellite were visible throughout its extent, it would appear as an ellipse whose major axis would be ten feet long; and it is clear that a very small elevation of the point of view above the plane of the orbit would make the minor axis of such an ellipse greater than the apparent diameter of Saturn's disc.* Hence it is only when the plane of the orbit passes through, or very near the sun, that this satellite can be eclipsed; and only when that plane passes through, or very near the earth, that an occultation or transit can occur. Further, as the period of this satellite is no less than 79 days, and as it is only for a brief interval in each revolution that the satellite is near Saturn's disc, the chance of an eclipse, occultation, or transit occurring is still further diminished. Saturn may pass the points of his orbit at which these phenomena are possible while the satellite is near its easterly or westerly elongation, and fourteen years must then elapse before Saturn is again so situated that an eclipse, occultation, or transit is possible. Thus these phenomena occur very seldom, and as they may take place by daylight or in weather unfavourable for observation, centuries may elapse before any one of them is actually visible from the earth.

Similar remarks apply with nearly equal force to Titan and Hyperion. The latter satellite is hardly ever visible owing to its minuteness. Eclipses, occultations, and transits of Titan, though uncommon, happen occasionally.† As already mentioned, an eclipse of Titan, and also the transit of his shadow across Saturn's disc, were observed by Mr. Dawes in the years 1861–1862, when the

* If the line of sight were inclined 58′ to the plane of the orbit, the minor axis of the ellipse would exceed Saturn's apparent diameter.

† The mean distance of Titan from Saturn's centre is 759,990 miles. On the scale of Plate I., Titan's orbit (if it were visible throughout its extent) would appear as an ellipse having a major axis 3½ feet long; and if the line of sight were inclined to the plane of Titan's orbit in an angle of about 3° 11′, this ellipse would be altogether clear of Saturn's disc.

plane of the ring (which is, to the sense, the plane of Titan's orbit) passed very near both to the sun and the earth. Eclipses, transits, or occultations of Titan are only possible when the plane of his orbit about Saturn is so situated.

Eclipses of the remaining satellites are not uncommon occurrences. They increase in frequency as the distances, and consequently the periods, diminish. The two inner satellites very frequently transit Saturn's disc, and are as frequently eclipsed or occulted. These phenomena are not very often observed, however, the satellites themselves being so difficult to detect.

The eclipses of Saturn's satellites may be considered in another manner. Since the sun's diameter is 854,928 miles, Saturn's mean distance from the sun 874,321,000 miles, and his mean diameter 68,965 miles, it may easily be calculated that the cone * of total shadow cast by Saturn extends to a mean distance of about 76,718,000 miles. The axis of this cone is not in general coincident with the orbit-planes of Saturn's satellites, but passes on one side or the other of those planes. The intersection of each orbit-plane with the conical surface of the shadow is therefore an ellipse,—that part of each such ellipse which lies beyond Saturn being in darkness. If the orbit corresponding to any such ellipse (that is, the orbit whose plane meets the surface in such ellipse) lies without the darkened part of the ellipse, the satellite cannot be eclipsed; but so long as the orbit falls within the ellipse, the satellite is eclipsed at each revolution. The planes of the seven inner satellites are (to the sense) coincident with the plane of the ring, and parts of the darkened portions of the ellipses in which the plane of the ring intersects the conical shadow of the planet are represented in the eight figures of Plate XII. These figures correspond to Saturn's positions at eight different periods:—thus, fig. 1 corresponds to Saturn's position when the plane of the ring passes through the sun; fig. 8 corresponds to his position one quarter of a Saturnian year later, or when the sun is at its greatest possible elevation

* Since neither Saturn nor the sun is perfectly spherical, the space beyond Saturn which (neglecting the refraction of Saturn's atmosphere) receives no light from the sun, is not a cone, but is bounded by a surface of a less simple form.

above the plane of the ring; the six intermediate figures correspond to his position at six intermediate epochs separated by equal intervals of time.* If these ellipses were completed, and the orbits of the seven inner satellites traced at their proper distances from Saturn's centre on the scale of the figure, it would be found that the orbits of all the seven satellites intersect the shadow in fig. 1 (which extends indeed more than eighty times as far as the orbit of Hyperion); the orbits of the four inner satellites intersect the ellipse of fig. 2, the orbit of the fifth just passing clear of it; the orbits of the first two satellites (Mimas and Enceladus), intersect the ellipse of fig. 3, the orbit of Dione being not very far beyond it; the orbit of Mimas alone intersects the ellipse of fig. 4; and all the orbits lie beyond the ellipses of the remaining figures. As about 384 days elapse before the ellipse changes from the form shown in any of the figures to that shown in the following figure, it appears that for about a year after the passage of the plane of the ring through the sun's centre, the fifth satellite is eclipsed at each revolution; the same must have happened for a similar time *before* the plane of the ring reached the sun's centre:—thus, twice in every Saturnian year the fifth satellite is eclipsed at each revolution (that is, every $4\frac{1}{2}$ days), during an interval of about two of our years. The corresponding intervals increase as we proceed successively to the fourth, third, second, and first satellites. In the case of Mimas each such interval contains about seven years.

Amongst the numerous eclipses of the different satellites in the course of a Saturnian year, several must be partial, the satellite merely grazing the shadow of the planet. Such phenomena, however, would hardly be detected by the observer on earth, even in the case of Titan, still less in the case of the smaller satellites.

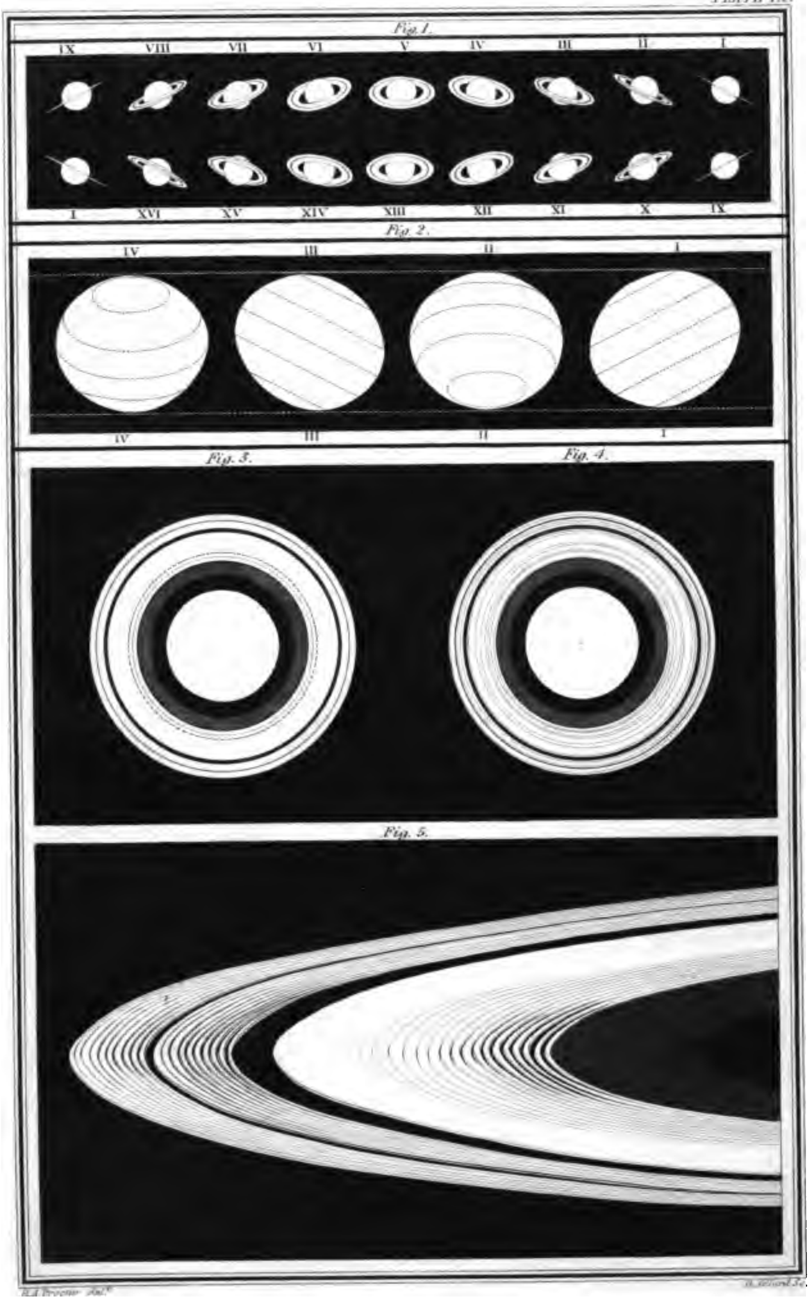
The eclipses of the outer satellite might be treated in a similar manner, replacing the plane of the ring by the orbit-plane of that satellite. It would be found that the satellite would pass through the shadow only for a few days before and after the passage of the orbit-plane through the sun's centre.

Occultations of the satellites by the ring, and transits of the

* Each interval is therefore about a seventh part of the quarter of a Saturnian year, or about 384 days.

satellites across the rings can happen, in the case of the seven interior satellites, only when the plane of the ring passes very nearly through the earth. At this time, as already mentioned, the satellites may be detected travelling, 'like golden beads on a wire,' along the line of the ring; they are therefore either partially occulted by the ring or transiting it. When the plane of the ring passes through the sun the satellites would present the same appearance to an observer in the sun, and therefore must, at certain parts of their orbits, be partially eclipsed by the ring, or partially eclipse the ring's edge. Such partial eclipses would not be easily detected, however, by the observer on earth. The outer satellite, whose orbit is inclined to the plane of the ring, may be eclipsed or occulted by, or transit, the ring: these phenomena, however, must be more rare even than the eclipses and occultations of this satellite by the planet.

PLATE IX.

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# CHAPTER V.

## NATURE OF THE RINGS.

If we consider the vast size and singular conformation of the Saturnian rings — appendages altogether unique in the solar system, and, so far as is known, in the universe itself—it will not appear surprising that they should have been the subject of many speculations and hypotheses ; or that the most wild and fanciful ideas should from time to time have been broached concerning them. Thus, Maupertuis considered that the tail of a comet passing near Saturn had been attracted from its course by the planet's mass, and has since continued to circle as a ring around him. It is singular that Buffon, who had himself conceived the fanciful theory that planets are portions of the sun's mass which have been struck off by passing comets, refused to accept Maupertuis' hypothesis of the cometary nature of Saturn's ring-system. One would have thought that in such a view he would have found a confirmation of his favourite theory. He might have argued, that, as it will happen that a strong wrestler, overbearing his foe, may yet be forced to follow him in his fall, so the comet that had dashed from the sun's globe the mighty mass of Saturn, and carried it on through space to so vast a distance from the parent orb, was unable to free itself from its massive burden, and gradually cooling, formed the vast ring that still circles about Saturn. Such an explanation is altogether unphilosophical, it is true, but not more so than the theory on which it is founded. The explanation actually offered by Buffon was different, however. He considered that Saturn's equator originally extended as far as the outer edge of the ring, and that the equatorial regions have been thrown off by centrifugal force, the rest of the planet gradually contracting to its present dimensions. Mairan supposed that the rings are the remains of outer shells about Saturn, broken up by some vast convulsion.*

Until very lately, whatever explanations might be offered as to the first formation and original state of the rings, the opinion that the system is at present solid and continuous was universally accepted. Such an opinion seems, at first sight, to be countenanced by the continuous appearance of the rings, and their general permanence of form. On closer examination, however, it will be found that the most serious difficulties attend the supposition of the solidity of the system.

In the first place, it has been shown in Chapter III. that the rings frequently exhibit traces of division, but that such traces are not permanent, sometimes varying in position, at others disappearing altogether. It is not easy to explain these changes on the supposition that the rings are solid. The approach of two rings, originally concentric, might, it is true, remove all trace of division at the point of approach, or in its immediate neighbourhood; but a wider gap would thus be left at the opposite part of the division's circumference, and the trace of division thus disclosed would be at once recognized as forming part of the division first seen,—that is, as belonging to a particular circle concentric with the great division:—on the contrary, the traces of division seen at different times belong to distinct circles. It is still more difficult to explain the appearance of non-permanent mottled or dusky stripes concentric with the rings, on the supposition of the solidity of the system. A division between the rings, whether permanent or not, allowing the dark sky beyond to be seen, should appear perfectly black like the great division; so that mottled or dusky stripes would seem to indicate only semi-transparency in those parts of the rings along which they are traceable. If we accepted such an explanation, we should have to account for the following mysterious conditions in Saturn's ring system:—In solid flat rings non-permanent concentric divisions open at different times along different circles,

* The theory that the earth itself is composed of several crusts liable to separate destruction was maintained by several distinguished astronomers of the eighteenth century; and, earlier, by Kepler.

while variable concentric bands become at different times semi-transparent; again,—the divisions close, and the transparent bands resume their opaqueness, after variable intervals.

It may be urged, however, that these lines are not necessarily traces of division; that ranges of hills upon the rings would throw black shadows, while rough districts would appear mottled or dusky, like the stripes seen by Mr. Dawes. Yet how inexplicable in either case that such irregularities should lie always in circular arcs concentric with the rings! And to what cause should the non-permanence of these irregularities be ascribed? Why should they disappear along one circle to be thrown up presently along another?

The presence of an atmosphere bearing clouds over the surface of the rings, and thus concealing the traces of division, may appear, at first sight, a plausible explanation of the phenomena we are considering. But the disposition of such an atmosphere necessary to produce the observed effects would be so artificial that, on this account alone, we might well be permitted to reject the supposition; and further, the cloudy regions imagined should at least, one would suppose, be as distinctly visible as the zones on Saturn's disc; indeed, lying over a flat surface, their outlines would probably be more distinct than those of Saturn's belts. But, except the permanent difference of tint observed in the two rings, the telescope has revealed no appearances that could be attributed to the existence of an atmosphere surrounding the rings; and even if that difference of tint be assigned to atmospheric causes, yet, being permanent, it does not avail to explain the variations we are examining. It might, indeed, be urged that the mottled lines on the rings indicate the presence of an atmosphere; that they are either clouds or breaks in the cloudy envelope of the rings; or, perhaps, the shadows of clouds themselves invisible. Their appearance is not favourable, however, to these suppositions; nor are such long narrow circular arcs the forms into which we should expect cloud-bands to arrange themselves, or the openings in clouds to appear, under the conditions to which the surface of the system is subject. A uniform distribution of light and heat must prevail over the whole of that surface except where the vast shadow of the planet actually falls, or has lately past, and the disturbing effects of this shadow (see Plate XII.) must operate across the breadth, or a great part of the breadth, of the rings, not along narrow arcs concentric with their edges. The argument against the solidity of the rings drawn from the varying traces of divisions in the system appears, then, to be little if at all impaired by the assumption that the rings are surrounded by a varying atmospheric envelope. Such an explanation is altogether inapplicable to the objections we have next to consider.

We have seen that one of our most accurate observers has seen traces of division in the dark ring, which also appears at times to be separated from the inner circumference of the neighbouring bright ring by gaps of considerable length. These appearances may be passed over, or simply viewed as confirmations of the argument drawn from similar non-permanent traces in the bright rings; but there are phenomena connected with the dark ring which appear altogether inexplicable on the supposition that this formation is solid. In the first place, there is the singular circumstance already recorded that this ring was not visible seventy years ago through one of the most powerful telescopes ever constructed; whereas since its discovery it has become gradually more and more conspicuous, until in 1856 it was visible in telescopes of very moderate power. Secondly, as already stated, this ring is transparent, and the edge of the planet's disc seen through it is not distorted. If the substance of the ring were a transparent solid (or even fluid) possessing properties similar to those of all transparent substances, solid or fluid, with which we are acquainted on earth, the edge of Saturn's disc seen through it would be distorted by the refraction of light passing through such a medium. Too much stress, however, must not be placed on this argument; for if the plane faces of the dark ring are parallel the distortion would be very small (its amount depending on the thickness of the ring) and probably not traceable even with the most powerful telescopes yet constructed. The great, and I think unanswerable, arguments against the solidity of Saturn's dark rings, are drawn from the facts, that so vast a formation should be transparent, that its transparency should once have been such that it was mistaken for a belt on the body of the planet, and finally that it should be continually growing more and more opaque, so that it becomes more clearly visible every year.

Let us next consider the dimensions of the rings. We have seen that the thickness of the system is very small compared with its other dimensions. A small ring of iron constructed on the scale of one of the rings, or even a ring of iron whose width should be proportioned to that of the complete system of rings, would be a flimsy body, easily bent or broken. But in considering the strength of bodies constructed of any substance, it is not sufficient that we should know their *proportions*, we must know also the *scale* on which they are constructed. Thus, if an engineer, who proposed to erect a bridge of iron of given length and to support a given weight, should construct a model a few inches long of the same kind of iron, and should determine the proportions of the bridge itself from the proportions he found necessary to support a proportionate weight in the model, he would probably erect a bridge hardly strong enough to support its own weight. The larger the scale on which a model in iron of the rings of Saturn should be constructed, the flimsier (in proportion to its size) it would become. If, then, it were possible to imagine a ring of iron constructed of the same dimensions as the Saturnian system of rings, it would be utterly unfit to bear the immense strains to which, as we shall see, these rings are subjected. If, further, we imagine such a ring divided into numerous concentric rings, the system thus formed would be still less fit to bear strain or pressure. But this is not all. We have arrived at the conclusion that the rings are about 100 miles thick, from the supposition that the mean density of the substance of which they are composed is equal to the mean density of Saturn's mass, or ·75.* Now the density of forged iron is about 7·7, or more than ten times as great as Saturn's mean density; and it is not probable that any substance (unknown on earth) could have the same strength and tenacity as iron with a much smaller density—say with a density less than 3·75, or five times that of Saturn. If we assume the mean density of Saturn's rings to be 3·75, then, instead of arriving at the conclusion that the thickness of the system is 100 miles, we deduce a thickness of only 20 miles. With such a thickness a model of the rings on the scale of Plate I.

* The density of water being as 1.

would be thinner than tissue-paper. Undoubtedly a solid iron system of such proportions, and of such vast absolute dimensions, 'would be not only plastic, but semi-fluid, under the forces it would experience.' *

The change that has taken place in the dimensions of the rings during the last two hundred years affords a still stronger argument against the solidity of the system. We have seen that the measurement of the width of the ring given by Huygens or Pound differs considerably from that given by Herschel, and that again from the results of the most trustworthy modern measurements. We cannot, perhaps, place much reliance on the absolute dimensions of the ring or planet determined by the earlier observers. Owing to the immense distance of Saturn from the earth, the determination of these dimensions is a task of great difficulty even to observers using the wonderfully delicate instruments of the present day. Far more reliance, however, can be placed on proportional measurements, and only such measurements are involved in the question; unless we suppose—which will hardly be considered probable—that the dimensions of Saturn's globe have undergone alteration during the interval we are considering. All the measurements that have been taken of the rings, from the time of their first discovery to the present day, have been carefully revised and examined by M. Otto Struve. He not only considered the result obtained in each case, but the method of measurement applied, the nature and quality of the instrument used, and the skill and general trustworthiness of the observer. He arrived at the following conclusions:—The width of the system of *bright* rings is increasing, gradually but continuously, by the approach of its inner edge towards Saturn's equator; both the rings have partaken in this change, but the inner ring has increased in width more rapidly than the outer. The dark ring, as already stated, has increased considerably in width during the comparatively short period that has elapsed since its discovery.

The increase in the width of the system of rings must, of course, have been accompanied by a corresponding decrease in

* 'Essay on the Stability of the Motion of Saturn's Rings,' by J. Clerk Maxwell, M.A.

thickness. Let us examine the extent of both these changes in Herschel's time and in our own. For simplicity, we may treat the system of bright rings as a single ring, and neglect all consideration of the dark ring. The outer diameter of the rings is 166,920 miles, the equatorial diameter of Saturn 72,250 miles. Now, the measurement of Huygens made the width of the ring equal to the breadth of the space between the ring and planet, and the measurement of Pound made the width of the ring somewhat less. Taking the first measurement (as the least favourable to our case), it appears that the width of the ring was 23,667 miles in Huygens' time; and it is easily calculated that the extent of either flat surface of the ring was upwards of 10,652,100,000 square miles: the outer dotted line in fig. 3, Plate IX., represents the inner edge of the bright ring at this epoch. Again, Herschel found that the width of the ring, in his day, bore to the breadth of the space between the ring and the planet's equator the proportion of 5 : 4; this gives to the ring a width of 26,297 miles, and a surface of 11,617,500,000 square miles: the inner dotted line in fig. 3, Plate IX., represents the inner edge of the bright ring at this time. Lastly, the best modern measurements give to the ring a width of 28,300 miles, and therefore a surface of 12,324,300,000 square miles. Thus it appears that in the first interval of about 120 years, the absolute increase in the width of the rings was 2,630 miles; and in the second interval of about 70 years, the rings increased in width 2,003 miles. The average annual rate of increase in the first interval is nearly 22 miles, in the second nearly 29 miles, so that the *rate* of increase in the width of the ring would appear to be itself increasing. Further, it appears that the surface of the ring was greater, and therefore the thickness of the ring less, in Herschel's time than in Huygens', in the proportion of 116,175 : 106,521 (or about 12 : 11); and in our own time the surface of the ring had increased, and the thickness of the ring therefore diminished, in the proportion of 123,243 : 116,175 (or about 35 : 33) as compared with those dimensions in Herschel's time, and in the proportion of 123,243 : 106,521 (or about 8 : 7) as compared with the corresponding dimensions in Huygens' time. Thus, if we assume the present mean thickness of the rings to be 100 miles, it appears that in Huygens' time the rings must have had a mean thickness of 114 miles, and have been narrower than at present by no less than 4,633 miles. It is hardly necessary to point out the difficulty of reconciling these changes of form with the supposition that the formation is solid.

Let us next discuss the results of more exact and systematic inquiries.

The question whether a solid flat ring could remain in equilibrium, under any circumstances, about a vast central orb like Saturn, attracting according to the law of gravity, was first discussed by Laplace, towards the end of the last century. This celebrated mathematician established three important points, but contented himself with offering an hypothesis respecting the stability of the system.

Laplace first proved that such a ring must rotate about the central globe. The enormous attractive force of an orb so vast as Saturn must in some way be counterbalanced. When a satellite revolves about a planet, the attraction between the planet and satellite is continually used up—so to speak—in changing the direction of the satellite's motion. If that motion were suddenly checked, the satellite would approach the planet; if the motion were stopped, the satellite would fall on the planet. Now, every portion of the ring is subjected to the immense attractive force of Saturn's mass, and also to the attractive force (by no means insignificant) of the rest of the ring. The first force drags the ring towards the common centre of the ring and planet. The second force has a different effect; it operates to drag the outer parts of the ring inwards, the inner parts outwards: the influence of this force would chiefly lie in its effect in weakening the ring, and thus rendering it more than ever unfit to resist the tremendous influence of the first force. Thus, if the ring were not rotating it would inevitably give way under these forces, and crumbling up—like an arch beneath a load too great for its strength—would fall in ruins about the planet's equator.

We are led immediately to Laplace's second conclusion. At what rate should the ring revolve? That all strain should be removed, each particle of the ring should move as if it were a free satellite revolving about Saturn: the greater part of the strain would be removed if each particle of the ring revolved at the rate with which a satellite at the same distance from Saturn would revolve in a circle about him. It is clearly impossible that either kind of motion should be found in each particle of a solid flat ring: if the outer parts of such a ring had the rate of motion corresponding to the second case, the inner parts would be revolving too slowly, and be dragged inwards; if, on the other hand, the inner parts had such a rate of motion, the outer parts would be revolving too fast, and be whirled outwards. It is clear that the supposition most favourable to the existence of the ring is, that it should revolve at the rate due to a satellite at the mean distance of the particles of the ring from Saturn's centre. But, even in this case, the outer parts of the ring have too great, the inner parts too small, a velocity. Thus, the outer parts, if not constrained by the cohesion of the ring, would travel in a larger orbit than that in which they actually move; while the inner parts would seek a smaller orbit. Now the cohesion of a flat ring, of the dimensions of Saturn's ring, would be altogether insufficient to resist these tendencies. The inner and outer rims of such a ring would be stripped off, probably in irregular fragments, and proceed to describe eccentric orbits. Such effects are due to the width of the system; the cohesion of a narrow ring would be sufficient to resist the comparatively small strains to which the parts of such a ring would be subjected. Hence Laplace concluded that Saturn's ring must be divided into several concentric rings. He calculated the rate of motion due to each part of such a system, and his conclusions were soon confirmed, so far as the outer rim was concerned, by the observations of the elder Herschel.

Laplace next proved that a perfectly uniform solid ring, of moderate width, might rotate for ever around a perfectly uniform planet, if subjected to no disturbing influences; but that if such a ring were once disturbed, however slightly, equilibrium would never afterwards be restored. The approach of one part of the ring towards the planet would cause a preponderance of attraction on that part of the ring; thus it would continue to approach the planet with constantly increasing velocity, and would finally fall upon the planet's equator. Now, in the first place, Saturn's ring is subject to numerous disturbing influences: even if we suppose Saturn's globe and the ring itself free from irregularities (which, however, is utterly incredible), yet the attractions of the satellites constantly varying in position, the attractions of the different members of the solar system, of the sun itself, of stars, of comets,—all these are disturbing influences, and any one of them would be sufficient to destroy the balance of the ring and effect its destruction. But, secondly, an eccentricity in Saturn's position with respect to the ring (due no doubt to the above-named causes) is not unfrequently palpable to observation. Since the destruction of the ring has not resulted, as must have happened if the ring were solid and uniform, it follows that the ring, if solid, is not uniform. I say the *destruction* of the ring, because it is clear that when once the ring had assumed an eccentric position, the proper rate of motion to prevent destruction would be different for different parts of the ring, and the actual motions of the ring (rotating, and falling towards the planet) could no longer give to each part its just rate of motion—some parts would be moving too fast, others not fast enough; and, finally, when the eccentricity of the ring's position became sufficiently great, the ring would be broken into fragments—like an arch pressed beyond its strength, inwards at some points and outwards at others.

Laplace lastly considered the case of a solid non-uniform rotating ring. He did not, however, subject this part of the question to the same searching mathematical inquiry that he had applied to the others. He contented himself by suggesting that the irregularity of such a ring, properly disposed, and combined with an eccentricity of position, might prevent the destruction of the ring. He considered that the breadth of any ring composing the system might vary in different parts of its circumference, so that the centre of gravity might be at a considerable distance from the centre of figure; and that the centre of gravity of such a ring might revolve about Saturn somewhat in the manner of a satellite, and with a period equal to that of the ring. There was one obvious difficulty in the way of this supposition. Under the different disturbing influences to which the rings are subjected, they would be liable to leave the plane of Saturn's equator, the plane of each ring moving with a slow precessional movement about Saturn. Now this movement would be different for each ring, and thus the rings would no longer be found in one plane, and the system no longer present the appearance actually observed. Laplace considered, however, that if equilibrium could be secured to each ring of the system, the attraction of Saturn's bulging equatorial regions might be sufficient to overcome all such disturbing influences, and to compel all the rings to move in a single plane very nearly coincident with the plane of Saturn's equator. Thus it appeared to Laplace that the system of rings was probably composed in some such manner as that indicated in fig. 4, Plate IX.,—the bounding outlines of each ring being necessarily circular, since otherwise the motion of the ring would be impeded by collisions with its neighbours. But he saw that it was not sufficient for the stability of the system that the bounding outlines of each ring should be non-concentric circles. Such an arrangement would leave the centre of gravity too near the centre of figure of the ring.* He conceived that the centre of gravity might be thrown to a sufficient distance from the centre of figure, either by variations in the density and thickness of the ring, or by irregularities on the surface. This view was confirmed by Herschel's determination of the rotation of the outer ring. We have seen that he effected this determination by watching the motions of certain bright points which might be supposed to be irregularities upon the surface of the ring.

The conclusion arrived at by Laplace was for more than half a century accepted by astronomers as the only possible interpretation of the stability of the Saturnian rings. Of the value of Laplace's investigations of this, as of so many other problems of difficulty, there can be no question; yet the result he arrived at is unsatisfactory. In the following observation, Professor Nichol estimates Laplace's views at their just value:—'Worthy

* With such an arrangement (the thickness and density of the ring being uniform throughout) the centre of gravity could never be so far from the centre of figure as half the radius of the outer boundary, whatever the proportion of the radii of the two boundaries, and whatever the distance between their centres.

of every admiration amidst a display of mechanical toys, such hypotheses rarely constitute essential parts of the vast and simple arrangements of nature.'

The discovery of the dark ring roused new inquiries. In 1851, Professor Pierce, of America, examined the second point established by Laplace—the narrowness of the rings composing the system. He found that, if the rings are solid, the breadth of each must be much smaller than even Laplace had imagined, so that the number of rings must be considerable. The elements of confusion and insecurity that must exist under such an arrangement are self-evident.

On March 23rd, 1855, the University of Cambridge announced that the stability of the motions of Saturn's rings had been chosen as the subject of the Adams Prize Essay; and in 1857 the prize was adjudged to Mr. J. Clerk Maxwell. Taking up the question of a solid ring where it had been left by Laplace, Mr. Maxwell finally disposed of it. He showed that the irregularity of each ring should be such as to bring the centre of gravity more than nine times as far from the lightest as from the heaviest side of the ring; and that the eccentricity of position of each ring must be such that a system composed of such rings would present an appearance altogether different from that of the actual system. He showed, further, that even with such an arrangement the slightest cause would be sufficient 'to destroy the nice adjustment of the load, and with it the stability of the ring.' We have also seen that a solid ring very eccentrically placed would be broken into fragments.

We are compelled, then, finally to reject the idea that the system is solid.

The appearance of continuity presented by the rings leads next to the supposition that they may be fluid. The hypothesis seems at first sight an inviting one. The variations in the form of the system, the temporary divisions in the bright rings, and the transparency of the dark ring, no longer appear to offer insuperable difficulties. Yet the notion of an isolated ocean of such vast dimensions, and poised in so artificial and apparently precarious a manner, is not one that would be readily accepted save as a resource against the still more serious objections to the solidity of the formation. And further, if we accept some such view of the development of the solar system as that embodied in Laplace's Nebular Theory (and the arguments in favour of such an hypothesis appear irresistible),* we must place the formation of these rings in point of time between that of the satellite nearest to Saturn and that of the planet itself. As there is no reason for supposing either of these bodies to be otherwise than solid, we have at least a negative argument against the fluidity of the rings. But the strict examination, by Professor Pierce and Mr. Maxwell, of the stability of a system of continuous fluid rings, forces us to reject altogether the idea that the Saturnian rings form such a system. The various disturbing attractions to which the rings are exposed would inevitably lead to the formation of waves, under whose influence the fluid rings would be broken up into fluid satellites.

We are compelled, then, finally to assume that the continuous appearance of the rings is not due to real continuity of substance. The sole hypothesis remains that the rings are composed of flights of disconnected satellites, so small and so closely packed that, at the immense distance to which Saturn is removed, they appear to form a continuous mass.

An *à priori* argument in favour of such a supposition may be drawn from analogous instances in the solar system. In the zone of asteroids we have an undoubted instance of a flight of disconnected bodies travelling in a ring about a central attracting mass. The existence of zones of meteorites travelling around the sun has long been accepted as the only probable explanation of the periodicity of meteoric showers. Again, the singular phenomenon called the zodiacal light is, in all probability, caused by a ring of minute cosmical bodies surrounding the sun.† In the Milky Way and in the ring-nebulæ we have other illustrations of similar arrangements in nature, belonging, however, to orders immeasurably vaster than any within the solar system.

* See Appendix I., note B, Laplace's Nebular Theory.

† It has been suggested that the appearance of the zodiacal light in equatorial regions may be explained by supposing it to be a ring of minute satellites, surrounding the earth. The investigations in Chapter VII., which may be applied, *mutatis mutandis*, to a ring and globe of any dimensions, prove, however, that the zodiacal light cannot be due to such a cause, the appearance of the meteor in high latitudes being altogether different from that which would be presented by a ring surrounding the earth.

Let us consider in what light the difficulties met with when we supposed the rings to be solid and continuous, appear on the hypothesis that the system is composed of disconnected satellites.

The temporary divisions and mottled stripes are easily explained. It is conceivable, for instance, that the streams of satellites forming the rings might be temporarily separated along arcs of greater or less length by narrow strips altogether clear of satellites, or in which satellites might be but sparsely distributed. Divisions of the former kind would appear as dark lines, while those of the latter kind would present precisely that mottled appearance seen in the dusky or ash-coloured stripes. The transparency of the dark inner ring is easily understood if we consider the satellites to be sparsely scattered throughout that formation. The fact that this ring has only become visible of late years no longer presents an insuperable difficulty, for it is readily conceivable that the satellites forming the dark ring have originally belonged to the inner bright ring, whence collisions or disturbing attractions have but lately propelled or drawn them. The gradual spreading out of the rings is explicable when the system is supposed to consist of satellites only connected by their mutual attractions; while the thinness of the system is obviously a necessary consequence of such a formation, for the attraction of Saturn's bulging equatorial regions would compel each satellite to travel near the plane of Saturn's equator.

Another remarkable phenomenon—the elliptical shading at the ends of the apparent longer axis of the dark ring—must next be considered. These appearances have been called the 'shadows projected on the ring.' It is perfectly clear, however, that they are not shadows; for, in the first place, there are no luminous or light-reflecting bodies from which these parts of the rings are at any time concealed, while the brighter parts are illuminated; and, secondly, the fact that they are always seen in the same *apparent* parts of the ring, though the direction of the line of sight from the earth to Saturn is continually varying, shows conclusively that their appearance depends on the position of the observer on earth, whereas the motions of Saturn and of the ring are altogether independent of the earth's position in her orbit. There is no difficulty, however, in explaining these appearances, even on the supposition of the solidity of the system. Such explanation will serve to introduce and render intelligible the corresponding explanation on the hypothesis we are actually examining. Consider the great division in the rings: it is perfectly clear that if the rings were indefinitely thin, this division would appear to be bounded by two exactly similar and concentric ellipses, and it would therefore appear broadest at the ends of its longer axis and narrowest at the ends of its shorter axis. But now suppose the rings to be of appreciable and uniform thickness—then it is clear that this circumstance will operate to make the division appear narrower at the ends of the shorter axis, while it will not affect the apparent breadth of the division at the ends of the longer axis. For at the ends of the shorter axis the apparent breadth will be the angle between two lines of sight, one passing over the upper edge of the nearer boundary of the division, the other passing under the lower edge of the farther boundary; and it is clear that as the angle diminishes at which the ring is viewed, the apparent breadth of this part of the division would rapidly diminish, until at length the line passing over the upper edge of the nearer boundary would fall upon the opposite face of the division, so that the division would no longer be visible at this point. After this, as the angle at which the ring is viewed continued to diminish, the arc along which the division is invisible would gradually extend more and more towards the extremities of the longer axis of the apparent outline of the division; but until the angle became very small it is clear that the apparent breadth of the division would be very little affected at the ends of the longer axis, for here the lines of sight to the edges of the division would fall (approximately) along, and not across, the bounding faces of the division. Similar remarks apply to the division in the outer ring; but this division being so much narrower than the great division, would disappear much sooner at the ends of the shorter axis, as the ring closed, and the arc along which it is invisible would extend much more rapidly towards the extremities of the longer axis. Now, imagine the formation of the rings to be that exhibited in fig. 5, Plate IX.; that is, that each ring is formed of a number of concentric hoops of uniform thickness, but the breadths of which diminish, while the intervals between them grow gradually wider towards the inner boundary of each ring. Then it is clear, either from the considerations detailed above or from an examination of the figure, which represents the appearance of such a system of rings, that dark spaces must be visible at the ends of the longer axis of the inner boundary of each bright ring.* These shaded spaces would vary in form according to the manner in which the rings and the divisions between them varied in width, and might either be bounded by definite outlines or toned off by insensible gradations. It is clear, however, that if the width of the rings diminished, and the width of the spaces between them increased, by any uniform law, the shadings would present oval forms similar to those presented by the Saturnian system.

The explanation of these appearances on the supposition that the rings consist of flights of disconnected satellites, is similar to the above—though not so convenient for illustration—whether we suppose the satellites to travel in narrow rings, or, which is more probable, to be in general less regularly disposed. We have only to imagine that the satellites are strewn more densely near the outer edges of the bright rings, and especially of the inner bright ring, and that this density of distribution gradually diminishes inwards. For instance, we may conclude that along the inner edge of the inner bright ring the satellites are so sparsely strewn that, at the extremities of the apparent longer axis of that edge, the dark background of the sky becomes visible through the gaps between the satellites. If these gaps were separately visible we should find, as the eye travelled *across the breadth* of the bright ring at this part, that they became smaller and less numerous as the satellites became more and more densely crowded; but as the eye travelled *round* the ring we should find the gaps becoming smaller and less numerous from another cause. For a satellite would

* We must suppose these narrow rings to be so numerous, and, therefore, the divisions between them so narrow, that neither rings nor divisions would be separately visible even in the most powerful telescopes.

appear of the same size at whatever part of the ring it appeared, and thus, if separately visible, would occupy a much smaller part of the breadth of the rings when seen near the longer axis, where this breadth is greatest, than when seen near the shorter axis, where this breadth is least. Hence a flight of satellites which, in a telescope of sufficient power, might be resolvable into its component satellites when in the former position, might, from such foreshortening, become irresolvable in the latter, though the separate satellites maintained their relative positions unchanged. If such a flight of satellites could be traced in its motion from the longer to the shorter axis of the system, the discs of the component satellites would be seen gradually to approach, then to overlap each other, until, finally, all the dark spaces between them would disappear. If the satellites were not separately visible, such a flight would appear dusky in the former position, and would become gradually smaller and brighter, until in the latter position it would be as bright as the outer parts of the bright ring. Now the ring may be considered as made up of flights of satellites; and though the members of such flights in no case maintain their relative positions unchanged, even for a few seconds, yet the general average of density along any band of the ring remains tolerably uniform. Hence we can readily understand that there should be a gradual increase in the brightness of the rings, whether the eye travels across their width from within outwards, or along any circle concentric with the outlines of the rings from the longer to the shorter (apparent) axis of the system. Further, as it appears impossible to offer any other explanation of these shaded spaces, we may conclude that in the inner bright ring, and probably in each member of the outer double bright ring, the distribution indicated actually prevails—that is, that the component satellites are crowded along the outer boundaries of the bright rings, and more sparsely distributed along the inner boundaries; and that, although there may be local irregularities—such as strips, along which for an interval satellites are more or less crowded than in the neighbouring spaces—yet, on the whole, the density with which the satellites are strewn increases gradually outwards in each bright ring.

The appearances observed by Mr. Wray and M. Otto Struve, which seem altogether inexplicable on either of the hypotheses before considered, may be readily explained on the supposition we are examining at present. For it is conceivable that the disturbing attractions of Saturn's outer satellite may draw the satellites composing the ring from the plane of Saturn's equator (or the mean plane of the ring), so that when the edge of the ring is turned to the observer the satellites thus disturbed present the nebulous appearance described. Furtner, tne more densely the satellites composing any part of the ring are crowded, the more efficient will be their common action to check such disturbances; so that the gradual increase in the width of these nebulous appendages, as they (apparently) approach the disc of the planet, is, perhaps, a further indication of the diminution of density inwards mentioned above. But this phenomenon may be satisfactorily explained in another manner:—The number of satellites at a given distance from the central plane of the ring must rapidly diminish as that distance increases; thus, when this distance is very small, the disturbed satellites may be strewn with sufficient density to become visible near the extremities of the ansæ, where the line of sight passes through a small range of satellites; but that the sparsely strewn satellites at a greater distance from the central plane of the rings should become visible, it may be necessary that the line of sight should pass through a much greater range,—that is, should fall much nearer the disc of the planet. Thus, clearly, the apparent breadth of these appendages would be greater near the planet's disc, even though there were not an increase inwards in the numbers of satellites disturbed from the mean plane of the ring. It is very probable, however, that there is such an increase, and that the effects resulting from both causes combine to render the peculiar apparent shape of these appendages more distinct than it would be if either cause operated alone.

The investigation of the motions of a crowd of satellites traveling in rings about a central attracting globe, is a problem of too great complexity to be exactly resolved. If the motion of our moon is of so complex a nature that even yet all its inequalities have not been exactly determined, it will readily be conceived that a problem which deals with the motions of hundreds of moons, disturbed by and disturbing each other, must lie far beyond the range of our most powerful modes of mathematical analysis. Even if we knew the exact size, shape, and position of each satellite, and the rate and direction of its motion at any instant, the exact investigation of the subsequent motions of the system would still lie utterly beyond the grasp of the acutest human intellect. But of all those elements we are ignorant. All that we know certainly is that the bodies constituting the system are very numerous; we may also conclude from the analogy of other parts of the solar system that they are not uniform either in size or density.

Notwithstanding the difficulty of the problem, and the uncertainty of all its conditions, highly interesting general results may be deduced from its consideration.

And first, while we cannot assert that such a system is actually permanent, it is undoubtedly safe from sudden destruction. We speak of the orbits of our earth and of the planets as permanent, because, though they undergo various changes, these are oscillatory, and produce no lasting effect. But rings of satellites, subject like all the members of the solar system to numerous disturbing attractions, and mutually disturbing each other, undergo changes of form that proceed continuously. Whether such development results in the destruction of the rings (as rings) is not certain. It appears probable, however, that under certain conditions the destruction of the rings might be indefinitely postponed.

We may consider separately two forms of disturbance, chiefly due to the varying attractions of Saturn's eight satellites, but partly to the attractions of the other members of the solar system: each form of disturbance also generates the other, or modifies disturbances already existing.

In the first place, the members of these rings will be subject to perturbations out of the general plane of the system. If it were possible to trace the motion of a single satellite, it would be found that its orbit has its ascending and descending nodes on the ring's plane, and (at each instant) a definite inclination to that plane. These elements of the satellite's orbit would be found to be continually changing; the nodes at one time advancing, at another regreding—the inclination now diminishing, now increasing.

Considering the whole system, the result of these extra-planar motions and their variations would be a series of waves, wrinkling (so to speak) both surfaces of the ring. These waves would vary in extent, and would move with various velocities—travelling neither directly across nor in circles concentric with the rings.* They would not of themselves produce any marked or permanent effects upon the extent of the rings,—that is, on their diameters internal and external. Their effects on the development of the system would arise chiefly from their influence in generating the form of disturbance next to be considered. But their effects on the appearance of the rings when the edge is turned towards the earth are, as we have seen, very observable; for it is undoubtedly to such waves as these that the changes observed by M. Maradi and others at the disappearance of the ring, and the nebulous appearances already considered, are to be attributed.

Secondly, the members of these rings will be subject to variations in their distances from the centre of their gigantic primary. If a single satellite were tracked as before, it would be found that its orbit has its peri-saturnium, and its apo-saturnium, and (at each instant) a definite eccentricity. These elements, like those just considered, would be found to vary continually; the line of apsides advancing at one time and regreding at another, the eccentricity now diminishing and now increasing. Considering the whole system the result of these variations would be a series of waves of concentration and dispersion.† These would travel

* It must be remembered that it is not the motions of the satellites themselves that are here spoken of, but the motions of the waves of disturbance resulting from irregularities in the motions of those bodies. The two kinds of motion are as distinct as the motion of a wave on the ocean from the motion of the particles of the ocean; the wave itself may travel hundreds of miles, while the particles whose successive motions form the wave may not be displaced more than a few yards.

† It is not to be understood that waves of this kind, and waves of the kind before considered, exist separately, and separately travel across or round the ring; they are only considered separately to avoid confusion, but are in reality commingled, and their motions are varied and interchanged in inextricable combinations. If it were possible to view the rings from their common centre, waves of the kind first considered would be visible, apparently travelling round the ring; to an eye placed anywhere in the plane of the rings the same kind of waves would be seen, and their motions round and across the ring would both be visible. If the rings were viewed from a point in the axis of Saturn produced (so that they appeared as in the figures of Plate XII.)

neither directly across nor in circles concentric with the rings; but it appears probable, from the formation of the system of rings, that there would be a continual tendency in waves of the kind we are considering to assume the form of circles concentric with the rings and travelling across their breadth inwards and outwards. Their effects on the appearance of the rings, viewed from the earth, would depend partly on the intensity attained by the wave, and partly on the density with which the satellites are strewn in the particular zone of the ring across which the wave is travelling. If the intensity of the wave is great and the satellites not very densely crowded, the transparent phase of the wave may be traceable in a temporary division or dusky stripe.* Analysis shows that waves of this kind would produce a gradual but continuous increase in the breadth of the system of rings—the inner edge travelling inwards, the outer edge travelling, but much more slowly, outwards. These changes do not, of course, operate only at the edges, but throughout the breadth of the rings; † probably their effects are smaller at the edges than elsewhere: however, it is clear that the only marked change visible to us must be the increase in the breadth of the system.

waves of the second kind would be visible as waves of transparency and opaqueness, travelling, in general, concentrically across the ring, inwards and outwards. A tolerably exact notion of the disturbances to which the rings are subject may be obtained as follows:—Let a semi-transparent fluid be poured into a large circular plate of uniform colour until the bottom of the plate is just hidden; if now this fluid be disturbed in any manner waves will be seen travelling across the surface, crossing and interlacing as they are reflected from the edges of the plate; if the fluid be viewed, however, from above, these disturbances will appear as waves of colour (the colour of the plate and the colour of the fluid); if a motion of steady rotation be communicated to the plate by suspending it from a twisted string, the rotation of the rings, considered as a system, will be illustrated; and it will be found that disturbances can be as readily communicated to the rotating fluid as to the fluid at rest.

* It might be interesting to examine whether the temporary marks that appear on the rings have any motion across the breadth of the system in the intervals during which they remain visible.

† It may be suggested as possible that in the great division of the rings we have the indication of a zone along which, at an early stage in the development of the system, the parts of the ring spreading outwards were separated from those spreading inwards. This division may possibly be still increasing in width. The division in the outer ring seems certainly to be increasing in width, since it becomes more distinctly visible as the rings successively attain their greatest opening.

Let us èxamine the effects of such increase at the inner and outer edges of the system, respectively. It is clear that both changes operate to increase the extent of the rings, and consequently, as the changes proceed, the satellites have more and more space for their movements; but it also appears obvious that among satellites near the inner edge seeking smaller orbits collisions must be much more frequent than among satellites near the outer edge seeking larger orbits. Further disturbance would thus be continually generated among satellites near the inner edge. The satellites no doubt move in the same general direction about Saturn, so that it is only the difference of the velocities of two impinging satellites that comes into play at a collision;* but the eccentricities of the orbits of the satellites may be very importantly affected in this manner, and it is clear that a satellite which once begins to move in an orbit of considerable eccentricity must continually cause fresh disturbances, until either its orbit is altered to a form of less eccentricity or it falls upon the planet. The general effect of such collisions would be that (after the lapse possibly of many ages) numbers of satellites originally travelling in orbits nearly circular would pursue eccentric orbits. There would still remain a tolerably well defined inner edge; but these orbits would lie partly within and partly without its circle. It appears probable that after a time this process would be checked by the formation of a new ring within the original inner boundary of the system, and that the orbits of the satellites composing this new ring would gradually become less and less eccentric. After a further lapse of time, however, the inner edge of this ring would begin to undergo a series of like changes, ending in the formation of a new ring within it,—and so on continually, or until the process were checked or assumed new forms through the approach of the rings to Saturn's equator. The inner edges of outer rings would probably be liable to similar changes, proceeding, however, much more slowly.

* The satellites composing the system being bodies of imperfect elasticity, there is at every collision a loss of a part, however small, of the 'vis viva' of the system, and a corresponding generation of heat. The 'angular moment' of the system about Saturn is not, however, affected by collisions.

We have seen that the appearance of the rings, and their changes of form, correspond with the results detailed above. The interior diameter of the system is continually diminishing: two distinct rings are visible, and there are indications of the approaching formation of a third ring within them; the outer ring, also, is divided into two rings separated by a comparatively narrow interval. The exterior diameter of the system has not perceptibly increased. This, however, may be accounted for in two ways:—first, the change of this element would not be easily detected, since analysis shows that such change would be very small compared with the variation of the inner diameter of the system; and, secondly, it is not impossible that the existence of a resisting medium checks the outward and encourages the inward growth of the rings. It is probable that when the rings are again open to their full extent (or in the year 1870), the development of the dark ring will be found to have made great progress, and that the inner parts of the inner bright ring will appear much darker than at present. It is not impossible that the disintegration of the inner bright ring (the progress of which is shown by the gradual increase of the dusky elliptical spaces at the ends of the longer axis of the dark ring), may be found to have resulted in the formation of a new dark ring. The dark ring will probably be wider or brighter, possibly both; perhaps even traces may be discernible of the approaching transformation of the dark ring into a new inner bright ring separated from the neighbouring bright ring by a well-marked division.

## CHAPTER VI.

### THE 'GREAT INEQUALITY' OF SATURN AND JUPITER.

After the discovery of the three laws of Kepler, the motions of the planets were diligently watched by astronomers, and compared with the motions due to those laws. The comparison was conducted still more carefully when it became apparent that the law of gravitation could be established or confuted by such observations alone. Before long a singular discrepancy was detected in the motions of Saturn and Jupiter:—Saturn's period, instead of being constant, appeared to be continually diminishing; Jupiter's period, on the other hand, seemed to be continually increasing. It appeared, further, that Saturn's period was in excess of his mean period (calculated according to Kepler's laws, or, more strictly, according to the laws of gravity), while Jupiter's period was less than his mean period. Accordingly, the observed changes were operating to restore the two periods to their respective mean values. Until this restoration should be effected, it is clear that Saturn was gradually falling further and further behind, Jupiter getting further and further in advance of his calculated place. Near the end of the eighteenth century, the periods of the two planets were restored to their mean values;* and since that time

* See Table IX., Appendix II., and explanation. A slight error may be noticed in Sir J. Herschel's account of the great inequality. He describes Saturn's period as increasing during the seventeenth century, Jupiter's period as diminishing; and he adds—'In the eighteenth century a process precisely the reverse seemed to be going on.' It will readily be seen from Table IX. that the changes in the periods have proceeded in the same direction from the middle of the sixteenth century to the present time; they will continue to proceed in that direction for more than a century.

The correction applies to the first edition of the work in question. Probably this error, and one or two others mentioned in these pages, have been corrected in later editions.

Jupiter's period has continued to increase and Saturn's to diminish. Jupiter's period having thus become greater than his mean period, he has been continually losing more and more of his surplus progress in longitude; while Saturn, whose period has become less than his mean period, has been continually working off (so to speak) his arrears of longitude. Thus both planets are approaching, but many years will elapse before they actually reach, their mean places.

For a long time these changes threw doubt on the law of gravitation. Astronomers were aware that mutual attractions caused the planets to deviate from the simple elliptic orbits traced out by Kepler. But these disturbances—it appeared to them—must be oscillatory, and the period of any such oscillation not very great. Thus, when it appeared that the changes in the motions of Jupiter and Saturn were proceeding for centuries in the same direction, grave doubts began to arise as to the truth of a theory with which those changes seemed so discordant. To Laplace is due the honour of removing these doubts, or rather, of making the discrepancies from which they arose the means of confirming the Newtonian doctrine. He showed that the observed changes are due to a certain relation between the periods of Saturn and Jupiter, which will presently be pointed out. Before giving an outline of Laplace's explanation, however, it will be necessary to examine,—the various elements of a planet's orbit which admit of change, the effect (if any) of the change of each element in altering the period of the planet, and the natures of the disturbing forces that produce such changes.

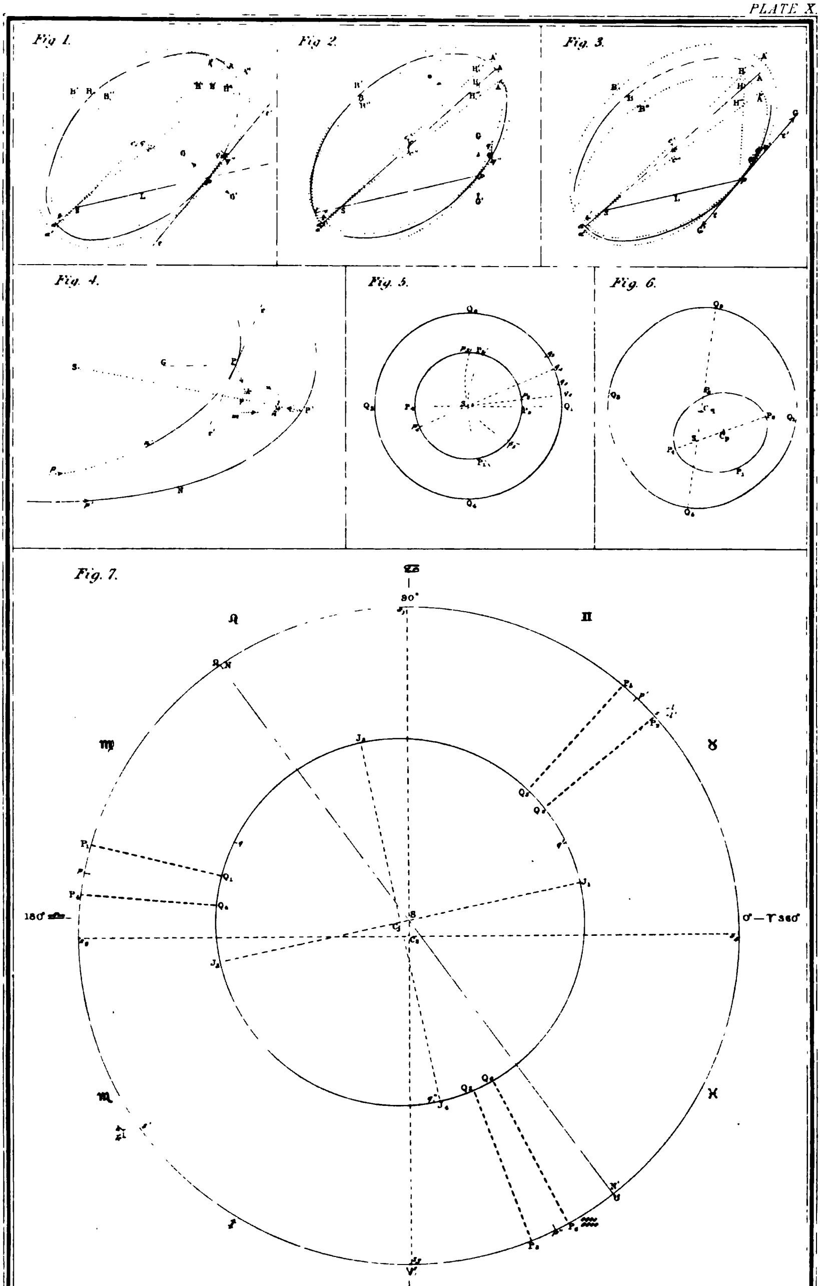
The elements to be considered in examining the orbit of a planet are,—the *major axis*, the *eccentricity*, the *position of the line of apsides*, the *inclination of the plane of the orbit to a fixed plane*, and the *position of the line of nodes on that plane*. That is, these elements being known, we know the orbit in which the planet would continue to move, if undisturbed. But, owing to the mutual attractions of the various parts of the solar system, every one of these elements is in a state of continual variation. The major axis, eccentricity, and inclination of each orbit, vary within narrow limits, by several variations, which have periods of different length. Again, the changes of each of these elements react on the others, the varying influences producing such changes commingle their effects, and the different cycles are blended and interchanged in apparently inextricable combinations. The positions of the apsidal and nodal lines are similarly subject to various changes, but the resulting variations in these elements are not confined within limits. On the whole, these lines travel continually round the orbit, and (except in the case of Venus) in opposite directions, but for short periods these motions are reversed. These changes also act on, and are reacted on by, the changes in the other elements.*

Now, it appears from the third law of Kepler that the period of a planet depends solely on the length of the major axis of its orbit. So long as this element remains unaltered, no variations in the eccentricity or inclination of the orbit, or in the positions of the nodal and apsidal lines, can have any effect in altering the period of the planet; and although, as has been stated, no variation can take place in any one of the elements without some influence on every one of the others, yet perturbing attractions, whose direct effect is to alter any of the other four elements, and which affect the major axis of the orbit only by the transmitted influence of such alteration, have little effect in modifying the period of the planet, compared with forces whose influence *directly* operates to alter the length of the major axis. Let us consider what are the elements of a planet's orbit on which perturbations of different kinds have the greatest influence.

Let the ellipse *a p* A B in each of the figures 1, 2, 3 of Plate X. represent the orbit of a body revolving according to the law of gravity about an attracting mass at S, a focus of the ellipse,—*a* A being the major axis, *c* the centre, and H the other focus of the ellipse.

* The secular inequality of the moon affords an excellent illustration of the interchange of effects alluded to in the above paragraph. Though the planets exercise but little direct control over the moon's motion, it is to their attractions this inequality is really due—the influence of those attractions on the eccentricity of the earth's orbit being propagated to the orbit of the moon. Singularly enough, the direct effect of this influence is scarcely perceptible, while the transmitted effect is so marked that it was detected long before its cause was recognised.

PLATE X.




*Orbits of Jupiter and Saturn Jan. 1st 1865.*

R.A. Proctor del.

E. Dillard Sc.

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First, let us suppose that when the body reaches the point $p$ it receives an impulse in the direction $p$ G (fig. 1) perpendicular to the tangent $t p t'$ at $p$, and in the plane of the orbit. This impulse, being applied in a direction perpendicular to the direction of the body's motion, cannot influence the body's velocity estimated in that direction, so that if at the end of a small interval of time the body undisturbed would have been at $q$, it will at the end of that interval, under the actual circumstances, be at some point $q'$, so situated that $q\,q'$ is parallel to $p$ G. Now, the impulse must be considered as sufficient only to produce a very small velocity in the direction in which it is applied compared with the original velocity of the body;* thus, $q\,q'$ (the displacement due to that impulse) is small compared with $p\,q$, and since $p\,q\,q'$ may be considered as a right-angled triangle, it follows that $p\,q'$ is very little greater than $p\,q$; and therefore the velocity of the body in moving along $p\,q'$, is very little greater than the velocity with which, if undisturbed, it would have moved along $p\,q$ in the same time. The full effect of the impulse, however, clearly operates in altering the direction of the body's motion. Now, one of the properties of elliptic motion under the influence of gravity is, that, if the velocity is known with which a body is moving when at a given distance from the centre of motion, the major axis of the ellipse in which the body moves is determined—altogether irrespectively of the *direction* in which the body may be moving at the time. It plainly follows that any disturbing force which only influences the direction of a body's motion will not at all affect the length of the major axis of the body's orbit; and a disturbing force which chiefly influences the direction of motion will very little affect the length of the major axis. Thus the period of the body whose motion we are considering is very little modified by an impulse in the direction $p$ G. It follows in the same manner that an impulse in the direction $p$ G$'$, which would make the body move in the direction $p\,q''$, would have very little effect on the period of the body. The influence of such disturbances chiefly affects the eccentricity of the orbit and the position of the apsidal lines. In the first case

* Since the perturbations which the effects of such impulse are intended to illustrate are small compared with the actual motions of the planets.

considered the body would proceed to describe an orbit $p$ A′B′$a'$ of less eccentricity than $p$ A B $a$, and having its line of apsides $a'$A′ in advance of $a$ A ; in the second case the orbit $p$ A″B″$a''$ subsequently described by the body would be more eccentric than $p$ A B $a$, and the line of apsides $a''$A″ would be behind $a$ A.* Into such changes, however, it is not necessary for us now to inquire, since the period of the body, with which alone we are at present concerned, is not affected by them.

Secondly, let us suppose that when the body reaches the point $p$ it receives an impulse in the direction $p$ G (fig. 2) perpendicular to the plane of the orbit $a\,p$ A. In this case, as in the former, the impulse does not influence the velocity of the body estimated in the direction in which it was originally moving; so that if at the end of a small interval of time the body undisturbed would have been at $q$, it will, under the effect of the impulse, reach some point $q'$ (at the end of that interval of time), such that $q'q$ is parallel to $p$ G. And, as in the former case, since $q\,q'$ must be small compared with $p\,q$, and since $p\,q\,q'$ is a right-angled triangle, $p\,q'$ must be very nearly equal to $p\,q$, and therefore the velocity of the body is very little affected by the impulse received. Hence such an impulse very little affects the length of the major axis of the orbit, or (therefore) the period of revolution. And, similarly, an impulse applied in the direction $p$ G′ would have very little effect on the period of revolution of the body. The influence of such disturbances will chiefly affect the plane of motion of the orbit. In the first case considered the body would proceed to revolve in some orbit $p$ A′B′$a'$, of which the part $p$ A′$r$ ($p\,r$ being a straight line through S) would lie above the plane of the original orbit; and in the second case the body would move in an orbit $p$ A″B″$a''$, of which the part $p$ A″$r$ would lie below the original orbit. In both cases the line $p$ S $r$ would be the line of nodes of the new plane of motion on the

* Such are the changes if the body is at $p$ when the respective impulses are applied. The effects of such impulses on the eccentricity of the orbit would vary with the position of the body at the moment they were applied. It may easily be shown that the points $c$, $c'$, $c''$, the centres of the three orbits, lie on the circumference of a circle about L, the bisection of S $p$, as centre; and the points H, H′, H″, foci of the orbits, on the circumference of a circle about $p$ as centre.

original plane: and clearly such changes could not take place without affecting the inclination of the plane of the orbit to any fixed plane adopted as a plane of reference, and the position on such a plane of the line of nodes of the orbit. With such changes, as they would not affect the period of the body, we are not at present concerned.

Lastly, suppose that when the body is at $p$ it receives an impulse in the direction $p$ G (fig. 3) along the tangent line $t p t'$. In this case, since the impulse is applied in the direction of the body's motion, its whole effect operates to increase the velocity estimated in that direction. Thus, if at the end of a small interval of time the body undisturbed would have reached the point $q$, it will, under the effect of the impulse, reach some point $q'$ (in that interval of time), such that $q q'$ is parallel to $p t'$; therefore the velocity of the body must have been increased, since the arc $p q'$ is clearly greater than the arc $p q$. Thus the effect of such an impulse is to increase the major axis of the orbit, and therefore the period of revolution. The body will proceed to describe some orbit $p$ A′B′$a'$, to which the line $t p t'$ will still be a tangent; and $a'$A′, the new major axis, will be greater than and in advance of $a$ A. Similarly it may be shown that if the impulse were applied in the direction $p$ G′, the body would proceed to describe some orbit $p$ A″B″$a''$, having a major axis $a''$A″ less than and behind $a$ A, and to which $t p t'$ would still be a tangent.* So far as the position of the line of apsides is concerned, these effects would vary with the position of the body at the moment the impulse is applied; but as regards the length of the major axis, it is clear that, whereever the body may be in its orbit, an impulse applied to it in the direction of its motion increases the major axis of the orbit, and therefore the period of revolution of the body, while an impulse applied in the opposite direction diminishes the major axis and the period of revolution. Thus we have the apparently paradoxical result that an impulse whose immediate effect is to accelerate the

* In the case illustrated by fig. 2, the points $c'$,$c$, $c''$, and H′,H, H″, lie in lines very nearly straight and perpendicular to the plane of the orbit. In the case of fig. 3, the points $c'$,$c$, $c''$, lie in a straight line with L, the bisection of S$p$; and the points H′,H, H″, in a straight line with $p$.

motion of the body diminishes its mean angular velocity about S (since the period is increased), while an impulse retarding the body's motion increases its mean angular velocity about S.*

Let us apply these results to the actual forces operating to disturb the planetary orbits. Let $p\,n'P$, $p'N\,P'$ (fig. 4, Plate X.) represent parts of the orbits of two planets (which we may call, respectively, P and P′) revolving about S, the sun; and let the plane of the orbit $p\,n'P$ intersect the plane of the orbit $p'N\,P'$ in the line $S\,n'N$, so that the part $n'P$ of the first named orbit lies above, the part $n'p$ below, the orbit $p'N\,P'$; suppose further that both the planets are moving in the same direction (indicated by the arrows at $p$, $p'$). Let P, P′, be simultaneous positions of the two planets and join P P′; then the attraction of the planet P′ operates in direction P P′, and is inversely proportional to the square of the distance P P′. Now we are seeking to learn the effect of the attraction of the planet P′ in disturbing, not the *actual* motion of P, but the orbital motion of P about S. Plainly, therefore, we must take into account the attraction of P′ on S, for it is only the difference of P′'s attractions on P and S respectively, that can affect the orbital motions of P about S. In the actual configuration represented P′ is farther from S than from P, and therefore the attractive force of P′ is less on S than on P, in the inverse proportion of the squares of the distances S P′ and P P′. If, then, we represent the attractive force of P′ on P by the line P P′, the attractive force of P′ on S will be correctly represented by the line $P'l$, along P′S, if $P'l$ bears to P P′ the same proportion that the square of P P′ bears to the square of S P′. Let $t'\,P\,t$ be the tangent at P to the orbit $p\,n'P$, and draw P G perpendicular to $t'\,P\,t$ in the plane of that orbit; from P draw $P\,k$ perpendicular to the same plane; from P′ draw $P'm$ parallel to P G to meet the plane $t'P\,k$ in $m$, and from $m$ draw $m\,k$ parallel to $t'\,P\,t$. Then the attractive force of P′ on P, represented by the line P P′, may be resolved into three forces, represented respectively by the lines $P\,k$, $k\,m$, and $m\,P'$; the first of these forces is of the kind illustrated in

* Encke's comet affords an illustration of the effect of a force acting at every instant in the direction of the tangent to the orbit of a body. The motion of this comet is continually retarded (probably owing to the resistance of the æther occupying space), yet its mean angular velocity about the sun is continually increasing as its period of revolution decreases.

fig. 2, that is, is perpendicular to the plane of the orbit; the second is of the kind illustrated in fig. 3, that is, is tangential to the orbit; and the third is of the kind illustrated in fig. 1, that is, is normal to the orbit. Similarly the force represented by the line $l$ P′ may be resolved into three $l\,o$, $o\,n$, and $n$ P′, respectively parallel to P $k$, $k\,m$, and $m$ P′. Thus, in the configuration represented by fig. 4, the actual *disturbing* force perpendicular to the plane of P's orbit is represented by the difference of the lines P $k$, and $l\,o$, and acts downwards; the tangential disturbing force is represented by the difference of the lines $k\,m$ and $o\,n$, and retards P's motion; and the normal force* is represented by the difference of the lines P′$m$ and P′$n$, and tends to draw P outwards, or *from* S. In a similar manner we can determine the disturbing forces in any other configuration: they are found to vary both in magnitude and direction, according to the configuration, and there are certain points at which one or other disappears altogether. It is not necessary, however, to examine all these variations, for we have seen that it is chiefly with the tangential disturbing force that we have to deal in examining the changes of a planet's period. Further, since the inclinations of the planes of the planetary orbits to each other are small, we can simplify the preliminary inquiry, without introducing sensible error, by supposing our illustrative orbits to lie in one plane.

Suppose, first, that both the orbits are circular (fig. 5, Plate X.), and consider the effect of the tangential disturbing force of a planet (which let us call Q), supposed to be stationary at $Q_1$, while a planet (which we may call P) performs a revolution about S in the orbit $P_1\,P_2\,P_3\,P_4$. Let $Q_1\,P_2\,S\,P_4$ be a straight line; and let a circle about $Q_1$ as centre, and at distance $Q_1$ S meet the circle $P_1\,P_2\,P_3\,P_4$ in the points $P_1$, $P_3$. Then, in the first place, it is clear that the tangential force vanishes when the planet P is at $P_2$ or $P_4$, since the tangents at these points are at right angles to the line $Q_1\,S\,P_4$; and in the second place, the tangential force vanishes when the planet P is at $P_1$ or $P_3$, since at these points Q exerts an equal attractive force on the sun at S and on the planet P,† and further, the lines

* Since P G always passes very near S in the planetary orbits, which are very nearly circular, the third force may be called the *radial* force.

† It must be remembered that in this case, and in all similar cases, the masses of

S $Q_1$ and $P_1Q_1$, or $P_3Q_1$, are plainly inclined at equal angles to the tangent at $P_1$, or $P_3$, respectively, and thus the tangential parts of those equal attractive forces are equal, and their effects (so far as P's orbit relatively to S is concerned) neutralise each other. Again, in the arc $P_1P_2$, the tangential disturbing force plainly *accelerates* P's motion, for P is nearer than S to $Q_1$, and the angle at which S $Q_1$ is inclined to the tangent at any point of this arc, is greater than the angle at which the line from $Q_1$ to P is inclined to such tangent;* thus the tangential disturbing force on P is greater than that on S, and as it plainly acts towards $P_2$, P's motion about S is accelerated in the arc $P_1P_2$. In exactly the same manner it may be shown that the tangential force is greater on P than on S while P moves from $P_2$ to $P_3$, and as it acts towards $P_2$, P's motion is retarded in the arc $P_2P_3$. In the arc $P_3P_4$, P's motion is accelerated; for P is farther than S from $Q_1$, and the line from $Q_2$ to any point of this arc is inclined at a greater angle than the line $Q_1S$ to the tangent at that point; † thus the tangential disturbing force is greater on S than on P, and as this force on P plainly tends from $P_3$, or acts as a retarding force, while the force on S acts in the opposite direction, as far as P's motion about S is concerned, or acts as an accelerating force, the latter effect predominates and P's motion is accelerated in the arc $P_3P_4$. Lastly, it may be shown in exactly the same manner that P's motion is retarded in the arc $P_4P_1$.

Now it is perfectly clear that in describing any number of complete revolutions about S, the accelerations and retardations of P would neutralise each other, and that thus on the whole P's period would not be affected by the attraction of Q. For in moving

the attracted bodies have not to be considered, any more than the mass of a falling body has to be considered in determining the time of falling. The *amount* of the attractive force actually exerted by Q on S (in the case supposed) would of course greatly exceed the amount exerted by Q on P; but the mass of S exceeding the mass of P in the same proportion, the effects of those unequal attractive forces are exactly equal.

* This will plainly appear if a tangent be drawn at any point P of the arc $P_1$ $P_2$; the angle that S $Q_1$ makes with this line is the exterior angle of a triangle, of which the angle that P $Q_1$ makes with the same line is an interior and opposite angle.

† This will appear by drawing lines as described in last note. To avoid confusion these lines are not introduced into figure 5. In the case of a point in the arc $P_3$ $P_4$ the line from P to $Q_1$ forms with the tangent at P the exterior angle of a triangle, of which the lines $Q_1S$ and the same tangent form an interior and opposite angle.

from $P_1$ to $P_2$,* P would be accelerated and his orbit disturbed; in moving from $P_2$ to $P_3$ he would be retarded, and the original form of his orbit restored; and similar counterbalancing effects would be experienced in the arcs $P_3P_4$ and $P_4P_1$; so that on the whole P would arrive at P in the same time as if his orbit had not been disturbed, and if Q were then suddenly removed, P would proceed to describe a circular orbit. This exact counterpoise of effects will not be disturbed if we suppose Q, instead of remaining fixed, to move at a uniform rate in the orbit $Q_1Q_2Q_3Q_4$. The only effect of such motion is an equal increase of each of the arcs $P_1P_2$ and $P_2P_3$, and a greater but still equal increase of each of the arcs $P_3P_4$ and $P_4P_1$. Thus, supposing P to start from $P_1$, and Q from $Q_1$, at the same instant; that they are in conjunction along the line $Sp_2q_2$; and that Q at $q_3$ is equidistant from S and P, P being then at $p_3$;—then, from the uniform motions and circular orbits of P and Q, it follows that the arcs $P_1p_2$ and $p_2p_3$ are equal, and that the accelerating effects of Q's action on P in the first arc is exactly counterbalanced by Q's retarding action on P in the second arc. And similarly if (continuing their motions) Q and P are in the same line with S when at $q_4$ and $p_4$, respectively; and if Q at $q_5$ is equidistant from S and P (at $p_5$); the arcs $p_3p_4$ and $p_4p_5$ are equal, and the accelerating and retarding effects of Q on P in these arcs, respectively, exactly counterbalance each other.

But now let us suppose one, or other, or both of the orbits to be elliptic. If, first, we suppose Q at rest, the orbit of P elliptic, and the perihelion or aphelion at $P_2$, we plainly get the same counterbalancing series of effects as in the former cases. But if the line of apsides have any other position than $P_2P_4$ the disturbing effects of Q on P will not be equally balanced in the course of a complete revolution of P about S. Thus, suppose that $p_5$ is the perihelion of P's orbit: then, in the first place, the tangential disturbing force no longer vanishes at the points $P_1$, $P_2$, $P_3$, and $P_4$;

* P would not actually move from $P_1$ to $P_2$, but in a disturbed orbit that would carry him to a point between $P_2$ and $Q_1$; and so of the other arcs, the orbit of P relatively to S passing through the points $P_1$ and $P_3$, but outside the points $P_2$ and $P_4$. Thus the more exact mode of expression would be, 'in moving from $P_1$ to the line $P_2Q_1$,' and similarly for the other arcs. Such a mode of expression would, however, be inconvenient, and that adopted in the text is, as explained, sufficiently intelligible.

for the tangents to the orbit of P at the points $P_2$ and $P_4$ are not at right angles to the line $Q_1P_2S\,P_4$, and the lines $Q_1P_1$ and $Q_1P_3$ are not inclined equally with $Q_1S$ to the tangents at $P_1$ and $P_3$ respectively. But neglecting this consideration (as we may safely do if the orbit of P is supposed to be very nearly circular, so that the actual points at which the tangential force vanishes lie very near the points $P_1$, $P_2$, $P_3$, and $P_4$) we have in the circumstance of P's elliptic motion a more serious disturbing cause. For P passes his perihelion point in the arc $P_1P_2$, and thus moves more rapidly over this arc—in which, as we have seen, his motion is accelerated—than over the arc $P_2P_3$ in which his motion is retarded. Thus these disturbing effects no longer compensate each other, the retardation exceeding acceleration. On the other hand, since the aphelion of P's orbit lies in the arc $P_3P_4$, P passes over this arc in which his motion is accelerated, more slowly than over the arc $P_4P_1$ in which his motion is retarded. The acceleration in the arc $P_3P_4$, therefore, exceeds the retardation in the arc $P_4P_1$. Thus, whereas in moving over the arc $P_1P_2P_3$, P's motion on the whole was retarded, in moving over the arc $P_3P_4P_1$, his motion on the whole is accelerated.* These effects acting in opposite directions, produce a partial, but (unless P's orbit be exactly adjusted in a certain mean position) not a complete compensation; and wherever we suppose the perihelion of P's orbit to lie, an outstanding retardation or acceleration would remain after each revolution, and these disturbances always operating in one way, P's period would continually increase or diminish, and his orbit would be continually more and more disturbed from its original figure.

But now let us suppose that Q, instead of remaining stationary, travels uniformly round the circle $Q_1Q_2Q_3Q_4$, P describing several revolutions in the meantime in his elliptic orbit. Then it is clear that for a single revolution of P the above considerations hold good, and that P's period is retarded or accelerated by Q's attrac-

* These effects are partially modified by changes of P's distance from S. It may be mentioned that any difference in P's distance from Q due to this cause produces its full effect in modifying Q's disturbing influence; whereas, in the next case considered, only a part of the effect of Q's change of distance from P's orbit operates: for in approaching P's orbit Q is also approaching S, and *vice versâ*; and it is to the difference of Q's attractions on P and S that Q's disturbing effects are due.

tion, and his orbit modified. But in the course of several revolutions different parts of P's orbit are successively presented to Q as Q travels onward in his orbit; hence Q's disturbing effects are altered, accelerations replacing retardations, or *vice versâ*; and on the whole, when Q has completed a revolution, P's period and orbit are but little disturbed. A small outstanding disturbance, however, necessarily remains, since Q's conjunctions with P have happened opposite *particular* points of P's orbit, and though compensating effects have taken place in different synodical revolutions, the compensation (save under an exceptional adjustment of P's orbit) is not *exact*. Now, if at the beginning and end of such a single revolution of Q, Q and P are in conjunction, then in the next revolution of Q a similar series of disturbing effects will be produced by Q on P's orbit and period, and so on continually (or at least until the modification of P's period prevents the uniform recurrence of conjunctions along or near the same line); thus P's orbit and period would in this case, also, be permanently modified, though not so rapidly as when Q is supposed stationary. Similar permanent effects would be experienced if P and Q returned to conjunction after two, three, four, or any exact number of revolutions of Q. The greater the number of Q's revolutions in such a cycle the smaller would be the outstanding disturbance of P's orbit at the end of the cycle. But in the course of many cycles such disturbances, acting always in the same way, must produce permanent and observable changes in P's period and orbit. In the case only in which the periods of P and Q are incommensurable, so that these bodies never return to conjunction along the same line, no permanent disturbance will accrue.

Next, let us consider the effect of an ellipticity in Q's orbit, and suppose P's orbit circular. Let the perihelion of Q's orbit be at $Q_1$, and Q's motion near perihelion such that as P moves from $P_1$ to $p_2$, Q moves from $Q_1$ to $q_2$. Then, since Q's motion gradually becomes slower as Q moves from perihelion, it is clear that the arc $p_2p_3$ passed over by P before P and S are equidistant from Q at $q_3$, does not so greatly exceed the arc $P_2P_3$ (similarly passed over by P when Q was considered stationary), as it would if Q moved over the arc

$q_2q_3$ with the greater mean velocity belonging to Q in the arc $Q_1 q_2$; thus the arc $p_2p_3$ is less than the arc $P_1p_2$: and further, $Q_1$s distance from P's orbit is continually increasing as Q leaves $Q_1$. On both accounts, the retarding effect of Q's action in the former arc is less than the accelerating effect in the latter arc. Since Q's motion continues to diminish, and his distance from P to increase, it is clear that Q's retarding effect as P moves over the arc $p_4p_5$, will be less than Q's accelerating effect as P moved over the preceding arc $p_3p_4$; or again there remains a balance of retardation. And so long as Q's motion continues to diminish, and his distance to increase—that is, until Q has reached aphelion at $Q_3$—P's motion will be accelerated in each synodical revolution of the two bodies. By parity of reasoning, it follows that so long as Q's motion continues to increase, and his distance from P's orbit to diminish, after aphelion passage—that is, until Q is again at $Q_1$—P's motion will be retarded in each synodical revolution. The final result would not, however, be a complete compensation in a single revolution of $Q_1$, in this, any more than in the former case. Some outstanding acceleration or retardation would remain at the end of each revolution of Q, and permanent disturbing effects on P's orbit and period would accrue in this case as in the last, unless the periods of P and Q were incommensurable.

Similar reasoning holds when the orbits of both P and Q are elliptical; but the tangential disturbances which operate according to the varying positions of P and Q, are somewhat more varied and complex. The effects due to the ellipticity of P's orbit may either cooperate with or partly neutralise those due to the ellipticity of Q's orbit; but there will not be a complete compensation of effects, either in any single revolution of P, or in several revolutions of P taking place during a single revolution of Q. And, further, if the periods of P and Q be commensurable, so that after a certain number of revolutions they return to the positions they had respectively occupied at first, there will remain an outstanding disturbance of P's period at the end of such cycle of revolutions, whose amount will depend partly on the eccentricities of the orbits of P and Q, and partly on the number of revolutions of P and Q, respectively, which may occur in each cycle. Thus, if $P_1P_2P_3P_4$ and $Q_1Q_2Q_3Q_4$

(fig. 6, Plate X.) are the orbits of P and Q about S, and $C_p$ and $C_q$ the respective centres of those orbits, it is clear that the *irregularities* of the tangential disturbance will depend on the distances S $C_p$ and S $C_q$ or rather on the proportions borne by these distances to $C_pP_2$ and $C_qQ_2$, the respective major semi-axes of the orbits of P and Q; and consequently the *outstanding effects* resulting from those irregularities after a given number (supposed very great) of revolutions of Q, during which such irregularities have been sometimes acting one way, sometimes another, more or less effectively—must also depend in some degree on the eccentricities of the orbits of P and Q. But the circumstance on which that effect mainly depends is the relation between the periods of P and Q. If these are commensurable, then after one, two, or more revolutions of Q, the series of disturbances that had been operating during such revolutions, and which had left a certain outstanding effect, will be repeated, and so on continually, so that the resulting outstanding effects are accumulated, and P's orbit and period permanently affected. The greater the number of revolutions of P and Q that occur before such exact reproduction of a series of disturbances, the smaller will be the outstanding effect of such a series, for there must occur a greater variety in the modes in which Q is presented to the orbit of P. Thus, if at the end of only one revolution of Q, P and Q return to conjunction along the line from which they had started, the effect outstanding will be greater than if two revolutions of Q occur before such exact coincidence; the effect in the latter case will be greater than if three such revolutions occur; and so on continually. And again, in any of these cases the effect will diminish as the number of revolutions made by P in each cycle increases.*

* It may be remarked here that even if two planets were moving at any instant so that their periods would be exactly commensurable if they were not disturbed by their mutual, or by extraneous, attractions; yet, being so disturbed, their periods would no longer remain commensurable. Thus, even if some simple relation of commensurability existed between the periods of two planets at any instant, it is quite possible that disturbances which would at first be accumulative, each cycle adding to the amount, would at length effect their own removal, by destroying the simple relation of commensurability to which they were due. The period necessary to effect such a change would, however, be far greater than the greatest cycles (so far as our system is concerned) with which astronomers have to deal; and it is questionable whether the

We have been considering hitherto the disturbing effects of a planet external to the disturbed planet. This case is more convenient for illustration than the case of a planet disturbing an external planet, but the reasoning in the latter case is exactly similar. There is no occasion, however, to consider this case separately: for, since action and reaction are equal and opposite, the internal planet exerts precisely the same force to retard or accelerate the external planet as the latter exerts to accelerate or retard the former. The *effects* of such equal and opposite forces, so far as changes of orbits and periods are concerned, may be very different, since such effects will plainly depend on the relative masses and orbits of the two planets;* but whatever outstanding effects of disturbance may appear after a given time in the orbit and period of one, corresponding opposite effects will appear in the orbit and period of the other. Thus we are able to apply the results just obtained to disturbances of the period either of Saturn or Jupiter, produced by the mutual attractions of these planets.

No simple relation of commensurability exists between the periods of any two planets; † but in one or two instances we meet

amount of disturbance accumulated before such change began to operate would not so far modify the orbits thus related that the inhabitants of the two planets would be affected injuriously, if not destroyed.

* We were able in considering the disturbing effect of one body on each of two others to neglect the masses of these latter; but in considering the effect on each of two bodies of the mutual attraction between them the masses must be taken into account. In the former case the attraction of the disturbing body on the disturbed bodies varied as their masses. In the latter case, the same force is exerted on each—namely, their mutual attraction: the effect of such attraction will plainly be greater on the body of smaller mass. As an instance of the kinds of action considered:—One man can pull a given mass at the same rate as ten men, of the same strength as the first, can pull a mass ten times as great; but if one man were to pull at one end of a rope while ten men of equal weight pulled at the other end, on a smooth and horizontal surface, the ten would prevail against him by superior weight, even though his strength exceeded their united strength, for the united strength of the eleven produces a tension along the rope which acts equally on the unequal masses at the two ends of the rope, and therefore prevails on the smaller. Obvious as such considerations may appear, they are frequently lost sight of by the student of astronomy, and a difficulty is felt in conceiving why, in one case, the mass of a body is not considered at all, while in another case it is one of the chief points of inquiry.

† It is not correct to say that the periods of the planets are absolutely incommensurable: a sét of quantities which, like the planetary periods, undergo continuous (however small) changes of increase or diminution, must at times have commensurable

with an approach to such a relation, and consequently find an approach to those progressive perturbations which, as we have seen, would result from simple relations of commensurability. In the periods of Jupiter and Saturn there exists an approach to the following very simple relation:—That two periods of the exterior planet should be equal to five periods of the interior planet. The statement of the actual relations of the periods of Jupiter and Saturn is generally presented somewhat as follows:—Five periods of Jupiter amount to 21,662·9240 days, and two periods of Saturn amount to 21,518·4394 days; the former interval exceeds the latter by 144·4846 days. Hence, supposing the two planets to start from conjunction, Saturn would reach this line the second time (that is, after passing it once) 144·4846 days before Jupiter reached it the fifth time (that is, after passing it four times). In 144·4846 days Jupiter describes 12° 0′·7 about the sun, so that when Saturn reached the original line of conjunction Jupiter is about 12° behind. On the other hand, Saturn in 144·4846 days describes 4° 50′·4 about the sun, so that when Jupiter has reached that line Saturn is not quite 5° in advance. Thus the two planets are very near, but have not quite reached, conjunction. Jupiter's daily mean motion of 4′ 59″·3 exceeds Saturn's daily mean motion of 2′ 0″·6 by 2′ 58″·7,—this is Jupiter's daily (mean) angular gain; Saturn has a start of 4° 50′·4, and this angle contains 2′ 58″·7 rather more than 97½ times: thus, Jupiter will overtake Saturn, or they will be in conjunction, 97½ days after the passage by Jupiter of the original line of conjunction, or 21,760·4 days from the time of that conjunction.* In this interval of 97½ days, Jupiter, with a mean daily motion of 4′ 59″·3, describes 8° 6′·4 about the sun, by which angle, therefore, the line of this conjunction is in advance of the original line of conjunction. This mean value will be of use presently in deter-

values. The *true* mean periods of the planets *may* be absolutely incommensurable, but they are not known to be so, since they are not exactly determined. It is sufficient, however, to prevent permanent or injurious changes in the planetary periods that no such simple relation as that approximated to in the cases of Jupiter and Saturn, Venus and the earth, should subsist exactly.

* Since there have been two conjunctions in the interval, or three synodical revolutions of Saturn and Jupiter, we obtain at once their mean synodical period by dividing 21760·4 by 3, giving 7253½ days, nearly.

mining the period of the cycle of disturbances. In the meantime let us proceed to a more exact inquiry into the motions of Saturn and Jupiter. The investigation given above presents a sufficiently accurate view of the general features of those motions, and is further useful in determining the mean angle of progression of successive third conjunctions; but it will be seen that it does not accurately present the true relations of Saturn and Jupiter. In fact, if it did, the inequality we are inquiring into would not exist, for the uniform progress of each set of successive third conjunctions could only result from the uniform motions of Saturn and Jupiter in circular orbits.

Fig. 7, Plate X., represents the orbits of Jupiter and Saturn about the sun at S. If we suppose that Jupiter's orbit $J_1 J_2 J_3 J_4$ lies in the plane of the paper, then the plane of Saturn's orbit $s_1 s_2 s_3 s_4$ must be supposed to intersect this plane in the line N N′,* the part N s′N′ of Saturn's orbit lying above, the part N′s N lying below, the plane of Jupiter's orbit; the points at which Saturn's orbit attains its greatest departure from the plane of Jupiter's orbit lie at *s*′ and *s*, and their respective distances above and below that plane are represented on the scale of the figure by the lines *k k*′ and *l l*′. $J_1 J_3$ is the major axis of Jupiter's orbit, $J_1$ being the perihelion; $C_j$ is the centre, and $J_2 J_4$ the minor axis: $C_j J_1$ is 494,256,000 miles; $C_j S$. 23,854,000 miles. Similarly $s_1 s_3$ is the major axis of Saturn's orbit, $s_1$ being the perihelion; $C_s$ is the centre, and $s_2 s_4$ the minor axis: the dimensions of Saturn's orbit have been given in Chapter II.

The last conjunction of the two planets took place on the 28th of December, 1861, at about a quarter past seven in the evening, the heliocentric longitude of each planet being 166° 51′ 17″ at the moment of conjunction. Thus, Saturn and Jupiter were situated as at $P_1$ and $Q_1$ respectively, the points $P_1$, $Q_1$, and S, being in a

* The longitude of the rising node of Saturn's orbit on the plane of Jupiter's orbit is 126° 32′ 41′; these planes are inclined to each other at an angle of 1° 15′ 41″. It must be remarked that the point marked ♈ in fig. 7, represents the first point of Aries at the commencement only of the motions considered. During the interval (more than 99 years) in which the six conjunctions occur, the first point of Aries regredes (that is, approaches N′) by nearly 1° 23′. Changes, less marked but still not unimportant, occur also in the forms of the orbits of Jupiter and Saturn, and in the position of the line N N′.

straight line inclined at an angle of 166° 51′ 17″ to S ♈. Starting from this line of conjunction, Jupiter has been continually gaining on Saturn—so that, for instance, at the present instant (January 1st, 1865, 0h. 30m. P.M.) Jupiter is 47° 20′ 52″ in advance of Saturn. Thus, when Jupiter again arrives at $Q_1$, Saturn will not have advanced much beyond the point N′, and continuing their motions, they will be in conjunction along the line $P_2Q_2$, the arc $P_1N'P_2$ being about two-thirds * of the complete orbit of Saturn. Now, by what has been already shown, if Saturn and Jupiter moved uniformly with their respective mean motions, the arc $P_1P_2$ (or, which is the same thing, the arc $Q_1Q_2$) should exceed an arc of 240° by one-third of 8° 6′·4,—or $P_1P_2$ should be an arc of 242° 42′·1. This is not the case, however, under the actual circumstances. Saturn's mean daily angular motion is 2′ 0″·6; his maximum daily motion (when he is in perihelion) is 2′ 15″·3; his minimum daily motion (when he is in aphelion) is 1′ 41″·2: again, Jupiter's mean daily angular motion is 4′ 59″·3, while his greatest and least daily motions are respectively 5′ 30″·5 and 4′ 32″·3. It is clear that these variations are sufficient to introduce very important modifications into all the circumstances of the motions of the two bodies. In moving from $P_1$ to $P_2$ Saturn passes his aphelion point, and thus his mean motion during the interval is less than his mean motion in a complete revolution. Jupiter's motion in the interval may be divided into two parts: first, the complete revolution beginning from the point $Q_1$—in this, of course, he may be considered to move with his mean motion, or 4′ 59″·3; secondly, the motion through the arc $Q_1Q_2$, comprising the semi-orbit $J_3J_4J_1$ in which Jupiter passes from aphelion to perihelion (and therefore may be considered to move with the same mean motion as in a complete revolution), and the two nearly equal arcs $Q_1J_3$ and $J_1Q_2$, one next to aphelion, the other next to perihelion, Jupiter's small velocity in the former being compensated by his greater velocity

* In Herschel's 'Introduction to Astronomy' the second conjunction is made to take place 123°, the third 246° and the fourth 368° 6′ from the first. It is clear that if this were the case, Saturn would perform one revolution while Jupiter performed four, which, as we have seen, is not the true relation between their motions. A similar mistake occurs in the description of the motions of Venus and the earth, in Mitchell's 'Popular Astronomy.'

in the latter. Thus, on the whole, during the interval between the two conjunctions, Jupiter may be considered to move with a mean velocity almost exactly identical with his mean velocity in a complete revolution. Saturn, as we have seen, moves with a mean velocity less than his mean velocity in a complete revolution. Thus Jupiter gains on Saturn more rapidly than in the case first supposed,—that is, of uniform mean motions. Accordingly, the arc $P_1P_2$ is less than the arc of 242° 42′·1, obtained on that supposition,—and not by a small or scarcely appreciable difference, but by some eight or nine degrees.

In precisely the same manner it may be shown, that the next conjunction falls on the line $Q_3P_3$; that in the interval Saturn moves with a mean velocity slightly greater than his mean velocity in a complete revolution (passing over a complete half-orbit from perihelion to aphelion); and that Jupiter moves with a mean velocity less than his mean velocity in a complete revolution (passing his aphelion in moving from $Q_2$ to $Q_3$ after his first complete revolution). Thus the arc $P_2P_3$ is greater than the mean arc 242° 42′·1.

In moving to the fourth conjunction along the line $P_4Q_4$, near to, but in advance of $P_1Q_1$, Saturn passes the perihelion of his orbit, and moves with a mean velocity greater than his mean velocity in a complete revolution. Jupiter also, after a complete revolution, passes his perihelion in moving from $Q_3$ to $Q_4$, and moves with a mean velocity greater than his true orbital mean velocity. Here, then, the variations from mean uniform motion partly compensate each other. But in this case we need not examine the circumstances of motion in the interval between the two conjunctions, to determine the position of the line $P_4Q_4$. For, since the first conjunction in this neighbourhood, Saturn will have completed two revolutions and the small arc $P_1P_4$, while Jupiter will have completed five revolutions and the small arc $Q_1Q_4$; thus the arc $P_1P_4$ will only differ from the arc of 8° 6′·4 (determined from the consideration of uniform mean motions) by the effects of the variations from mean motion in the passage of Saturn over the small arc $P_1P_4$, and of Jupiter over the small arc $Q_1Q_4$. It might appear that such effects, though possibly appreciable, must be very minute; in reality they are important, as will appear from the following calculation:—

Saturn (as already shown) will arrive at the point $P_1$ 144·4846 days before Jupiter reaches the point $Q_1$. Now Jupiter at $Q_1$ has a daily motion of 4′ 34″·9, but his mean motion during 145 days preceding his arrival at the point $Q_2$ of his orbit is somewhat greater (since he is throughout approaching aphelion) and is approximately 4′ 37″·7. With this mean velocity, it is easily calculated that in 144·4846 days he passes over an arc of 11° 4′ 52″. Thus when Saturn is at $P_1$ Jupiter will be at $q$, $q\,Q_1$ being an arc of 11° 4′ 52″.* Again Saturn at $P_1$ has a daily motion of 2′ 4″·5, but his mean motion during 145 days following his arrival at the point $P_1$ is somewhat less (as he is approaching aphelion) and is approximately 2′ 4″·0. With this mean velocity he moves in 144·4846 days over an arc of 4° 58′ 36″, so that when Jupiter is at $Q_1$ Saturn is at $p$, $P_1p$ being an arc of 4° 58′ 36″. Now Jupiter moves from $Q_1$ with a daily angular velocity of 4′ 34″·9 about the sun, to overtake Saturn, which moves from $p$ with a daily angular velocity of about 2′ 3″·5 ; hence immediately after passing $Q_1$, Jupiter gains 2′ 31″·4 daily. But before the two planets are in conjunction, the velocity of each is diminished (since both are approaching aphelion), Jupiter's velocity more than Saturn's; thus the mean daily gain of Jupiter in the interval is approximately 2′ 30″. Since the angle 4° 58′ 36″ contains the angle 2′ 30″ 119·4 times, Jupiter will overtake Saturn—that is, the two planets will be in conjunction—119·4 days after Jupiter has passed the point $Q_1$ and Saturn the point $p$. Jupiter's mean daily motion in the interval being about 4′ 33″·5, $Q_1Q_4$ is an arc of 9° 4′ 16″. Thus the line of conjunction $P_4Q_4$, instead of falling 8° 6″·4 beyond $P_1Q_1$ the original line of conjunction, falls nearly one degree farther forward.

In exactly the same way the conjunctions falling near $P_2Q_2$ and $P_3Q_3$ can be determined. It will appear that $Q_2q'$, corresponding to $Q_1q$, is an arc of nearly 12° 55′·5 ; that $P_2p'$, corresponding to $P_1p$, is an arc of nearly 5° 8′·5 ; and that the line of conjunction $P_5Q_5$ falls about 8° 24′ beyond $P_2Q_2$. Lastly, $Q_3q''$, corresponding to the arcs $Q_1q$ and $Q_2q'$, is an arc of about 12° 4′ ; $P_3p''$, corresponding to

* The determination of the arc $q\,Q_1$ is not necessary to the inquiry ; the calculation is introduced to illustrate the effects of the non-uniformity of the motions of the two planets. It will be remembered that on the supposition of uniform motions the corresponding arc was shown to be 12° 0′·7.

the arcs $P_1p$ and $P_2p'$, is an arc of 4° 22′·5; and the line of conjunction $P_6Q_6$ falls 6° 51′ beyond $P_3Q_3$.*

Thus the irregularities in the motions of Saturn and Jupiter arise from several sources.

In the first place, there are irregularities due to the forms and positions of the two orbits. For instance, consider the conjunction which takes place along the line $P_1Q_1$:—This line falls above $C_j$ the centre of Jupiter's orbit, and thus divides that orbit into two unequal parts, the upper (in the figure) being the smaller; hence, considering Saturn as the disturbing body, it is clear that the radial part of Saturn's disturbing effect has a smaller purchase (so to speak) on Jupiter as he moves through the arc $J_2\,Q_1$ to conjunction, than as he moves on through the corresponding arc, beyond $Q_1$, from conjunction: on the other hand, the tangential disturbing effect is greater in the former arc than in the latter. Jupiter's motion would (on this account), be accelerated in the former arc more energetically than it would be retarded in the latter; or which is the same thing, Saturn's motion would be more powerfully retarded as Jupiter moved through the former arc, than it would be accelerated as Jupiter moved through the latter arc. But here another circumstance must be considered. As Jupiter moves from $J_2$ to $Q_1$ his motion is continually diminishing, and it continues to diminish till Jupiter reaches his aphelion at $J_3$. Further, Saturn's motion as he approaches, and after he passes $P_1$, is continually diminishing. Since Saturn's motion is so much smaller

* The relations between these angles and those resulting from uniform mean motions may be thus exhibited:—

| | Arc corresponding to $Q_1\,q$. | Arc corresponding to $P_1\,p$. | Arc corresponding to $P_1P_4$, or $Q_1Q_4$. |
|---|---|---|---|
| near $P_1Q_1$ | 11° 4′ 52″ | 4° 58′ 36″ | 9° 4′ 16″ |
| near $P_2Q_2$ | 12 55·5 | 5 8·5 | 8 24 |
| near $P_3Q_3$ | 12 4 | 4 22·5 | 6 51 |
| mean | 12 0·7 | 4 50·4 | 8 6·4 |

These results, it must be remembered, are not strictly correct. They have been obtained on the supposition that Saturn and Jupiter perform their revolutions in their respective mean periods, which, as already mentioned, is not the case. They are, however, sufficiently accurate for the purposes of our inquiry. The errors in fig. 7, plate X., arising from this cause would not be appreciable on the scale of that figure.

than Jupiter's, so also the variation of his motion in a given time is smaller in amount; thus throughout the approach and separation we are considering, the relative motion of the two bodies—that is, the excess of Jupiter's angular velocity over Saturn's—is continually diminishing. Hence the period of separation after conjunction is longer than the period of approach before conjunction; and therefore, of course, the accelerating effects on Jupiter in the latter period would, on this account, be less than the retarding effects on Jupiter in the former period; and similarly of the opposite effects operating on Saturn. Here, then, the effect of the forms and positions of the orbits, and the effect due to the varying motions of the planets in those orbits, are conflicting. If we analysed the corresponding effects at the conjunction along $P_2Q_2$, it would appear that the two effects are of the same kind as in the case just considered, and are therefore conflicting, but the effect due to rate of motion is much more marked than in the former case (since Jupiter is leaving, Saturn approaching, perihelion). If we considered the conjunction along $P_3Q_3$, it would appear that both the effects are of opposite kinds to those operating in the other cases, and therefore are still conflicting, but the effect due to the form of Jupiter's orbit* is more marked than in either of those cases, since the line $P_3Q_3S$ is nearly perpendicular to $SC_j$.

Corresponding variations in the effects of the mutual action of the planets in moving over arcs preceding and following opposition may be considered in the same manner. To avoid confusion, the lines along which the planets are in opposition, during the interval of 100 years illustrated by the figure, are not indicated. They occupy positions intermediate to those occupied by the lines of the preceding and following conjunctions.†

Secondly, we have seen that the angle between successive lines of conjunction is variable, sometimes considerably exceeding, at others falling considerably short of the mean angle 242° 42′·1. In

* Since Saturn only passes over a small part of his orbit while the planets are near conjunction, the effects due to the form of Saturn's orbit need not be considered here; they will fall under the consideration of the varying distances of the planets at different conjunctions.

† Thus, at the opposition occurring between the conjunctions along $P_1Q_1$ and $P_2Q_2$ Jupiter is near $J_2$, and Saturn near $P_3$.

this, and in the corresponding variation of the interval of time between successive conjunctions, we have new sources of irregularity, serving considerably to diminish the approach to compensation which would result from a more symmetrical adjustment of these arcs and periods. Similar irregularities occur, of course, in the angles between successive lines of opposition, and in the intervals of time in which such angles are swept out by the two planets.

Thirdly, the distance between the planets at successive conjunctions (and oppositions) is variable. It is clear from the figure that $P_1Q_1$ is less than $P_2Q_2$, $P_2Q_2$ less than $P_3Q_3$; and similarly $P_4Q_4$ is less than $P_5Q_5$, $P_5Q_5$ than $P_6Q_6$. Since the mutual action of the two planets varies inversely as the square of the distance, this is a very effective source of irregularity.

Fourthly, it appears that at, and in the neighbourhood of $Q_1P_1$, Jupiter gains more slowly on Saturn than near $Q_2P_2$, and here again more slowly than near $Q_3P_3$. In other words, the two bodies hang longer in the neighbourhood of each other near $P_1Q_1$ than near $P_2Q_2$, and near $P_2Q_2$ than near $P_3Q_3$. The disturbing effects due to their mutual attractions necessarily vary in amount according to the time during which such attractions are in operation.

Fifthly, it has been shown that the more nearly the line of any conjunction approaches to exact coincidence with the line of some former conjunction, the more nearly will the disturbing effects operating near the two conjunctions resemble each other. Now the arc $P_1P_4$ is greater than the arc $P_2P_5$, and the arc $P_2P_5$ is considerably greater than the arc $P_3P_6$. Thus the effects operating to disturb the period (or any other element) of Saturn or Jupiter, near the conjunction along $P_3Q_3$, and those operating near the conjunction along $P_6Q_6$, resemble each other more closely, than do those operating near the conjunctions along $P_2Q_2$ and $P_5Q_5$, and these again resemble each other more closely than do those near the conjunctions along $P_1Q_1$ and $P_4Q_4$. Thus that *accumulation of perturbations operating in the same way*, on which, as has been shown, the permanent, or long continued modification of the elements of either orbit depends, is more marked in the series of third conjunctions falling near $P_3Q_3$ than in the series falling near $P_2Q_2$, and in this than in the series falling near $P_1Q_1$.*

Sixthly, the gradual change in the periods of the two bodies affects the approach to commensurability on which it is itself dependent. For instance, during the series of conjunctions considered, Saturn's period is continually diminishing and Jupiter's increasing; in other words, the two periods are continually approaching more nearly to the simple relation of commensurability already indicated. Thus the repetition of disturbances operating continually in one direction becomes more and more marked during the series of conjunctions considered, and for several following sets of triple conjunctions.

Seventhly, the inclination of the planes of the two orbits has an influence, though a small one, on the effects we are considering. The greater the inclination of the line joining the centres of the two planets to the plane of either orbit, the larger is that resolved part of the mutual action which acts perpendicularly to such plane, and thus the resolved part *in* that plane is correspondingly diminished,—in other words, the perpendicular force gains at the expense of the radial and tangential forces. This diminution can never be very great, since the angle in question is always very small. It is plainly greatest in the series of conjunctions near the line $P_2Q_2$, and nearly vanishes in the series near $P_3Q_3$.

Other causes of disturbance and of variation in disturbance might be added; these, however, are sufficient to indicate the complexity of the problem. It is necessary, however, to notice that all the causes considered operate to produce variations in the efficiency of the radial and perpendicular disturbing forces.† Hence there arise changes in the eccentricities and inclinations, and in the positions of the perihelia and nodal lines of the orbits of the two planets. These effects cannot take place without influencing, in some degree,

* It does not follow that the absolute effects of those series are to be placed in the order indicated; for a series of closely-resembling small disturbances may be less efficient than a series of more marked disturbances less closely resembling each other.

† Not necessarily in the same way or to the same extent; for instance, a cause operating to diminish the tangential may increase the radial force, and *vice versâ*. Diminution of distance increases all the forces.

the action and effects of the tangential force, the variations of which depend, as we have seen, on those elements of the two orbits. A more marked effect of such changes in lengthening the cycle during which the periods of Saturn and Jupiter undergo their respective variations, will be considered presently.

To ascertain the exact amount of disturbance due to each of the causes here mentioned, and thence to determine whether, in any set of three conjunctions, acceleration prevails over retardation, or *vice versâ*, can only be effected by rigid mathematical analysis, in which the absolute quantities concerned—as the masses of the sun, Saturn and Jupiter, and the eccentricities, inclinations, and other elements of the orbits of the two planets—are fully taken into account. The results of such analysis correspond with the results of observation. For a series of conjunctions, there will remain—after each set of three conjunctions—an uncompensated acceleration of one planet and retardation of the other: then during a similar series of conjunctions, these effects are reversed, and the accumulated results of the former series gradually worked off, until an almost exact compensation is effected at the close of the complete cycle corresponding to the inequality we are considering.

The angle 8° 6′·4 is contained 44·408 times in 360°. Hence, if we were to mark the places of successive third conjunctions from that along $P_1Q_1$, we should eventually (after marking down 44 such conjunction-lines) arrive at one falling between $P_1Q_1$ and $P_4Q_4$. Thus the conjunction-line would not return exactly to the position $P_1Q_1$. But we may consider the cycle during which this conjunction-line travels completely round the circumference, to be 44·408 times the mean period of three conjunctions,—that is, 44·408 times 21,760·4 days, or about 2,645·74 years. But it is clear that in travelling round the circumference this conjunction-line must pass the lines $P_3Q_3$ and $P_2Q_2$, and when it is at or near either of these positions the two other conjunction-lines must have also travelled round, so that the three conjunctions in each triple set take place at or near the three original conjunction-lines, $P_1Q_1$, $P_2Q_2$, and $P_3Q_3$. In other words, a complete cycle of disturbances will have taken place in one-third of the period just obtained, or in 881·91 years, if the elements of the two orbits have undergone, meanwhile, no alteration. This, however, is not the case:—The perihelion of Saturn will have been advancing at the rate of more than 19″ yearly, the perihelion of Jupiter at the rate of nearly 7″; the eccentricity of Jupiter's orbit will have been increasing, that of Saturn diminishing, yearly; and the inclinations and nodal lines (on the ecliptic) of the two planets will also have varied. The relations between the orbits at the commencement of a cycle are never exactly reproduced; but it has been calculated that the series of disturbances is completed, and the periods of the two planets are restored to their original values and *states* (increasing or decreasing) in about 918 years: in each such cycle there are rather more than 46 conjunctions. Thus the disturbance in the periods of the two planets, or the *inequality*, as it is termed, attains its maximum amount either way at intervals of about 459 years, in each of which are included seven or eight sets of three conjunctions. The inequality, at such times, amounts to about 48½′ of retardation or acceleration in the longitude of Saturn, and about 20′ of acceleration, or retardation, respectively, in the longitude of Jupiter.* At present Saturn is about 41½′ behind his mean place, Jupiter about 17′ in advance of his mean place. But in each set of three conjunctions Saturn's actual motion is on the whole retarded, so that his period is diminishing and his mean motion in longitude increasing, while Jupiter's actual motion is accelerated, so that his period is increasing and his mean motion in longitude diminishing. In other words, the two planets are (at present) being gradually restored to their mean places.

Saturn's period also undergoes alterations, due to the disturbing influence of Uranus, Neptune, and even of our earth and the other comparatively minute planets that revolve within the zone of asteroids. The alterations due to the attractions of Uranus and Neptune are small, however, the others altogether insignificant, compared with those due to the attraction of Jupiter. For, in the first place, the mass of Jupiter greatly exceeds that of all the other

* The proportion between the variations in the longitudes of two planets, due to their mutual disturbances, is the inverse ratio of their masses multiplied by the square roots of their mean distances from the sun.

members of the solar system; and in the second place, no approach to any simple relation of commensurability exists between the period of Saturn and that of any of those bodies; so that alterations in one direction are compensated by corresponding alterations in the contrary direction.

The eccentricity of Saturn's orbit and its inclination to the ecliptic undergo oscillatory variations in cycles of great length, the perihelia and nodal lines shifting, meanwhile, round the ecliptic. The latter series of changes is not important: the former would be injurious, if not destructive, if the limits within which the variations took place were considerable. This, however, is not the case. Lagrange has shown that, amidst all variations, the following relations hold between the masses of the various members of the solar system, and the mean distances, inclinations, and eccentricities of their orbits:—

(i) If the mass of each planet be multiplied by the square root of its mean distance from the sun, and the product by the square of the eccentricity of the orbit, the sum of all such products taken throughout the system is invariable.

(ii) If the same product (of the mass by the square root of the mean distance) is multiplied by the square of the tangent of the angle at which the plane of the orbit is inclined to a fixed plane, the sum of all such products taken throughout the system is invariable.

The products corresponding to Saturn and Jupiter are by far the most important in either series.* As these products correspond to but moderate eccentricities in the orbits of these two planets, it is clear that the eccentricity of either orbit could never become very great, even if it were possible for the eccentricity of the other absolutely to vanish. This, however, can never happen. If Jupiter and Saturn were the only members of the system the effects of their mutual attraction would be that in a cycle of 70,414 years the eccentricity of Saturn's orbit would vary between

* The sum of the masses of Saturn and Jupiter is fully 11½ times as great as the sum of the masses of all the remaining members of the solar system (yet known); and the sum of the masses of Neptune, Uranus, Saturn, and Jupiter, exceeds the sum of the masses of Mercury, Venus, the Earth, Mars, and the asteroids, more than 200 times.

the limits 0·08409 and 0·01345, that of Jupiter's between the limits 0·02606 and 0·06036,* 'the greatest eccentricity of one orbit corresponding to the least of the other, and *vice versâ.*' And what is of far more importance, so far as the inhabitants of the earth or of the other minor planets are concerned, no large part of the product corresponding to either of the two giant members of the system, or to Uranus and Neptune, can ever be imparted to the smaller bodies.† An interchange of eccentricities and inclinations is always taking place between every pair of members of the system, whether near or distant, or whether either or both is small or great; but analysis shows that this interchange is so distributed that the eccentricities and inclinations of the minor orbits can never become very great.

* Sir J. Herschel's 'Introduction to Astronomy,' 1st Edition, p. 368.

† A very simple calculation will suffice to show that if the orbits of the four exterior planets could become circular simultaneously, so that the whole of the eccentricity-products corresponding to them were distributed among the minor planets, all these bodies would be destroyed by falling into the sun, their orbits becoming too elongated to allow them to clear his globe.

# CHAPTER VII.

## HABITABILITY OF SATURN.

When we consider the analogy of our own planet, it seems impossible to doubt that Saturn is inhabited by living creatures of some sort. *Here* we find, not only the earth, but the fathomless depths of ocean, not only the temperate zones, but the scorched regions of the tropics, and even the solid ice within the arctic and antarctic circles, crowded most abundantly with living creatures. Here also we find that, not merely while the conditions now holding have subsisted, but throughout millions of ages, during which the earth has undergone variations of the most marked and startling nature, the same abundance of life has been found upon its surface. That a globe so stupendous as Saturn, and surrounded by a system so magnificent and elaborate, should be devoid of inhabitants, can hardly, then, be reasonably imagined: but what manner of creatures subsist on Saturn—whether it is inhabited, as yet, by comparatively rudimentary races, or whether it is already peopled by reasoning and responsible beings, capable of appreciating the wonders that surround them, and adoring their Almighty Creator—it is not given to us to know.

On our own planet we find creatures of every race admirably adapted to the conditions that surround them. Whether we suppose such adaptation to be the result of express creative acts of the Almighty, or—which appears more probable—that, in His infinite wisdom, He has appointed laws under whose action species are modified with the varying conditions that surround them, analogy points to the conclusion that on the other members of the solar system the same perfect adaptation prevails. It is, therefore, merely by way of comparison, that I propose to examine, in this chapter, the adaptation of the physical conditions of the planet Saturn to the wants of beings constituted like the inhabitants of our earth: it is not necessary to establish the subsistence of conditions so adapted, in order to prove that the planet is the abode of living creatures.

If an inhabitant of our earth could be placed on the surface of another planet, it is probable that the first circumstance in his new condition that would attract his attention would be the change in his own weight. If he were removed to Jupiter, he would find his weight more than doubled, and would be unable to move without pain and difficulty. If, on the other hand, he were removed to Mars or Mercury, he would find his weight diminished by more than one half, and his activity and apparent muscular power correspondingly increased. If he were removed to Saturn, the change in his weight would vary with the latitude of the spot to which we suppose him to be conveyed. Owing to the compression of Saturn's globe, his vast size, and his rapid rotation on his axis, gravity varies with latitude in a much more marked manner than on our earth. If Saturn were not rotating, the weight of a terrestrial pound would be about 1·19 lbs. at Saturn's pole, and 1·17 lbs. at his equator. But the centrifugal force at Saturn's equator is about 0·164,* by which amount gravity is still further diminished. Thus, a man weighing 12 st. on earth, would weigh only a few ounces over 12 st. at Saturn's equator, but would weigh more than 14 st. at Saturn's pole. The difference of weight in the former case would hardly be appreciable: in the latter it would prove a heavy burden; but its effect would be somewhat diminished by perfect adjustment, since it would be distributed over the whole body. A Saturnian would find his weight increased in nearly the same degree, if he travelled from Saturn's equator to either pole: we shall presently see that this is not the only circumstance in which the physical conditions of Saturn's arctic and temperate zones present a marked contrast to those prevailing at and near his equator.

* By this is meant that a mass weighing 1 lb. on the earth and revolving about a centre with the same velocity and at the same distance as points on Saturn's equator about Saturn's centre, would require, to retain it in its orbit, a force sufficient to counterpoise a weight of 0·164 lbs.

It has been considered probable that the appearance of Saturn's surface differs greatly from that of our earth. And this for two reasons:—In the first place, it is urged that his density being so small, he must be composed of materials very much lighter than, and therefore very different from, those composing our earth; and, in the second place, that fluids upon his surface must either be of less density than the planet, and therefore very different from our oceans, or if of greater density, must all be collected in one hemisphere.* Saturn's globe may, however, be hollow, and the mean density of the materials of this hollow globe not very different from the mean density of the materials composing our earth.† And, again, it has not been established by rigid mathematical inquiry, that oceans upon a planet of Saturn's figure, would necessarily be collected wholly, or almost wholly, in one hemisphere, if their density exoeeded that of the planet. On the contrary, it appears probable that fluid masses on the surface of such a planet would tend to form two vast polar oceans, since gravity is so much greater

* Whewell's 'Astronomy and General Physics:' (Bridgewater Treatise.)

† Whether the earth is solid throughout or merely a spherical shell is a question on which the world of science is divided. The increase of heat as we proceed inwards seems to indicate that at no very great depth the heat must be so intense that all known substances would be converted into fluids. On the other hand, Mr. Hopkins has shown that the precession of the earth's pole is not such as it would be if the earth were a shell of such comparatively small thickness containing a vast fluid mass. Arguments of some force have also been urged to show that the above-mentioned increase of heat is not to be considered as an indication of a fluid nucleus; and it is certain that man has penetrated the earth's crust to a distance absolutely insignificant compared with the dimensions of the earth's globe. Yet it seems clear that the balance of probability is largely in favour of a continual increase of heat inwards, in even a greater ratio than that observed near the surface. And it appears not improbable that at a depth of a thousand miles the heat should be so intense that all known substances would at ordinary pressures be converted into vapour. But the pressure exerted by a vapourous nucleus on the surface of the fluid shell next to it, and by transmission on the solid shell, must be so immense that the interior parts of the solid and liquid shells must owe their solidity and fluidity respectively to the intensity of such pressure, and not to the insufficiency of the heat in those parts to change respectively solidity into fluidity, and fluidity into gaseity, at ordinary pressures. Thus the thickness of either shell may be far greater than would appear from any calculation founded on ordinary pressures. It is also conceivable that the immensity of the pressure exerted by the gaseous nucleus would be sufficient to modify the motions of the fluid shell, and that by combining the effects of such modification with the increased thickness of the fluid crust deduced from the consideration mentioned above, the prec ion of the earth's pole might be accounted for as exactly as on the supposition of the solidity of the whole mass of the earth's globe.

at Saturn's poles than at his equator. But even if it were proved that the former arrangement must inevitably subsist, what, after all, is such an arrangement but an almost exact counterpart of what is observed on our own earth? * It is true that tidal waves could not sweep round such an ocean, as round the oceans that surround the earth, but an ocean whose tides are ruled by eight satellites, and restrained by the attractions of a stupendous ring, may require arrangements altogether different, in this respect, from those prevailing on our earth. The appearance of Saturn, however, is not favourable to the supposition that the ocean masses on his surface are confined to a single hemisphere; for the bright bands on

* This arrangement on the earth is modified by the tendency towards the poles that might be expected from the earth's form; so that while the southern hemisphere (or more exactly the hemisphere of which New Zealand forms the central region) is evidently that towards which the main body of the water is attracted, the northern polar regions are also occupied by a vast ocean, connected with the southern ocean by Behring's Straits and the Atlantic. From this arrangement and the conformation of the land, it is obvious that *at present* the centre of gravity of our globe lies nearer the southern than the northern pole. M. Adhèmar has suggested that this displacement of the centre of gravity from the centre of figure is due to the vast masses of ice collected at the southern pole, and that as the duration of the antarctic summer is now continually increasing, those masses will diminish, and the frozen masses at the arctic pole increase, until the centre of gravity is nearer the northern than the southern pole, when the great southern ocean will rush northward. He conceives, in fact, that a vast flood takes place twice in every revolution of the vernal equinox (that is, twice in 25,868 years), the ocean masses rushing alternately from pole to pole; and he imagines that the successive states of submersion and emergence undoubtedly passed through by every part of the earth's surface may be better explained in this way than by the supposition of alternate elevations and depressions from internal causes. The close observer of nature will not readily accept the idea of such cataclysmal floods, destroying all living creatures on the face of the earth at each eruption. It is not altogether improbable, however, that the ocean masses may oscillate from pole to pole in a more gradual manner, and that during such oscillations inundations might take place, insignificant when compared with the universal floods imagined by Adhèmar, but sufficient to constitute tremendous local catastrophes, and to leave lasting traces of their effects. The results of such oscillations would differ in no respect from those of elevations and subsidences of continents. It may be remarked that while elevations and depressions of large tracts of the earth's surface have undoubtedly taken place, it appears improbable that whole continents should be so raised or depressed; and the expression sometimes met with in works on geology, that a whole hemisphere may be elevated by internal forces while the opposite hemisphere is depressed, is simply an absurdity. Such changes are inconsistent with the simplest law of mechanics, that 'action and reaction are opposite and equal.' Forces tending to elevate one hemisphere must *bear upon* and therefore tend to elevate the opposite hemisphere,—must tend, in fact, to lengthen that diameter of the earth along which their resultant acts.

Saturn's disc, which are probably vast belts of clouds drawn from oceans upon his surface, are found equally in the northern and southern hemispheres, and extend completely round Saturn's globe.

The climatic conditions on the surface of Saturn undoubtedly differ in the most striking manner from those which prevail on the earth. We may consider three points on which these conditions depend; namely:—the distance of Saturn from the sun; the inclination of his axis to the plane of his orbit; and the respective lengths of the Saturnian day and year.

We have seen that Saturn's mean distance from the sun is more than $9\frac{1}{2}$ times as great as the mean distance of the earth. Thus the diameter of the sun's disc appears less to the Saturnians than to us in the proportion of 2 to 19; while the apparent surface of the solar disc, which varies as the square of the apparent semi-diameter, appears diminished to about $\frac{1}{90}$th part of the apparent surface of the disc visible to us. The quantity of light and heat received on any part of Saturn's surface is therefore only $\frac{1}{90}$th part of the quantity received on a part of the earth's surface of equal extent, and equally inclined to the solar rays. In fact, notwithstanding the immensity of Saturn's globe, the whole of the light and heat received upon it, when Saturn is at his mean distance from the sun, is considerably less than the light and heat similarly received on the earth. It does not necessarily follow, however, that the climate of Saturn is so bleak and frigid as that of the earth would be under a corresponding diminution of the solar heat; for, independently of the consideration that the climate of any planet may be greatly affected by internal heat, there can be no doubt that the amount and density of the atmosphere that surrounds a planet has a most important influence on the climatic conditions that prevail upon its surface.* That Saturn has a very extensive, and therefore (at his surface) a very dense atmosphere, seems probable from the appearance presented by his disc in powerful telescopes, as well as from his vast absolute dimensions. Such an atmosphere can, of course, have no effect in increasing the

* If the atmosphere of our earth were suddenly subjected to such a change that heat radiating from the earth passed through the air as freely as the sun's direct heat, the earth would no longer be habitable by such races as now exist upon its surface.

amount of heat received upon any part of Saturn's surface, or rather, tends somewhat to diminish that amount; but by preventing radiation, it may serve to maintain a mean temperature as high as the mean temperature of our globe, or even considerably higher.* The amount of light received would not be increased by such an arrangement, except by the comparatively small amount refracted towards the planet by the atmosphere, and the consequent lengthening of the Saturnian twilights. That the surface of Saturn is illuminated with considerable brilliancy, however, may be inferred from the brightness of his disc. Although it is less splendid than the discs of planets nearer the sun,† there is no approach to the sombreness and gloom that one would expect from a diminution of the solar light to so small a fraction of that received upon our earth. It has been calculated, however, that under such a diminution the sun would still supply 560 times as much light as the moon at full—a calculation confirmed by the small loss of light in partial eclipses of the sun. There is therefore little reason for supposing that the quantity of light received by Saturn would be insufficient even for such forms of life as are found upon our earth; still less reason is there for supposing that no forms of life whatever could subsist on Saturn's surface.‡

* Mr. Hopkins has calculated that if the atmosphere of our earth were increased in height by about 40,000 feet, the earth would be maintained at its present temperature, *if exposed only to the radiation of space, in the total absence of the sun.* See Nichol's 'Cyclopædia, Appendix, Atmospheres of Planets.' This result, however, can hardly be considered as satisfactorily established.

† Reference is not here made to absolute splendour, which depends on the magnitude of a planet and its distance from the earth, but to the *intrinsic* brilliancy of the disc, which is independent of those relations. The faintness of Saturn's light compared with that of the moon was very observable at the reappearance of Saturn on the moon's bright limb after the occultation of May 8th, 1859.

‡ It is probable that our own earth once received much less light than at present. This is indicated by the size of the eye-orbits in many extinct species of animals, and by the development attained by creatures of the bat kind, which now form an insignificant class of the earth's inhabitants. Thus, Hugh Miller, speaking of the remains of animals of the secondary division, says, 'enormous jaws, bristling with pointed teeth, gape horrid in the stone, under staring eye-sockets a full foot in diameter;' and again, 'here we see a winged dragon,' the Pterodactylus Crassirostris, 'that, armed with sharp teeth and strong claws, had careered through the air on leathern wings like a bat.'—'Testimony of the Rocks,' Lecture III. The pterodactyles of the greensand exhibit not uncommonly a spread of wing of eight or nine yards! See also Note B, Appendix I.

Saturn's axis is inclined at an angle of 26° 49′ 28″ to the plane of Saturn's orbit. Thus the Saturnian seasons, so far as they depend on this element, are not very different from those which prevail on the earth. In Table XI. (Appendix II.), the points of the horizon at which the sun rises and sets, the elevation of the sun at noon, the diurnal arc traversed by the sun, and the length of the Saturnian day, are given for the equinoxes and solstices of latitudes 0°, 5°, 10°, . . . . . 60°; 65°, on Saturn's globe. These parts of Table XI. are calculated just as similar tables would be calculated for the earth. Thus:—suppose we require to obtain these arcs for a latitude of 42° upon Saturn's surface. Let $H_s Z H_n$ fig. 1, Plate XI., represent the projection of the celestial hemisphere visible from a place in such a latitude (supposed north of Saturn's equator); $H_s O H_n$ being the projection of the horizon-circle, whose south point is at $H_s$, its north point at $H_n$, its east and west points at O.* Then $H_s Z H_n$ is part of the meridian of the place; and $P_n$, the north pole of the heavens, is elevated above $H_n$ in an arc of 42°. Let O Z be perpendicular to $H_s H_n$, so that Z is the zenith. Draw O S perpendicular to O $P_n$; then O S is the projection of the sun's path at either equinox. Again, take arcs S S′ and S S″, each of 26° 49′ 28″, on either side of S, and draw S′M and S″M′ parallel to S O. Then S′M and S″M′ are the projections of the sun's path at the summer and winter solstices, respectively: thus, the arcs $H_s$S, $H_s$S′ and $H_s$S″, give the *mid-day altitudes* of the sun at the equinoxes, at the summer solstice, and at the winter solstice, respectively. Now imagine the horizon-circle turned about the diameter $H_s H_n$, so that the east point moves (along O Z) from O to Z; then the points at which the sun rises at the summer and winter solstices, respectively, move along the lines M *m* and M′*m′* (parallel to O Z) to the points *m* and *m′*. Thus the arcs $H_s$*m* and $H_s$*m′* give the *azimuths* (measured from the south point) of the rising or setting sun, at the summer and winter solstices respectively.† Again, M S′ is the projection of part of a small circle of the sphere $H_n Z H_s$. If we imagine this circle turned about the line S′M, it would, when fully open,

* The east point between O and the eye, the west point beyond O.

† The arcs Z *m* and Z *m′* are respectively the northern and southern *amplitudes* of the sun at the summer and winter solstices respectively.

*Fig. 1.*

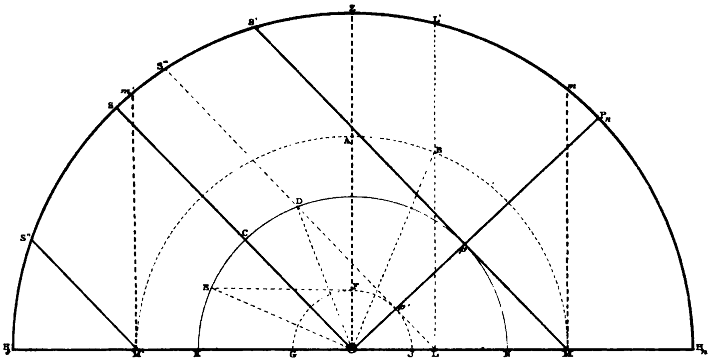


*Fig. 2.*

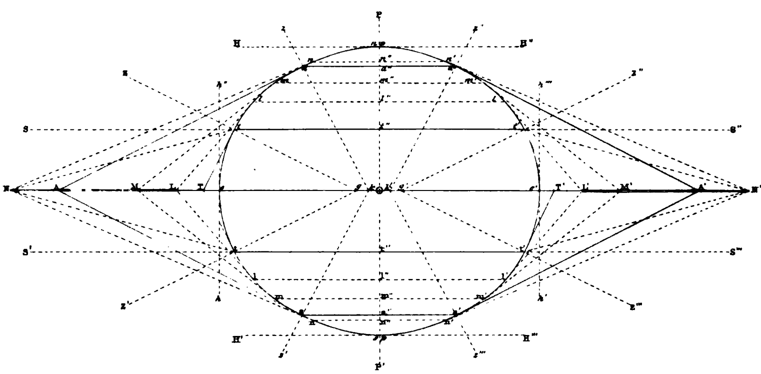
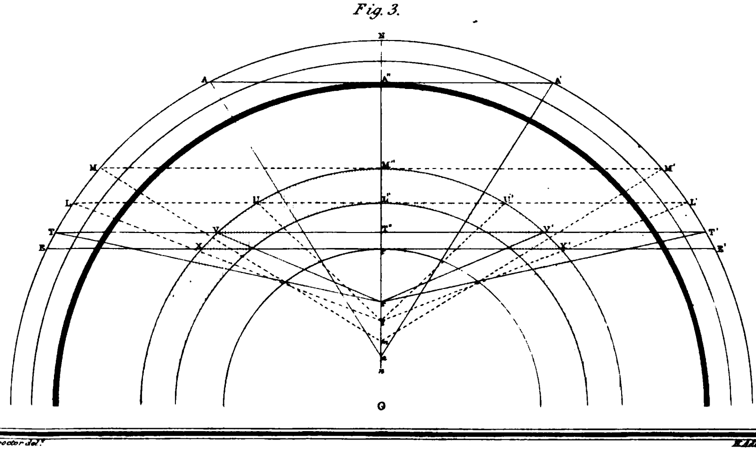


*Fig. 3.*



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appear as a circle about the point *p* as centre, and with a radius equal to *p* S′; the points corresponding to the rising and setting sun would move along a line through M at right angles to S′M, and this line would divide the circumference of the circle into two unequal arcs, the larger representing the *diurnal*, the smaller the *nocturnal arc* traversed by the sun at the summer solstice. The lengths of the day and night are, of course, respectively proportional to the lengths of the diurnal and nocturnal arcs traversed by the sun; and, further, the length of the day at the summer solstice is equal to the length of the night at the winter solstice, and *vice versâ*.

The elements considered in the preceding paragraph may also be determined for any given part of a Saturnian year. Thus, let A O B be the angle swept out by Saturn about the sun from the vernal equinox at the given period. Describe a circle M A M′ about O as centre, with O M or O M′ as radius; let O B meet this circle in B. Then, if L′B L be drawn perpendicular to $H_{,}H_{n}$, and L D S‴ parallel to O S, L S‴ represents the projection of the sun's diurnal path on the celestial sphere at the period considered, $H_{,}$S‴ is the sun's *meridian altitude*, $H_{,}$L′ the sun's *azimuth* (measured from the south point) at rising or setting, and the *diurnal arc* traversed by the sun is obtained, as before, by supposing the circle of part of which L S‴ is the projection, turned about L S‴. A similar construction applies for all latitudes, and for all parts of the Saturnian year—the line O $P_{n}$ assuming all positions from O $H_{n}$ to O Z, and the line O B sweeping from A through the complete circle of which M′A M is the semicircle.*

* The following construction is more convenient:—Let *p* be the point in which O $P_{n}$ meets the line S′M, and describe a circle K *p* N with centre O and radius O *p*; then if O D be drawn so that the angle D O C is equal to the angle swept out by Saturn about the sun from the vernal equinox, S‴D L through D parallel to O S, and L L′ perpendicular to $H_{,}H_{,}$ are the lines obtained by the former construction: for, if S‴L (as before obtained) meets O $P_{n}$ in *p*′, we have O *p*′ : O *p* :: O L : O M; that is, O *p*′ : O D :: O L : O B, or the right-angled triangles D O *p*′ and B O L are similar, and the angle O D *p*′ is equal to the angle O B L, that is, the angle D O C equal to the angle B O A. When the arcs corresponding to $H_{,}$ S‴ and $H_{,}$ L′ are required for different latitudes, the following is the most convenient construction:—The circle K D N is plainly a fixed circle for all latitudes (since S S′ is an arc of fixed length); thus, if E O be drawn so that the angle E O K is equal to the angle D O C or A O B, then E F perpendicular to O Z gives the radius

Again, if the azimuth of the sun at rising or setting (in any given latitude) is known, it is easy to determine the meridian altitude of the sun. Thus, if the arc $H_sL$ is known, we have only to draw $L'L$ perpendicular to $H_sH_n$, and $L\,s'''$ parallel to $O\,S$; then $H_ss'''$ is the altitude required. Further, if $L'L$ meets the circle $M\,A\,M'$ in $B$, then $A\,O\,B$, or its complement, is the angle swept out by Saturn from the vernal equinox about the sun. Similarly, if the altitude $H_ss'''$ is known, then by drawing $s'''L$ parallel to $S\,O$, and $L\,L'$ perpendicular to $H_sH_n$ we obtain the azimuth $H_sL'$, and the angle $A\,O\,B$, or its complement, as before.*

The Saturnian year, as already stated, contains about 29½ of our years, or nearly 10,759¼ of our days. The contrast between the Saturnian year and our own is rendered more marked by the shortness of the Saturnian day, of less than 10½ hours. Thus, the Saturnian tropical year contains 24,618½ Saturnian days.†

The most remarkable effect of Saturn's rapid rotation on his axis must be the rapid apparent motions of celestial objects. The diurnal motion of the sun, viewed from the earth, varies with the position of the sun on the ecliptic; at the equinoxes, when this motion is greatest, the sun moves over one degree of arc in 4 minutes of time, or over a space equal to his own apparent diameter in about 2 minutes. Viewed from Saturn, the sun (at the Saturnian equinoxes) moves over one degree of arc in about 1¾ minutes of time, or over a space equal to his own apparent diameter in about 6 seconds. All the celestial objects near the Saturnian

$F\,O$ of a circle $G\,F\,J$, whose intersection ($p'$) with $O\,P_s$ determines the lines $s'''L$ and $L\,L'$ in whatever direction $O\,P_n$ is drawn; that is, for all latitudes.

* It is assumed throughout that the diurnal path of the sun is a declination-parallel, which is not exactly the case, since the sun's declination is continually varying. In the case of Saturn, however, the day is so short compared with the year, that the sun's daily path differs inappreciably from a declination-parallel. Thus the part of Table XI. which states the meridian altitudes of declination-parallels having given azimuths on the horizon, and the azimuths on the horizon of declination-parallels having given meridian-altitudes, may be supposed to refer to the diurnal paths of the sun at different seasons in given latitudes, *where the given azimuths and altitudes fall within the range traversed by the sun in the given latitudes.* See note (4), Table XI.

† The sidereal Saturnian year contains 24,620½ Saturnian days; but owing to a slow precessional movement of the Saturnian equinoctial points, the tropical year is somewhat shorter. See Tables VII. and VIII., and the explanation of these tables and of Table X.

equinoctial line, except the satellites, have about the same apparent diurnal motion. The diurnal motions of the satellites are diminished by their own motions round Saturn, which take place in the same direction as Saturn's motion of rotation upon his axis: for instance, since Mimas revolves once around Saturn in about 22½ hours, the apparent motion of Mimas upon the celestial sphere is little more than one-half the apparent motion of the sun. The apparent motions of the satellites composing the ring (if these are separately visible) must be very slow, since these bodies revolve about Saturn in periods very little greater than his own period of rotation.

If our year and day were changed to the Saturnian year and day, respectively, the change in the year would undoubtedly produce far more important effects than the change in the day. The effects of the former change on the vegetable world would be not merely injurious, but destructive, except possibly within the tropics. All classes of animals, man included, would also suffer greatly in a winter or summer lasting nine years, and still more from the destruction of vegetables, plants, and trees. It is therefore probable that the vegetable and animal worlds on Saturn's surface, are, in general, very differently constituted from those which are found on our globe. Near the Saturnian equator there are two summers in each Saturnian year, and the variations of the seasons are not very marked. Here, therefore, if anywhere, the races existing upon Saturn may resemble those found on our earth. We shall see also, presently, that the intensity of the Saturnian winters in the zones corresponding to our temperate zones, is aggravated by long eclipses of the sun's light by the rings, whereas near the equator the corresponding eclipses take place near the equinoxes and are of comparatively short duration.

Saturn's orbit is more eccentric than that of the earth, and consequently the light and heat received by Saturn vary in a more marked manner than with us. Thus, the light and heat received by Saturn at perihelion, mean distance, and aphelion, are respectively as 46, 41, and 37. Let us consider the effects of this arrangement. The light and heat momentarily received by a planet vary inversely as the square of the planet's distance from the sun. From the equable description of areas, it follows that the momentary angular velocity of a planet about the sun also varies inversely as the square of the planet's distance from the sun. Thus, the light and heat momentarily received by a planet vary directly as the planet's angular velocity about the sun. It clearly follows, therefore, that the light and heat received by a planet in any time are directly proportional to the angle swept out about the sun in that time. Now, neglecting the precession of the equinoctial points, Saturn sweeps out the same angle about the sun (though not in the same time), in moving from the vernal to the autumnal equinox, as in moving from the autumnal to the vernal equinox of either hemisphere. He therefore receives the same amount of light and heat in each interval. Like our earth, Saturn passes his perihelion near the summer solstice of his southern hemisphere, and his aphelion near the summer solstice of his northern hemisphere; but the summer of the northern hemisphere lasts nearly sixteen years, the summer of the southern hemisphere less than fourteen years. The difference of time exactly counterbalances the difference of distance, so far as the light and heat received in the two intervals are concerned; but this arrangement clearly tends to equalise the seasons of the northern hemisphere, and to make their contrasts more marked in the southern hemisphere.*

It may easily be shown that the effect of the eccentricity of Saturn's orbit is to increase the total amount of light and heat

* Sir John Herschel in an article 'On the Astronomical Causes which may influence Geological Phenomena' (Geological Transactions, 1832), says, that the corresponding arrangement in the case of the earth exactly equalises the seasons of either hemisphere. This is not the case, however: the same amount of heat is received in the longer summer of the northern as in the shorter summer of the southern hemisphere; but it no more follows that such a compensation affords to each hemisphere 'an equal and impartial distribution of heat' than it would follow that two winters are equivalent in their effects to one summer because the same light and heat are received in the former as in the latter period. It is chiefly owing to the small eccentricity of the earth's orbit that the seasons of the northern hemisphere resemble so closely the seasons of the southern hemisphere; possibly the great southern ocean has a considerable influence in equalising the southern seasons. If the eccentricity of the earth's orbit were great the difference of summer and winter in the southern hemisphere would be greatly exaggerated; and the fact that the angular velocity of the earth about the sun varied accordingly, would not render the effects of the intense heat of mid-summer and cold of mid-winter less distressing to the inhabitants of that hemisphere.

received in a sidereal revolution of Saturn about the sun. Thus, let us compare the heat received by two planets (which let us call P and P′) revolving in orbits of different eccentricities, but whose major axes are equal. Let the minor axes of the two orbits be respectively $b$ and $b'$, $b$ being greater than $b'$. Then, since the periods of the two planets are equal (for their mean distances are equal), it follows from the equable description of areas about the sun, that the area swept out by P in any time : the area swept out by P′ in the same time :: the area of P's orbit : the area of P′'s orbit, or (by a well-known property of the ellipse) :: $b : b'$. Now, suppose both planets to be at their mean distances from the sun, that is, suppose they are equally distant from the sun; then the area swept out by P in any time : the area swept out by P′ in the same time :: $b : b'$; but considering the motion of each for a very small interval of time, it follows from the equality of the distances that the area swept out by either varies as the angle; hence, the angle swept out by P in any very small time : the angle swept out by P′ in the same time :: $b : b'$. In this time P and P′ receive equal amounts of heat from the sun, since they are equally distant from him; hence, from what was shown in the last paragraph, it follows immediately that the heat received by P in a sidereal revolution : the heat received by P′ in a sidereal revolution :: $b' : b$.* Or generally, the major axis of a planet's orbit remaining unaltered, the light and heat received in a sidereal revolution vary inversely as the minor axis of the planet's orbit. We have seen, however (Chapter II.), that though the eccentricity of Saturn's orbit is very observable, the minor axis is very nearly equal to the major axis, and thus the amount of heat and light received by Saturn is very little greater than it would be if Saturn revolved in a circular orbit at a distance equal to his present mean distance from the sun.

* Thus:—Let $h$ be the heat received by P and P′ in the short interval of time considered; $a$ and $a'$, the angles respectively swept out by P and P′ about the sun in that time; H and H′ the heat respectively received by P and P′ in a sidereal revolution: then from what was shown in the preceding paragraph, it follows that—

| | | | | | |
|---|---|---|---|---|---|
| | H : $h$ :: | 4 rt. angles | : | $a$ | ; |
| and | $h$ : H′ :: | $a'$ | : | 4 rt. angles | ; |
| ∴ (*ex æq. in prop. pert.*) | H : H′ :: | $a'$ | : | $a$ | :: $b' : b$. |

Of the electrical conditions of Saturn, as of the other planets, nothing is yet known. It is not altogether impossible, however, that as we approach the physical interpretation of the phenomena of terrestrial magnetism—phenomena no doubt chiefly due to cosmical causes—we may become acquainted with influences forming new and important bonds of union between the members of the solar system.*

* The connection between certain magnetic variations and disturbances in the solar atmosphere has been placed beyond a doubt; it remains, however, to be proved that the phenomena stand in the relation of cause and effect. Since it is highly probable that the solar spots affect the supply of light and heat, it is not improbable that they also influence electrical conditions. We may consider, therefore, that Humboldt was too hasty in contemptuously rejecting the idea of such influence. Both sets of phenomena, however, may be due to some cause yet undetermined. The correspondence of the decennial period of each with Jupiter's period of revolution has been noticed. It may be remarked, however, that if the influence of Jupiter depended on any other relation than distance, such influence would operate twice in each sidereal revolution; and his variations of distance (from the sun) are so small that no great influence can be attributed to them. The greatest number of solar spots are found also in years when Jupiter is not near his perihelion, as, for instance, in the year 1848, when he was nearer aphelion than perihelion. And, again, as far as terrestrial magnetism is concerned, we should expect any influence exerted by Jupiter to be greatest when he is in opposition, and no such variation has hitherto been noticed. It is not altogether impossible that in the successive conjunctions and oppositions of Saturn and Jupiter we may find a more satisfactory explanation of the decennial period in question; for these take place at intervals of about 3,627 days, or nearly ten years. It is conceivable that the presence of both these planets along the same line through the sun, whether in opposition or conjunction, would exercise a great influence over the zones of cosmical bodies revolving about the sun, on whose motions, in all probability, the supply of solar light and heat in great measure depends (see Appendix II., Explanation of Astronomical Terms, *Zodiacal Light*). It follows from the results of Chapter VI., that the conjunctions and oppositions of Saturn and Jupiter occur successively along lines inclined to each other at angles of about 58° 39′, each line of conjunction or opposition falling by that amount behind the preceding line of opposition or conjunction. Thus these lines complete the circuit of the two orbits in a retrograde manner in a period of about fifty-nine years. There must be a part of the circuit in which the influence of the planets in opposition or conjunction is most effective; and we should therefore expect to find the successive maximum disturbances going through a series of variations in a period of about fifty-nine years. Observation appears to indicate that such variations actually take place in a period of fifty-six years, though a long interval must elapse before the true period can be considered as established.

It has long been recognised that the disturbance prevailing along the equatorial zone of the sun's surface must be due to external causes, combining their effects with those due to the sun's rotation, 'which *alone* can produce no motions when once the form of equilibrium is attained.' External causes are clearly indicated also by the forms of the solar spots: their widest openings are outwards, which would not, probably, be the case

The physical conditions and phenomena of Saturn's globe must be affected in a marked manner by the rings that circle about him. These serve to illuminate the short summer nights, and to darken the short winter days of the Saturnian. Let us examine the nature of these effects, and the manner in which they are produced.

In the first place, neglecting all consideration of the illumination of the system, let us consider what parts of the rings are above the horizon, and what are the apparent outlines of such parts on the celestial sphere, for different latitudes on Saturn's surface. We may also, for the present, neglect the refractive effects of Saturn's atmosphere.

In fig. 2, Plate XI., let *n e* n′*e*′ represent a section of the planet's globe through the centre O; and let N L L′N′ represent the corresponding section of the ring,—A A′ being the sections of the division between the two bright rings, M L and M′L′ the sections of the dark ring. Let *n.p.* be the north pole and *s.p.* the south pole. Thus if the figure be supposed to revolve about the line P O P′ it would generate surfaces representing Saturn's globe and rings. Now imagine a spectator placed at the north pole *n.p.*:—the pole of the Saturnian celestial sphere is seen at his zenith, in the direction *p* P; his rational horizon is the celestial equator; and it is perfectly clear that the rings are altogether invisible to him whether he looks towards H or H″ or in any direction in the plane generated

if they were due to internal action. The theory of the dynamical source of solar heat explains at once the equatorial zone of disturbance, the form of the solar spots, and their spiral motions of rotation. A flight of cosmical bodies falling upon the sun would necessarily be converted by the resistance of the solar atmosphere into a spirally rotating whorl of intensely hot vapour. We may conceive such a vapourous whirlpool generating a rotatory motion in the solar atmosphere and so causing vast depressions similar to those indicated by the fall of the barometer in cyclones: see Nichol's 'Cyclopædia, Article Sun.' But a more plausible explanation may, I think, be suggested. Where the vapourous whorls approached the surface of the sun, their fierce heat would melt solid matter, turn liquids into vapours, possibly even vapours into some still more subtle form. The effects of such changes would correspond more closely than the effects of mere depressions with observed appearances. The outlines of the solar spots are sharply defined, and the spots change rapidly in form, especially before disappearing —phenomena which may not inaptly be compared to those presented when a stream of hot air is directed upon the surface of a sheet of ice covering water, till a large hole has been melted completely through the sheet, the ice being then allowed to form again.

by the revolution of H H″ about the axis P P′. Suppose now that he advances along the meridian *namlte*:—The point of the horizon towards which he is advancing becomes the south point as soon as he has left the pole, and the pole of the celestial sphere leaves the zenith-point towards what is now the north point of the horizon; as he advances, the south point of the horizon falls lower and lower below the celestial equator, until he reaches the point *n* (in which a tangent from N meets the ellipse *n e* n′*e*′) when the outer edge of the ring becomes visible at the south point of the horizon. Continuing to advance, more and more of the outer ring becomes visible, the outer edge appearing as an arch in the southern horizon; and soon after he passes *a* (on the northern arctic circle of Saturn's globe*) the inner edge of the outer ring becomes visible at the south point of the horizon. Soon after, the outer edge of the inner bright ring appears; and when he reaches the point *m* (in which a tangent from M meets the ellipse *n e* n′*e*′) the inner edge of this ring appears. At *l* the inner edge of the dark ring becomes visible; and after passing this point, parts of the complete system of rings are visible above the horizon, gradually rising higher towards the zenith, and extending farther and farther towards the east and west points of the horizon. Finally, at *e* he sees the inner edge only of the ring, extending as a zone of variable width on the celestial sphere; the prime vertical† divides this zone into two equal parts, and the width of the zone plainly diminishes from the zenith, where the ring's edge is nearest to the spectator, down to the horizon, where it is farthest from him. If

* The point *a* is determined by an inclination of *a* A, the tangent at *a* to the ellipse *n e* n′*e*′, at an angle of 26° 49′ 28″ to the line N O N′; *z a k*, the vertical at the point *a* of the Saturnian globe, is therefore inclined at the same angle to P O P′. In like manner, the point *t* on the northern tropical circle of Saturn's globe is determined by an inclination of Z *t g*, the vertical at *t*, in an angle of 26° 49′ 28″ to the line N O N′; *t* T the tangent at *t* is therefore inclined at the same angle to P O P′. The points t and a′ on the southern hemisphere are determined in a similar manner. It may be noticed that *z k* is parallel to t T, the tangent at t; and Z *g* is in like manner parallel to a A, the tangent at a. The vertical at any point of Saturn's globe meets *e* O *e*′ farther from O as the point is farther from either pole; thus at a point very near *e* or *e*′ the distance from O of the point in which the vertical meets *e* O *e*′, is $\frac{21}{121}$ths of the equatorial semi-diameter, or more than 6,000 miles, the compression of Saturn's globe being assumed at $\frac{1}{11}$.

† At this time the northern and southern poles of the heavens both appear in the horizon, towards *h*″ and *h* respectively, and the celestial equator is the prime vertical.

we suppose our spectator to advance from *e* towards the south pole, it is clear that the rings pass over to the northern half of the celestial hemisphere, falling gradually farther and farther away from the zenith, and disappearing in an order the exact reverse of that in which they appeared. The same succession of phenomena would be presented if the spectator travelled along the other half of the meridian *n e* n′*e*′, or along any other semi-meridian from the north to the south pole. Thus, if we imagine *n e* n′*e*′ to represent, not a section, but a hemisphere of Saturn's globe, then *n p n*′*n*″ and n *p* n′n″ represent halves of polar regions within which the rings are altogether invisible; *n a a*′*n*′ and n a a′n′ represent (approximately*) the halves of zones within which part of the outer ring only can be seen; *a m m*′*a*′ and a m m′a′ represent (approximately) the halves of zones within which parts of both the bright rings can be seen, but no part of the dark ring; *l t t*′*l*′ and l t t′l′ represent the halves of zones in which parts of all the rings can be seen; and finally *t* t t′*t*′ represents one half of a zone within which the inner edge of the dark ring appears above the horizon.†

It is easy to determine how much of each ring lies above the horizon of any point on Saturn's globe. Take, for instance, the point *a*:—it is clear that the horizon-plane at *a* (in other words, the plane touching the globe of Saturn at *a*) intersects the plane of the rings in a line through A, perpendicular to the line N O N′. Let fig. 3 represent part of the ring-system on the same scale as fig. 2, but viewed from above the plane of the rings; let O be the centre of the system, and draw a line O N crossing the edges of the rings at the points L″, M″, A″, and N, corresponding to the points L, M, A, and N, respectively, in fig. 2. Then, if we draw A A″A′ at right angles

* The tangent from the inner point of the division at A falls near *a* towards *m*.

† If Saturn were a perfect sphere it would be easy to determine the extent of these zones of his surface; since the surface of a zone of a sphere : to the surface of the sphere (or four great circles of the sphere) :: the distance between the planes of the bounding circles of the zone : the diameter of the sphere. In the case of an oblate spheroid, zones parallel to the equator bear a somewhat greater proportion than this to the surface of a sphere of diameter equal to the equatorial diameter of the spheroid: the reader acquainted with the elements of conic sections will easily see that a narrow zone represented by the line *t t*″*t*′ : a zone of equal width (measured from plane to plane of its bounding circles) on a sphere of diameter *e* o *e*′ :: the diameter conjugate to *t* o t′ : the minor axis of the ellipse *n e* n′*e*′.

to O N, it is clear that A A″A′ represents the line in which the horizon plane of the observer at *a*, fig. 2, meets the rings' plane; thus the segment A N A′ lies above the horizon of the point *a*. Similarly we obtain the segments M N M′, L N L′, and T N T′, lying above the horizons of the points *m*, *l*, and *t*, respectively; and corresponding lines and segments for any other points of Saturn's globe.*

Again, it is easy to determine where an edge of a ring intersects the horizon of any point on Saturn's globe. Thus, for the point *a*:—*a* A, in fig. 2, is the distance of *a* from the line through A corresponding to A A″A′ in fig. 3; hence, if, in fig. 3, we take A″*a*, equal to *a* A in fig. 2, along N O, and draw the lines *a* A and *a* A′, the equal angles A *a* A″ and A′*a* A″ are plainly the azimuths (measured from the south point) of the points at which the ring's outer edge crosses the horizon. In a similar manner, by taking the lines M″*m*, L″*l*, and T″*t*, in fig. 3, respectively equal to M *m*, L *l*, and T *t*, in fig. 2, we obtain the corresponding azimuthal angles M *m* M″ or M′*m* M″, L *l* L″ or L′*l* L″, and T *t* T″ or T′*t* T″. And if we draw *l* U, *l* U′, *t* V and *t* V′, we obtain the azimuths of the points at which the inner edge of the inner bright ring crosses the horizons of the points *l* and *t* respectively. We may obtain the corresponding azimuths in a similar manner for any point on Saturn's globe and for either edge of any ring.

It is also easy to determine the altitude of the point at which an edge of a ring crosses the meridian of any point on Saturn's globe. Take, for instance, the point *t* in fig. 2, and join *t* N; then the angle N *t* T at which *t* N is inclined to *t* T (the tangent at *t* to the ellipse *n e* n′*e*′) plainly gives the altitude of the point at which the outer edge of the ring crosses the meridian—in other words—of the point N. The angle Z *t* N is the zenith distance of the same point. If *t* S is drawn parallel to N O N′, it is clear that the angle S *t* T gives the meridian altitude of the celestial equator above the horizon of *t*; hence S *t* N gives the southerly declination of the point N of the ring's outer edge. In a similar manner the altitude of the point in which either edge of any ring crosses the meridian of any point on Saturn's globe may be determined.

* For a point on Saturn's equator we obtain the line E X *e* X′E′, cutting off the largest segment of the system that can lie above the horizon of any point on Saturn's surface.

Let us next consider the form of the arched outline of the edge of a ring. If it were possible to view the rings from any point in the line P O P′, it is clear that all the edges would coincide with declination-parallels on the celestial sphere, since the lines of sight from a fixed point in P O P′ to points in the circumference of the ring's edge, would be inclined at a constant angle to the line P O P′. But since all the points on Saturn's surface from which the rings are visible, lie at a great distance from P O P′, the edge of a ring (or, at least, that part of the edge which is visible) is viewed eccentrically. Now a circle viewed eccentrically from a point above its plane appears as an ellipse; and the greater the distance of the point of view from the perpendicular through the centre, in proportion to the diameter of the circle, the more eccentric such ellipse will appear. Hence the outlines of rings will not appear to coincide with declination-parallels; and the deviation will be more marked in the case of the outline of an inner, than in that of an outer ring. It is easily seen that a declination-parallel through the point in which an edge of a ring crosses the meridian, falls below the apparent outline of the ring's edge on each side of the meridian, touching that outline on the meridian. Take, for instance, the point *t*, fig. 2, Plate XI., and suppose a perpendicular let fall from *t* on the plane of the rings; with the foot of this perpendicular as centre, imagine a circle described in the plane of the rings through the point N; this circle will fall within the outer edge of the outer ring, touching that edge at the point N. Now this circle, viewed from *t*, would coincide with a declination-parallel on the celestial sphere; and the outer edge of the outer ring, viewed from *t*, would obviously appear to touch this circle, (and, therefore a declination-parallel) upon the meridian at N, and to pass above it on either side of the meridian. The same may be proved of the other edges of the rings, and from whatever point of Saturn's globe these may be viewed, except, of course, from points on Saturn's equator. It follows that the visible part of a declination-parallel passing through the points in which the outline of any ring meets the horizon, lies altogether above that outline.

In Table XI. the relations here considered are expressed for all latitudes within which any part of the rings can be seen. The meaning of the part of this table having reference to the rings may be shown as follows:—Take any latitude from the upper line, as, for instance, latitude 40° north, (corresponding nearly to the latitudes of New York, Madrid, Bokhara, and Pekin, on our globe). It appears from the table, that in this latitude the outer edge of the outer ring crosses the horizon at two points, 69° 36′ east and west of the south point; attains an altitude of 30° on the meridian, and an arc of 114° 2′—or somewhat less than one-third of the edge—lies above the horizon; similarly the inner edge of the outer ring crosses the horizon 65° 57′ east and west of the south point, and attains an altitude of 26° 11′ on the meridian, an arc of 104° 2′ lying above the horizon. Thus the outer ring covers two arcs of 3° 39′ on the horizon and an arc of 3° 49′ on the meridian. Although the arc on the horizon is nearly as great as the arc on the meridian, yet it is easily seen that the apparent breadth of the arch presented by the ring is much greater on the meridian, for whereas the meridian crosses the arch at right angles, the horizon crosses it at an acute angle; so that the apparent breadth of the arch near the horizon is much less than the arc of the horizon covered by it. Thus in north latitude 40° on Saturn's globe, the outer ring appears as a zone crossing the horizon towards the points E.S.E. and W.S.W., attaining a meridian altitude of 30°; and this zone increases in width from the horizon to the meridian, where its width is about seven times as great as the apparent diameter of our moon. Similarly it may be shown that the inner bright ring rises as a much wider zone towards the points S.E. by E. and S.W. by W. of the horizon; the upper edge of the zone rises to an altitude of 25° 22′ the lower to an altitude of 12° 22′, only, on the meridian; thus the greatest width of the zone is 13°, or more than 25 times the apparent diameter of the moon. The great division between the rings forms a zone between them, whose greatest width (where the zone crosses the meridian) is less than 49′,* or about three

* The arcs of the horizon and meridian covered by the rings are somewhat greater than the arcs given in the table, and the arcs covered by the division somewhat less; for the thickness of the rings has not been taken into account in the table. Thus in latitude 5° the division between the rings would hardly be visible if the thickness of the system were 100 miles. The corrections due to this cause amount, however, only to a minute or two of arc in general.

semi-diameters of the moon. The dark ring covers nearly the whole space within the inner edge of the inner bright ring; for the inner edge of the dark ring crosses the horizon 13° 55′ on each side of the meridian, and rises on the meridian to an altitude of only 46′, so that only a narrow strip of sky about 28° from point to point and about three semi-diameters of the moon in width on the meridian, is left uncovered by the rings on the southern horizon. Again, it appears from the table that the meridian-altitude of a declination-parallel through the point (called A in the table) in which the outer edge of the system meets the horizon, is 34° 31′, or 4° 31′ greater than the altitude of this edge where it crosses the meridian (at B); and that a declination-parallel through B crosses the horizon at distances 63° 29′ on either side of the south point, or 6° 7′ nearer than A to the south point: and similarly a declination-parallel through A′ crosses the meridian 7° 38′ above B′, and a declination-parallel through B′ crosses the horizon 12° 7′ nearer than A′ to the south point. Hence a star rising at A culminates 4° 31′ above B; a star culminating at B is altogether hidden by the bright rings, except for a very brief interval when it crosses the division between the rings; a star rising at A′ is altogether hidden by the inner bright ring, being 5° 22′ from the division between the rings even at culmination; and lastly, a star culminating at B is visible (through the dark ring) throughout its path above the horizon. Hence many stars must remain altogether invisible until the slow precessional motions of Saturn's equinoctial points so far alter the declinations of such stars as to remove them from the invisible zone of the Saturnian heavens.

In a similar manner the appearance of the rings for any latitude may be determined from Table XI. At Saturn's equator, the edge of the ring being turned towards the planet, it is probable, from the appearance of the rings when their edges are turned to the earth, that an irregular zone of variable appearance is turned towards the Saturnians. Assuming the dark ring to be only indistinctly visible, and the inner edge of the inner bright ring to be 100 miles in thickness, its appearance would be that indicated in note (3) Table XI. The width of the zone thus presented would at the zenith be nearly two-thirds, at the horizon about one-fourth, of the apparent diameter of the moon. The absolute extent of surface of the ring-system visible above the horizon is greatest for latitudes near the equator,* but the apparent surface of the celestial sphere covered by the rings attains its maximum extent in higher latitudes. It will be seen from Table XI. that the arcs of the meridian covered, respectively, by the outer ring, the division between the rings, the inner ring, and the dark ring, attain their maximum values in about latitudes 45°, 40°, 32° 30′, and 21°; while the arcs of the meridian covered, respectively, by the system of bright rings, and by the complete system of rings, attain their maximum values in about latitudes 35° and 29°.

It is clear that the bright rings are plainly visible from parts of that hemisphere, only, of Saturn which lies above their illuminated face. From the other hemisphere the rings are traceable in their effects in occulting the stars or other celestial bodies whose arcs above the horizon pass wholly or in part behind the rings. These rings may also reflect a faint light received from Saturn's moons. The dark ring may possibly be visible in both hemispheres, since the satellites composing it are probably separately visible from Saturn's surface. By day, the rings are either altogether invisible, or only appear as clouds of faint light below the sun's diurnal path. It might at first sight be supposed that the circumstance that these rings are composed of disconnected satellites, must have a marked effect, whether such satellites are separately visible or not; that the satellites in different parts of their revolution about the planet must exhibit such phases as our own moon, and that parts of the ring in which all the satellites are 'full' or nearly full, must present a much larger amount of illuminated surface to the planet, than parts in which all the satellites are 'new' or nearly new. A little consideration will show that this is not actually the case. The appearance of the system shows that the satellites composing it must be very numerous and closely packed: thus the effects of mutual eclipses and occultations among the satellites counter-

* It is easily calculated that the surface of either face of the ring-system above the horizon at the equator is equal to about $\frac{3}{10}$ths of the whole surface of either face, considered as extending from the inner edge of the dark ring to the outer edge of the outer bright ring, without regard to divisions.

balance the effects due to their phases, and the question of illumination may be considered precisely as it would on the assumption that the rings are solid bodies. Now the brilliancy of an illuminated surface (beyond the earth's atmosphere) does not vary with the distance of the observer, nor with the angle at which he views the surface; these circumstances affect the apparent magnitude of the object, and, in the same proportion, the total amount of light received by the observer, but the intrinsic brilliancy of the object remains unaltered. The apparent brilliancy of an illuminated surface varies, however, with the angle at which the illuminating body is elevated above that surface.* Hence the apparent brilliancy of the rings at any instant is the same throughout their visible extent, and (*cœteris paribus*) from whatever part of the hemisphere above their illuminated side they may be viewed; but such brilliancy varies with the sun's changes of declination, increasing gradually from the vernal equinox to the summer solstice, and thence decreasing to the autumnal equinox.

Between the vernal equinox and the summer solstice of either hemisphere, the shadow of the planet on the rings assumes suc-

* It appears from these considerations that Professor Challis is in error when he states that a 'distant spherical body shining by reflected light would appear equally bright at all points of the disc' (Article on the Indications by Phenomena of Atmospheres to the Sun, Moon, and Planets,—'Reports of the Astronomical Society,' June 1863). A self-luminous spherical body whose surface is uniformly brilliant would so appear, and therefore we may accept the diminution of brightness near the sun's periphery as an indication that the sun has an atmosphere; but in the case of a sphere shining by light received from a distant luminous body, the illuminated hemisphere is not uniformly brilliant, and therefore the disc presented by it exhibits corresponding variations of brilliancy. An atmosphere surrounding a planet, by tending to equalise the illumination of the planet's surface, would diminish rather than increase this variation; in fact, there is no resemblance between the cases of a self-luminous sphere and of a sphere shining by reflected light, as regards the effects to be attributed to the presence of an atmosphere.

The variation of brilliancy is sufficiently conspicuous in the discs of the planets Saturn and Jupiter; and though these planets never present a gibbous appearance, yet there is a perceptible difference in the illuminations of opposite sides of the disc when the planets are in or near quadrature; see the figures of Plate I. Similar variations of brilliancy are exhibited by Venus and the moon, when horned or gibbous. When the moon is full the variation is also traceable, but less clearly, owing to the irregularities of her surface. An examination of the *general* brilliancy of different parts of the lunar disc confirms the views of the moon's form (as respects her visible hemisphere) presented in Note C, Appendix I.

cessively all the forms indicated in Plate XII.* At the vernal equinox the edges of the shadow are straight, as in fig. 1; at and near the summer solstice the outline of the shadow is part of an ellipse, an extremity of whose longer axis lies within the outer edge of the rings; in all intermediate cases the outlines of the shadow are parts of an ellipse of considerable eccentricity. The interval of time in which the shadow changes from the form indicated in one figure to that indicated in the next is about 384 days. It is easy to determine the manner in which the vast shadow of the planet sweeps over the illuminated face of the rings. At sunset, at and near either equinox, the rings are illuminated throughout their visible extent in all latitudes. Near the equator the shadow of the planet rises in the east, as soon as the sun has set,† eclipsing at once the whole breadth of the rings near the horizon; in higher latitudes the shadow rises later, eclipsing first the outer edge of the rings. Later in the Saturnian year the curvature of the shadow shows its effect; the parallel of latitude within which the eclipse commences along the inner edge of the rings passing higher and higher, until it includes all latitudes within which the rings are visible. Near the summer solstice the outer edge of the outer ring is not eclipsed at all. The shadow also rises later and later to midsummer; but as the nights grow shorter and shorter, and as in high latitudes this change takes place at a greater rate than the change in the hour at which the shadow rises, it will happen that, in high latitudes, great parts of the ring are already in shadow when the sun has set. In all latitudes and at all seasons the central line of the shadow crosses the meridian at midnight. At this hour a very small part of the ring is visible, even from points near the equator, near the time of either equinox; but, for about three years, near the time of the summer solstice, the outer edge of the ring is not in shadow at midnight. At this time the system must present a magnificent appearance, as a vast double arch of light, indented by a broad elliptical shadow. Owing

* The point of view in these figures is supposed to lie in the axis produced of the planet; the lines A A′, L L′, M M′, T T′ and E E′, correspond to the lines similarly lettered in fig. 3, Plate XI.; and the circles *n n′n″*, *a a′a″*, *l l′l″*, *m m′m″*, *t t′t″* and *e e′e″* to the lines similarly lettered in fig. 2, Plate XI.

† Owing to refraction the shadow doubtless rises before sunset, just as the eclipsed moon is sometimes visible while the sun is yet apparently above the horizon.

*Fig. 1.*

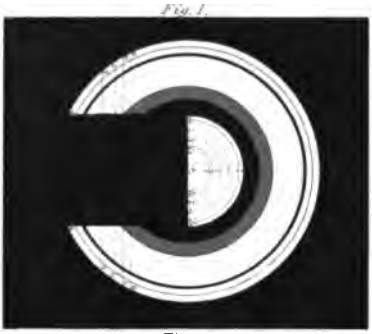


*Fig. 2.*

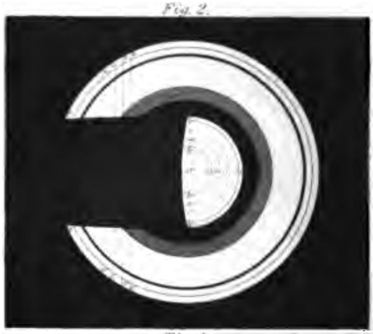


*Fig. 3.*

*Fig. 4.*

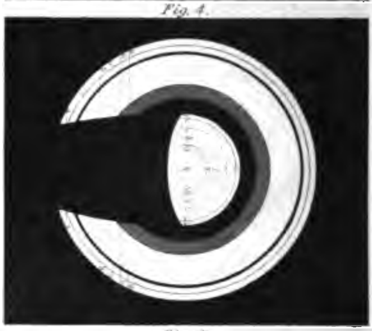


*Fig. 5.*

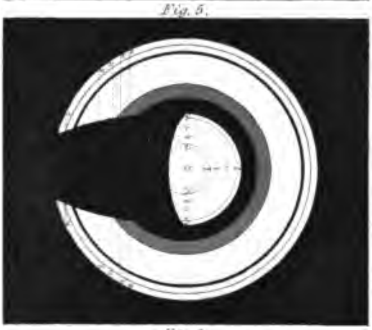


*Fig. 6.*

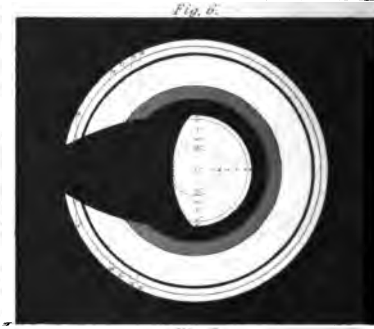


*Fig. 7.*

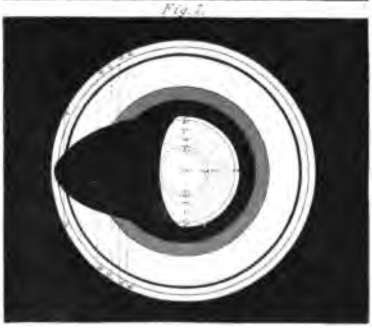
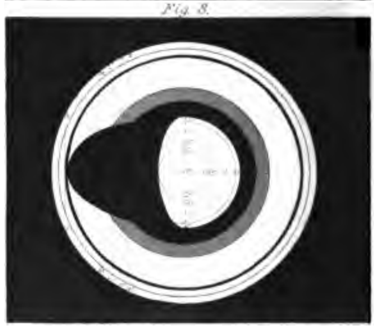


*Fig. 8.*



SHADOW OF THE PLANET ON THE RINGS
AT DIFFERENT SEASONS OF THE SATURNIAN YEAR.

*H. Adlard Sc.*

London: Longman & Co.

PLATE XIII.

*Fig. 1.*

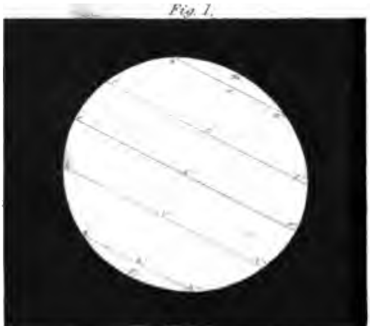


*Fig. 2.*

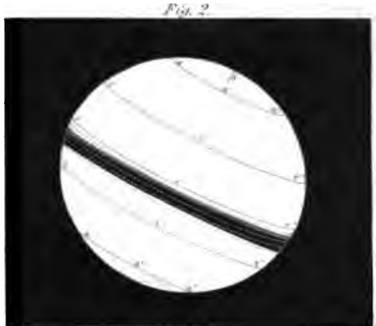


*Fig. 3.*

*Fig. 4.*

*Fig. 5.*

*Fig. 6.*

*Fig. 7.*

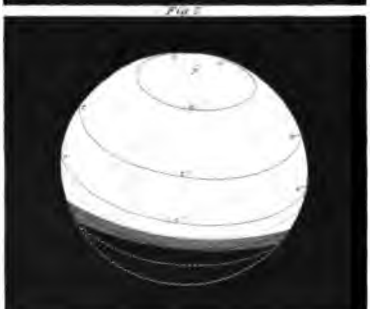


*Fig. 8.*

SHADOW OF THE RINGS ON THE PLANET
AT DIFFERENT SEASONS OF THE SATURNIAN YEAR.

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to the refractive effects of Saturn's atmosphere, the outline of the shadow is probably fringed with a wide ruddy or copper-coloured penumbra.* The central line of the shadow sweeps uniformly round the rings, the shadow disappearing, at or before sunrise, in the west, in the same manner (at all seasons and in all latitudes) as it had appeared in the east. The changes taking place from the vernal equinox to the summer solstice are repeated in reverse order from the summer solstice to the autumnal equinox.

In his 'Outlines of Astronomy,' Sir John Herschel states that 'the regions beneath the dark side' undergo 'a solar eclipse fifteen years in duration.' Dr. Lardner appears to have imagined that Herschel supposed the whole hemisphere beneath the dark side of the ring to undergo a total eclipse fifteen years in duration; and, in a paper read before the Astronomical Society in 1853, he endeavoured to show 'that by the apparent motions of the heavens produced by the diurnal rotation of Saturn, the celestial objects, including the sun and the eight satellites, are not carried parallel to the edges of the rings; that they are moved so as to pass alternately from side to side of these edges; that, in general, such objects as pass under the rings are only occulted by them for short intervals before and after their meridional culmination; that although, under some rare and exceptional circumstances and conditions, certain objects, the sun being among the number, are occulted from rising to setting, the continuance of such phenomena is not such as has been supposed, and the places of its occurrence are far more limited.'† It will appear, however, on a more exact examination, that Lardner was in error on nearly every point he imagined he had established.

There are two methods by which astronomers determine the occurrence and nature of a solar eclipse. In one, the apparent paths of the sun and moon on the celestial sphere are examined for short intervals of time before and after the time of new moon; and

* No attempt has been made in Plate XII. to indicate either the form of such penumbras or the twilight-circle bordering the parts of the planet in shadow. The extent of these depends altogether on the unknown extent and refractive powers of Saturn's atmosphere. The true penumbra, or that due to the apparent size of the sun's disc, is too small to be appreciable either in these figures or in the figures of Plate XIII.

† Dr. Lardner's 'Museum of Science and Art,' vol. i. p. 59.

the moments of first and last contact and of central eclipse are thence deduced. In the other, a spectator is supposed to view from the sun the passage of the moon's disc across the larger disc of the earth; the manner in which the given place on the earth's surface would appear to move, if viewed from the sun's centre during the time of passage, is easily determined; and the moments of first and last contact, and of central eclipse, are determined from the simple principle that if the given point on the earth is so situated that it would be invisible from a given point of the sun at any instant, then such point of the sun must also be invisible from the given point on the earth, or, in other words, is eclipsed.

The application of the first method to the eclipses of Saturn's surface by his ring-system is simplified by the consideration that the rings occupy an invariable situation on the celestial sphere of any given point on Saturn's surface. At the autumnal equinox of either hemisphere the sun has at rising an azimuth of 90° (in other words, the sun rises in the east), and attains a meridian altitude equal to the complement of the latitude. After the autumnal equinox the sun passes to the south of the celestial equator in northern, to the north in southern latitudes; and as his declination increases his meridian altitude and azimuth at rising diminish. At length the sun crosses the horizon at the points (called A in Table XI.) in which the outer edge of the outer ring meets the horizon. From this time the sun is eclipsed after rising and before setting for intervals of gradually increasing length, until he crosses the meridian at the same point (called B in Table XI.) as the outer edge of the outer ring. From this time the sun is eclipsed throughout the day (except, in certain latitudes, for two intervals of a few minutes, during which he is seen *between* the rings), until at rising and setting he crosses the horizon at the points (called A′ in Table XI.) in which the inner edge of the inner ring meets the horizon. From this time the sun is visible (through the dark ring), after rising and before setting for intervals of gradually increasing length, until he crosses the meridian at the same point (called B′ in Table XI.) as the inner edge of the inner ring. From this time he is visible throughout the day (neglecting the partial eclipses probably caused by the dark ring) until the winter solstice, and for a corresponding interval after the winter solstice. During the quarter of a Saturnian year from the winter to the vernal equinox a similar series of eclipses takes place in reverse order. In latitudes higher than 19° 50′ the sun does not reach the point B′; so that in these latitudes eclipses in the middle of each day continue to the winter solstice. Again, in latitudes higher than 35° 52′ the sun does not reach the point A′, so that in these latitudes eclipses lasting throughout the day continue to the winter solstice.

The intervals during which eclipses of each kind are continued can be roughly determined by construction in the method indicated at page 164. The last section of Table XI. contains the more trustworthy results of calculation. From this table it will be seen that, even at the equator, the sun is totally or partially eclipsed for several days; but that the periods of eclipse increase rapidly with the latitude. Thus, in latitude 40°, the eclipses begin when nearly three years have elapsed from the time of the autumnal equinox. The morning and evening eclipses continue for more than a year, gradually extending until the sun is eclipsed during the whole day. As the sun does not reach the point A′ in these latitudes, these total eclipses continue to the winter solstice and for a corresponding period after the winter solstice; in all for *6 years 236·4 days, or 5543·0 Saturnian days.* This period is followed by an interval of more than a year of morning and evening eclipses. The total period during which eclipses of one kind or another take place is no less than *8 years 292.8 days.* In a similar manner the eclipses for other latitudes are determined from Table XI. If we remember that latitude 40° on Saturn corresponds with the latitude of Madrid on our earth, it will be seen how largely the rings must influence the conditions of habitability of Saturn's globe, considered with reference to the wants of beings constituted like the inhabitants of our earth.

The second method of determining the extent and duration of solar eclipses—called the method of projecting eclipses—is less exact than the former, but better adapted for illustration. The figures of Plate XIII. represent Saturn as he would appear if viewed from the sun at the vernal equinox of the northern hemi sphere (fig. 1); at the summer solstice of the same hemisphere (fig. 8); and at six intermediate periods.* These epochs correspond with those of the figures of Plate XII., and, like them, are separated by equal intervals of 384 days. The arctic circles are represented by the lines $a\,a'a''$ and a a′a″, the tropics by the lines $t\,t't''$ and t t′t″, the equator by the line $e\,e'e''$, and the north pole by the point $p$, in each figure (see Chapter IV. pp. 95-99); the rings are supposed to be removed, and their shadows on the planet's disc thus rendered visible.† These shadows pass to the southern hemisphere at the autumnal equinox of that hemisphere, travelling rapidly southward at first, but more slowly as their width increases; and about two years before the winter solstice the lower edge of the black shadow passes beyond the lower edge of the disc. The five dotted parallels of latitude in each figure (except fig. 1) represent:—The parallel just reached by the lower edge of the black shadow; a parallel passed over by this edge; a parallel just within the upper edge of the black shadow; a parallel just clear of this edge; and a parallel just clear of the dusky shadow of the dark ring. Now, owing to the rotation of Saturn on his axis, any point on his surface would appear to an observer in the sun to travel along a latitude-parallel, appearing on the left edge of the disc (the moment of sunrise at the place), and disappearing on the right (the moment of sunset at the place). Hence, a place between the lowest pair of dotted parallels in any figure (that is, at the epoch represented by such figure) would be in shadow in the morning and evening, dipping below the shadow in the middle of the day; a place between the second and third dotted parallels (counting upwards) would be in shadow throughout the day; a place between the third and fourth would not be in the black shadow in the morning and evening, but would dip within it in the middle of the day; and, lastly, a place within the two upper dotted parallels would not be in the dusky shadow in the morning and evening, but would dip within it in the middle of the day. These results correspond with those already

* Fig. 1 corresponds to Saturn's position on the 18th of May, 1862; fig. 8, to his position in March, 1870.

† To avoid confusion the line of light corresponding to the division between the rings is omitted in the figures of Plate XIII.

obtained, and the figures of Plate XIII. sufficiently indicate the vast extent attained by the shadow near the time of the winter solstice, and the consequent long duration of eclipses, in latitudes not very near the equator. The shadow of the ring passes through the same changes of form in inverse order between the winter solstice and the vernal equinox of the southern hemisphere; but the pole of the planet passes to the left, so that at the latter period fig. 1 *inverted* represents the disc of the planet. The other figures inverted indicate the manner in which the shadow sweeps across the northern hemisphere, to the winter solstice of that hemisphere. The shadow, in returning, passes through the same changes, but in inverse order, and sloped towards the left instead of towards the right. Thus, at the end of the Saturnian year the appearance of the disc is again as in fig. 1.

All objects whose declinations are variable, such as the planets and the outer satellite, undergo a similar series of eclipses. The extent and duration of such eclipses for any celestial object will vary with the range of the object's changes of declination. Thus the outer satellite, whose declination never exceeds 15° north or south of Saturn's celestial equinoctial-line, may be totally eclipsed during the whole time it is above the horizon only in latitudes lower than 25° north or south; since the point B of the ring is more than 15° from the equinoctial in higher latitudes.

Since the seven interior satellites move very nearly in the plane of the rings, it is clear that in places very near Saturn's equator these satellites can only become visible when they reach their greatest departure from the plane of the rings. In all other parts of Saturn's surface these satellites can never be eclipsed by the rings. Their orbits being (approximately) circles concentric with the rings, would, like the edges of the rings, appear as ellipses to the Saturnians, and would lie altogether clear of the rings—just as the outer edge of a ring lies altogether clear of the inner edge.

The apparent magnitudes of the satellites vary with the point of Saturn's surface from which they are viewed, and with their own motions. The following table will serve to give an idea of the relations among the satellites in the latter respect; the satellites are supposed to be viewed from a place near Saturn's equator; in higher latitudes the satellites vary similarly in apparent magnitude, but within a narrower range of variation, as they traverse the sky from horizon to horizon:—

| Name | Assumed Diameter in miles | Apparent Diameter of Disc | | Same, mean diameter of Moon's disc as 1 | | Disc, mean disc of Moon as 1 | |
|---|---|---|---|---|---|---|---|
| | | on the Meridian | at the Horizon | on the Meridian | at the Horizon | on the Meridian | at the Horizon |
| Mimas . . | 1000 | 43′ 27″ | 31′ 24″ | 1·396 | 1·009 | 1·950 | 1·019 |
| Enceladus . | 1000 | 30 44 | 23 57 | 0·988 | 0·770 | 0·975 | 0·593 |
| Tethys . . | 1500 | 35 3 | 28 42 | 1·126 | 0·922 | 1·269 | 0·851 |
| Dione . . . | 1500 | 27 20 | 23 15 | 0·879 | 0·747 | 0·772 | 0·558 |
| Rhea . . . | 2000 | 23 34 | 21 7 | 0·757 | 0·679 | 0·574 | 0·461 |
| Titan . . . | 4000 | 19 0 | 18 7 | 0·610 | 0·582 | 0·373 | 0·339 |
| Hyperion . | 800 | 3 7 | 3 0 | 0·100 | 0·096 | 0·010 | 0·009 |
| Japetus . . | 3000 | 4 45 | 4 40 | 0·153 | 0·150 | 0·023 | 0·023 |

The eclipses of the satellites by the planet, and of the planet by the satellites, may be determined by the methods applied to the rings. These eclipses have been considered in Chapter IV.

Of the planets only Jupiter and Uranus are visible to the Saturnians supposing their eyesight as ours. Jupiter is an inferior planet; and its phases, viewed from Saturn, resemble those of Venus viewed from the earth. Although Jupiter approaches as near to Saturn, when in conjunction, as to the earth when in opposition, he is, in the former case, invisible to the Saturnians, since he rises and sets with the sun. Since in other configurations Jupiter is at a greater distance from Saturn than from the earth when in opposition, and since, further, only a part of his illuminated face is turned towards the Saturnians, he can never present an appearance even approaching in brilliancy the appearance he presents to the earth when in opposition. His mean synodical period with respect to Saturn is 7253·445 days. Uranus is a superior planet to Saturn, and must be distinctly visible when in opposition, being then removed by a distance only one-half that by which he is removed from the earth when in opposition. But before reaching quadrature Uranus must become invisible to the Saturnians, supposing their eyesight as ours. His mean synodical period with respect to Saturn is 16568·295 days.

The result of the examination of the probable physical conditions and phenomena subsisting on Saturn does not appear to favour the supposition that the planet is a suitable habitation for beings constituted like the inhabitants of our globe. The variation of gravity, the length of the Saturnian year, and the long-protracted eclipses caused by the ring, are the circumstances that seem to militate most strongly against such a supposition. Over a zone near the Saturnian equator these circumstances have less effect, however; and it is not impossible that arrangements unknown to us prevail on Saturn which may render other parts of his surface habitable as we should understand the term:—'The very combinations which convey to our minds only images of horror, may be in reality theatres of the most striking and glorious displays of beneficent contrivance.' *

On the *general* question of the habitability of the system that circles about Saturn, we have no means of forming an opinion. From the analogy of our moon it appears highly probable that no part of the system is habitable by such creatures as inhabit our earth. †

* Herschel's 'Outlines of Astronomy,' p. 286.

† See Note C, Appendix I., Habitability of the Moon.

# APPENDIX I.

# NOTE A.

## CHALDÆAN ASTRONOMY.

. . . . . . The Planetary Five
With a submissive reverence they beheld;
Watched from the centres of their sleeping flocks
Those radiant Mercuries, that seemed to move
Carrying through æther in perpetual round
Decrees and resolutions of the gods.

WORDSWORTH.

IN many parts of the East astronomical systems are found which bear obvious traces of a common origin. We find in all the same duodecimal division of the Zodiac;* the week used as a division of time; the planets associated in the same order with the days of the week and the hours of the day;† and other points of resemblance, sufficiently marked to leave little doubt that they are not accidental.

Meagre and imperfect as these systems now appear, there can be little doubt that the system from which they sprang was founded by astronomers of great ability, close observers and careful interpreters of nature. The origin of the system must be placed in a remote antiquity. The Egyptians, to whom the invention of astronomy is sometimes attributed, in all probability derived their system from a more ancient nation. The claims of the Chinese may be dismissed at once, if we consider the character of that people—apt to imitate but slow to invent. When we examine the claims of the Indians, Persians, and Babylonians, we are met by a singular circumstance:—*Their systems of astronomy belong to a latitude considerably higher than the latitudes of Benares, Persepolis, or Babylon.* For the Brahmins teach in their sacred books that the longest day in summer is twice as long as the shortest day in winter, which is not the case in any

* The signs of the Zodiac are not the same in all the systems. The signs we use—the Ram, Bull, Twins, &c.—appear to have been derived from the Dodecatemoria of the Chaldæans; the Chinese name the signs as follows:—The Mouse, Cow, Tiger, Hare, Dragon, Serpent, Horse, Sheep, Monkey, Cock, Dog, and Bear.

† This order is as follows:—The Sun, the Moon, Mars, Mercury, Jupiter, Venus, and Saturn for the days of the week; Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon for the hours of the day,—the first hour of each day being associated with the planet that ruled the day.

part of India; so also Zoroaster taught the Persians; and Ptolemy obtained ancient Babylonian records of star-risings, and these belong to latitudes certainly not lower than the 40th parallel. In the measurement of the earth's circumference adopted by ancient astronomers, we have a singular confirmation of this circumstance. It corresponds to a latitude of about 45°; and taking into account errors of observation, we must yet place the latitude of the country in which the measurement was effected somewhere between 35° and 55° north.

Struck by the singular circumstance that these nations should employ a system of astronomy with which the celestial phenomena visible to them do not even approximately correspond, Bailly was disposed to ascribe the invention of the system to an extinct race, whom he called the Atlantides. He placed the seat of this race in Tartary or Siberia; but the cradle of the race,—and, indeed, of mankind—the Atlantis of Plato's Timæus, he placed 'somewhere near the north pole.' Uncertain whether Iceland, Greenland, Nova Zembla, or Spitzbergen, were the true Atlantis, he avowed a preference for Spitzbergen as 'nearest to the pole.' He pleaded thus plaintively for these cheerless regions:—'Are not these countries miserable enough, deserted as they are by the sun; covered with ice which grows thicker every year; left desolate by the emigration of their inhabitants? Shall *we*, too, abandon them? Should we not rather console them for the losses they have sustained, and for their present state, by praising their former condition?' Accordingly he proceeded to claim so high a value for the scientific attainments of the race, as to give rise to the remark of D'Alembert, that 'the race appears to have taught mankind everything except their own name and existence.'

There is no occasion, however, to seek in these uninviting regions either for the origin of the human race or of science. We may accept the traditions which point to the Chaldæans as the first people who dwelt in cities and formed a nation. All that is necessary to reconcile their claim to the invention of astronomy with the facts stated above, is to suppose that they were not originally settled in Mesopotamia, where they afterwards appeared, but near Mount Ararat, and that the race spread themselves so far north as the Caucasian range. It would be easy to account for their removal from these regions. They were subdued by the Assyrians, and by the Babylonians; and the deportation of conquered races was the common practice of both these nations: Chaldæan astronomy was already famous, so that it is in no way improbable that the monarchs of Assyria and Babylonia, anxious to attach the credit of so much learning to their own courts,* would remove the Chaldæans (even from such distant regions)

* In the time of Daniel we find the Chaldæans set apart as a race devoted to the study of the stars and laying claim to the possession of magical powers. It was part

successively to Nineveh and Babylon. That Mesopotamia was not the original seat of the race is evident from the words of the book of Judith (undoubtedly written by a person well-informed respecting the Chaldæans): —'This people' (the Jews) 'is of the offspring of the Chaldæans. They dwelt first in Mesopotamia, because they would not follow the gods of their fathers, who were in the land of the Chaldæans': Judith, v. 6.

Two arguments have been urged by Bailly against the claims of the Chaldæans to the invention of astronomy. They were acquainted with the fact that comets are wandering members of our system, and Bailly finds a difficulty in understanding how the Chaldæans should discover what Hevelius denied all his life, and what Cassini long considered doubtful. He does not tell us why the Atlantides should be better informed than the Chaldæans. Possibly he considered that, having gone to the Poles to invent a nation, he could assign to them what attributes he pleased; or he may have wished to 'console them for their present state by praising their former condition;' otherwise, there is no nation of antiquity for whom comprehensive views of nature and a high amount of mental culture might be more justly claimed than for the Chaldæans. Dr. Prichard, the celebrated ethnologist, considered that the Syro-Arabian or Semitic branch of the human family has at all times equalled in mental development the most favoured races of the Indo-European branch. 'The Semitic nations,' says Humboldt, 'afford evidence of a profound sentiment of love for nature;' and in another place he says, 'a grand and contemplative consideration of nature was an original characteristic of the Semitic races.'

Secondly, Bailly argues that the Chaldæans were not sensible of the true value of the system they used. In support of this statement he urges that, although we learn from Berosus that the Chaldæans were acquainted with the period of 600 years, which Cassini thought so perfect,* they made little use of it. Now, whatever the advantages of the period of 600 years—and Cassini greatly over-rated them—it is slightly cumbrous, and we cannot wonder that the Chaldæans should employ their convenient Saros of 18 years, rather than a period of as many generations.

It may, however, be admitted that the Chaldæans, subdued successively under the despotic control of Assyrian, Babylonian, and Persian rulers, gradually lost the skill that had distinguished them in their days of freedom. The decay of science that usually follows the subjection of a nation, was hastened in the case of the Chaldæans by other circumstances. Poetical,

of their servitude to overawe the ignorant Babylonians; they revenged themselves by deceiving their equally ignorant rulers.

* 'Six hundred years,' Cassini wrote, 'is the finest period that ever was invented; for it brings out the solar year more exactly than that of Hipparchus and Ptolemy, and the lunar month within one second of what is determined by modern astronomers.'

fanciful, and, like all eastern nations, quick in tracing fanciful analogies, the genius of their race led them early to choose the heavenly bodies as types of the divine attributes, and, in later times, as objects of adoration. The planets were regarded with peculiar reverence, and the later Chaldæans reposed undoubting faith in the influence of these orbs on the destinies of men and nations.* Such views must have proved a serious check upon their progress. Astronomical discoveries came to be jealously guarded as sacred secrets, to be revealed only to the initiated, and to them only by symbols. The spirit of inquiry and speculation began next to be viewed with suspicion and distrust; astronomers contented themselves with handing down the records of observations, without discussing how far these tended to support or to modify the systems they had been taught. Experience has repeatedly shown that the effects of such a course are not merely repressive, but destructive. Men can no more succeed in stereotyping a system of science than they can arrest the development of a tree without destroying it.

On the whole there appears to be no valid reason for rejecting the traditions which attribute the origin of astronomy to the Chaldæans, and which assign a high value to the system they founded, and to the accuracy and extent of their observations. The records that have been handed down to us are mixed up, however, with much that is false and exaggerated: we have such fables as the tale of Ctesias, that the observations of the Chaldæans had been continued for 470,000 years, during which time they had calculated the nativities of all the children that were born†; they believed, also, we are told, that the earth is formed like a boat; that the earth would be overwhelmed by a flood when all the planets were conjoined in Capricorn, and destroyed by fire when such a conjunction took place in Cancer; and many other such fables have been handed down to us. On the other hand, many accounts of their observations, the periods they employed, and the discoveries they effected, agree very closely with the discoveries of modern times; and it is not probable that these accounts were invented by the writers who relate them—often themselves ignorant of astronomy. Thus we learn from Diodorus Siculus, and Apollonius Myndius, that the Chaldæans maintained that comets are bodies travelling in extended orbits, and were able to predict the coming of some of these meteors.‡ They were acquainted with the precession of the equinoxes,

* The Chaldæans asserted that astrology was founded 'not in reason and physical contemplations, but in the direct experience and observation of past ages.' Bacon's 'Advancement of Learning,' Book III., chap. iv.

† This fable is referred to by Cicero, lib. ii., *de Divinat.* c. 97. Other accounts make the number of years 270,000. *Proclus in Timæus*, lib. i., p. 31. *Diog. Laert. Procem.* p. 3.

‡ The account that the Chaldæans were able to predict earthquakes and inundations

making use of a tropical year of 365 days, 5 hours, 49 minutes, 11 seconds (only 25 seconds too great), and a sidereal year of 365 days, 6 hours, and 11 minutes (not quite 2 minutes too great). The Chaldæans were also acquainted, long before the Egyptians and Greeks, with the art of dialling. But the most remarkable evidence of their skill and ingenuity is undoubtedly the invention of the period called Saros* (or Restitution), by which they were able to predict lunar eclipses, and announce the days on which eclipses of the sun might be expected. This period is still used by astronomers, and is the best period of the kind ever invented. Its nature may be thus stated:—Eclipses of the sun and moon can only take place when the moon is new or full, and near one of her nodes; thus the recurrence of eclipses depends chiefly on the common and nodical lunar months: but the apparent magnitude of the moon, and her position in the sky, must plainly affect the nature and visibility of an eclipse; so that the recurrence of eclipses depends in part on the anomalistic and sidereal lunar months. Now, the Saros contains 223 lunar months; falls short of 242 nodical months by about 39 minutes, and of 239 anomalistic months by less than 5 hours; and lastly, exceeds 241 sidereal months by less than a day. Thus eclipses very nearly recur, take place nearly in the same part of the celestial sphere, and the magnitude of the moon is very nearly equal, in the corresponding eclipses of each successive cycle. Modern astronomers calculate the length of the Saros to be 6,585 days, 7 hours, 40 minutes, and 38 seconds; the Chaldæan value of the period was 6,585 days, 8 hours, exceeding the true period by only 19 minutes, 22 seconds.†

There are good reasons for supposing that the Chaldæans were acquainted with the true system of the universe. It has been mentioned in Chapter II. that the ancients were acquainted with the relative distances of the planets, a knowledge which could only have been obtained from considerations founded on the true system. Again, though Hipparchus had the advantage of Chaldæan records with which to compare his own observations, he deduced the tropical and sidereal years—in other words, calculated the precession of the equinoxes—with less correctness than the Chaldæans.

is possibly fabulous. It is not altogether impossible, however, that, close observers as they were of nature, and able to devote their whole time to watch her operations, they noted and recorded warning signs that escaped the notice of the less observant.

* Hesychius says, *Σάρος ἀριθμός τις παρὰ Βαβυλωνίοις.* The ancients were not well acquainted, however, in general, with the nature of the Saros. Abydenus and Berosus estimated the Saros at 3,600 years; Euseb. Chron., lib. I. p. v. 13, and p. vi. 37. Suidas came nearer the true value, estimating the Saros at 18½ years.

† They trebled the period to make the number of days exact, so that eclipses happened nearly at the same hour of the day in each successive triple-Saros.

Hence we may conclude that such accurate observers were not unacquainted with those irregularities which the Epicyclians were forced to explain by means of epicycles, eccentrics, and oscillating planes. Now, it was a part of the Greek character to frame systems on insufficient knowledge, and to explain false systems by false hypotheses,—

> Collecting toys
> And trifles for choice matters, worth a sponge;
> As children gathering pebbles on the shore.*

We can understand, then, that the Greeks should make the earth the centre of all celestial motions, should suppose these to take place necessarily in circles, and the like. But we have no reason for supposing that the Chaldæans, close and patient observers of nature, and disposed, like all the Semitic races, to seek grand and simple interpretations of natural phenomena, would make the earth the centre of motion, when observation had once proved that such a system could only be maintained by cumbrous and complicated hypotheses. On the contrary, from the reverence with which they regarded the planetary bodies, it seems little likely that they imagined these bound to move about a terrestrial centre.

It may be urged that, if the Chaldæans considered the sun to be the centre of the scheme, they must either have adopted some such modified system of epicycles and eccentrics as Copernicus, or else have preceded Kepler in the discovery of the elliptic motion of the planets. It appears, indeed, highly probable that their observations were conducted with sufficient accuracy to enable them to detect the ellipticity of the planetary orbits.† The account given by Philolaus of the opinions of Pythagoras seems clearly to point to knowledge of this kind. We have seen that the Chaldæans regarded the planetary motions as sacred secrets, not to be spoken of save in doubtful mysterious terms. Further, in the time of Pythagoras they were subdued under Cyrus; so that, fearful of offending the Fire-worshipping Persians, the Chaldæans would conceal their own religious opinions—or, in other words, their system of astronomy. If the opinions attributed to Pythagoras by Philolaus were really derived from Chaldæan

* 'Greek natural philosophers,' says Humboldt, 'were but little disposed to pursue observations, but evinced inexhaustible fertility in giving the most varied interpretations of half-perceived facts.' 'The Greeks,' wrote Bacon, 'by only employing the power of the understanding, have not adopted a fixed rule, but have laid their whole stress upon intense meditation, and a continual exercise and perpetual agitation of the mind.'

† 'Callisthenes,' says Porphyrius, 'sent to Greece observations of the planetary motions taken by the Chaldæans for 1,903 years before Alexander's entry into Babylon.' Aristotle, speaking of an occultation of Mars by the moon, adds, 'Such observations have been made on the other planets for many years by Egyptian and Babylonian astronomers; and many of these have come to our knowledge.'

astronomers, they effectually attained both ends by simply telling him 'that the earth and planets move in oblique circles about Fire.' Philolaus adds, 'as the sun and moon do,' from which we may conclude that *he*, at least, was not acquainted with the true system; nor is it probable that Pythagoras was better informed; but the very circumstance that Pythagoras probably knew little of astronomy makes it the more remarkable that he should not only attribute motion to the earth, but assign elliptic orbits to the earth and planets.

There is no evidence that Chaldæan astronomers were acquainted with the nature of gravity. They may have conceived the idea that the sun and planets exercise attractive influences varying with their distances and volumes, but there is no reason to suppose that they were able to deduce from the motions of the planets the manner in which such attraction varied, still less that they were acquainted with the general principle now known as universal gravitation. Dr. Gregory, however, considered that either Pythagoras himself or the astronomers from whom he derived his system were acquainted with the law by which gravity varies with distance. 'He observes that these philosophers spoke allegorically when they asserted that Apollo touched the seven-stringed lyre, which he supposes to represent the sun and the seven planets, and to indicate that the former retained the latter by attractive forces in harmonic proportion; and, because the tones obtained from chords of equal thickness are inversely proportioned to the squares of their lengths, he infers that the harmonic proportion alluded to is the inverse duplicate of the squares.'* We may adopt the opinion of the author from whom the above passage is derived, that the doctrines of Pythagoras did not lie quite so deep; and it is little likely that the Chaldæans concealed real knowledge under so obscure and fanciful an image.

Mr. Layard has shown that the Assyrians and Babylonians were skilful mechanicians, and particularly well acquainted with the nature of the various metals, and the best methods of working and alloying them. There can be little doubt from the account handed down to us in the Book of Daniel of the state of the Chaldæans under their Babylonian masters, that neither wealth nor skill was spared in erecting buildings that might serve as observatories, and in supplying these with astronomical instruments of the best workmanship. The terrace and pyramid of Belus, for instance, were used for astronomical among other purposes: here the Chaldæan astronomers pursued their labours, and thence they proclaimed the hours of the night. We learn from Callisthenes and Epigenes that they recorded astronomical observations on bricks and tiles. Many such tablets,

* 'The Earth and its Mechanism.' By H. Worms. Pp. 9, 10.

formed of clay, and bearing inscriptions in cuneiform characters, have been discovered by Mr. Layard and other eastern travellers. The inscriptions appear to have no reference to astronomical observations. Across the tablets, however, while yet moist, engraved cylinders were rolled, and the impressions thus stamped appear, in general, to be records of celestial phenomena. It may be conjectured that one of the duties of the Chaldæans was to superintend the construction of cylinders, the symbols on which should serve to indicate to the initiated the date corresponding to each tablet.*

When we consider the marvellous exactness with which the Chaldæan astronomers calculated the several periods their determinations of which have reached us, the question suggests itself whether they could possibly have attained such exactness without the aid of the telescope. The art of making glass was known to the Assyrians, who were also sufficiently acquainted with the science of optics to construct lenses. Sir David Brewster, speaking of a plano-convex lens of rock-crystal discovered by Layard at Nimroud, says:—'The convex side is tolerably well polished, and though uneven from the mode in which it has been ground, it gives a tolerably distinct focus, at the distance of 4½ inches from the plane side:' he adds, 'It could not have been intended as an ornament; we are entitled, therefore, to consider it as intended to be used as a lens, either for magnifying, or for concentrating the rays of the sun.'

If we were better acquainted with the nature of the Chaldæan system of mythology, and knew the planets with which their various deities were associated, we might be assisted in the inquiry whether they used telescopic aid in examining the celestial bodies. It is clear that little connection exists between the Assyrian and Greek systems of mythology. Now and then some attribute of an Assyrian reminds us of a Greek deity; but when we proceed to consider other attributes no further resemblance can be traced.† Various opinions have been expressed as to the celestial bodies with which the deities were severally associated; the most probable arrangement appears to be the following:—Nisroch, the great triune deity, was associated with the planet Saturn; Bel with Jupiter; Merodach with Mars; Mylitta

* It is doubtful whether all the symbols on these cylinders are astronomical, or only those around the principal figures and in the background. It has been suggested that the principal figures may represent constellations. Among symbols representing the sun, moon, and stars, Dr. Birch has detected figures corresponding to ten of the Zodiacal constellations.

† For instance, Mylitta, the chief goddess of the Assyrians, wife of Bel (chief of the twelve great gods presided over by Nisroch), corresponds so far with Heré; but in her other attributes more nearly resembles Aphrodité.

*Fig 1*

*Fig. 2.*

*Fig. 3.*

## ASSYRIAN DEITIES.

*Fig 4.*

TABLET WITH IMPRESSION STAMPED FROM
AN ASSYRIAN CYLINDER.

R.A.Proctor del.

H.Adlard Sc.

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with Venus; Nebo with Mercury; Ishtar with the moon; and Shamash with the sun.*

Fig. 1, Plate XIV., represents one of the forms in which the deity Mylitta appears in Assyrian sculptures. In her left hand she bears an emblem resembling the symbol still used by astronomers to represent the planet Venus; † in her right hand she bears a staff tipped with a crescent. Now, with moderate telescopic power, Venus is the only planet that ever presents the crescent form, Mercury only assuming that form when too near the sun to be seen without telescopes of great power, properly mounted, and directed to the calculated place of the planet on the celestial sphere. It may therefore be noticed as, at the least, a singular coincidence, that the crescent should be found associated, not with the deity representing the moon, but with the deity representing a planet whose appearance never affords any indication to the unaided eye of the crescent form.

Bel or Baal is variously represented in Assyrian sculptures. Among the winged globes, which probably symbolised at the same time a deity and the planet with which he was associated, we find one in which the central globe is surrounded by four others: this may possibly indicate the system of satellites attending on Jupiter. Bel is also represented with four horns; and a figure in Assyrian sculptures with four star-tipped wings probably represents the same deity.‡

Nisroch or Asshur (among other attributes) was the time-god or year-

* In one of the Phœnician dialects the sun is called Camosh, a term probably identical with the epithet Chomeus of Apollo. Humboldt mentions in his 'Views into Nature' that Camosi is a South-American word for the sun.

† This symbol ( ♀ ) somewhat resembles the Egyptian symbol of life (the *crux ansata*), which consists of a straight rod and circle separated by a cross-bar,—emblematic (I suppose) of temporal and eternal life separated by death. Mylitta 'was sometimes represented with the waters of life flowing from her breasts:' see Layard's 'Nineveh and Babylon,' p. 606; note also the first figure on page 605. The figures of Plate XIV. are taken from the above-named work, and from Layard's 'Nineveh and its Remains.'

The astronomical symbols of the planets have been derived, in all probability, from Chaldæan and Assyrian sources. The symbol of the planet Mercury ( ☿ ) is the *caduceus*, which, like the *petasus*, is an emblem of eastern origin. The symbol of Mars ( ♂ ) represents a shield and spear—the former being the circular shield 'with which Assyrian spearmen are constantly represented' (Layard's 'Nineveh and Babylon,' p. 194). The symbols of Jupiter and Saturn ( ♃ and ♄ ) are more doubtful; a resemblance has been traced between the former and the initial letter of the Greek Zeus; it appears to me far more probable, however, that the symbols of Jupiter and Saturn are simply the Syro-Arabic forms of the numbers 4 and 5, indicating the positions of these bodies in the scheme of the Planetary Five. The symbol of the earth ( ♁ ) is simply the inverted emblem of life, and possibly bears some reference to terrestrial corruption and decay.

‡ If the Chaldæans used telescopes even of moderate power they must have been acquainted with the vastness of Jupiter's bulk, and would therefore, in all probability,

god, beneath whom were the twelve great gods presiding over the twelve months of the year; and beneath these again some 4,000 deities who ruled over the days of the year and the various phenomena of nature. If the Chaldæan astronomers were acquainted with the fact that Saturn is girdled about by a ring, they could have chosen no more suitable representation of a deity who was supposed to sway 'the circling seasons and the flight of time.' For among all nations and in every age the ring has been chosen as the aptest emblem of time; and such names as *annus*, ἔνος, ἔτος ἐνιαυτός, *year*, * &c., indicate that the ring has been considered an especially appropriate emblem of the simplest and most marked recurring period known to man. Thus we find the serpent-ring among other emblems of Cronus or Saturn; and Homer constantly applies to Cronus the epithet ἀγκυλομήτης, an epithet plainly connected with the influence of this deity over the year.†

We have seen that Galileo was able to detect a peculiarity in Saturn's figure with a very low telescopic power. In the clear skies of eastern climes, it is probable that the same or even less power would distinctly exhibit the form of the ring. But, further, if we suppose, as we are justified in doing, that the changes observed in Saturn's rings during the 200 years that have elapsed since their discovery, form part of a progressive series of changes, the rings must have been very much narrower—and, therefore,

assign to him a proportionately vast attractive influence. Possibly some tradition of such knowledge is embodied in the well-known passage:—

Εἰ δ' ἄγε, πειρήσασθε θεοί, ἵνα εἴδετε πάντες,
Σειρὴν χρυσείην ἐξ οὐρανόθεν κρεμάσαντες,
Πάντες δ' ἐξάπτεσθε θεοί, πᾶσαί τε θέαιναι·
Ἀλλ' οὐκ ἂν ἐρύσαιτ' ἐξ οὐρανόθεν πεδίονδε
Ζῆν' ὕπατον μήστωρ', οὐδ' εἰ μάλα πολλὰ κάμοιτε·
Ἀλλ' ὅτε δὴ καὶ ἐγὼ πρόφρων ἐθέλοιμι ἐρύσσαι,
Αὐτῇ κεν γαίῃ ἐρύσαιμ', αὐτῇ τε θαλάσσῃ·
Σειρὴν μέν κεν ἔπειτα περὶ ῥίον Οὐλύμποιο
Δησαίμην· τὰ δέ κ' αὖτε μετήορα πάντα γένοιτο.
Iliad VIII. 18–26.

Compare also line 451 of the same Book, and Plato's 'Theætetus,' i. 153. The image in the above passage seems singularly infelicitous unless interpreted in some such way as is indicated above; such an explanation appears more natural than that commonly offered, which refers the image to subtle dogmas of physical influences and powers, associating together the various parts of the universe.

* From *yra*, to surround.

† The later meaning of the epithet appears little suited to a deity represented as a semi-idiotic old man swallowing stones for children. As the epithet βοῶπις of Heré was probably derived from the worship of the eastern original of Heré under the form of a cow (Müller 'Scient. Myth.,' p. 202), so the epithet ἀγκυλομήτης either refers, like the mythological account of Cronus, to his rule over the seasons, or is derived from the form under which the eastern original of Cronus was worshipped. See figs. 2 and 3, Plate XIV.

their ring-form much more easily detected—three or four thousand years ago than in Galileo's day.* It certainly does not appear incredible that mechanicians so ingenious as the Chaldæans, and not unacquainted with the laws of optics, should have been able to construct telescopes as powerful as Galileo's; and, with such telescopes, they could not have failed to detect Saturn's rings. It is certainly a singular coincidence that the god Nisroch should be represented in Assyrian sculptures *within a ring* (see figs. 2 and 3, Plate XIV.); and it seems difficult to account for the selection of such a remarkable figure to represent the supreme god, unless we suppose the Chaldæans acquainted with the peculiar conformation of the planet they associated with that deity. On such a supposition, however, one can readily understand, that they should conceal their knowledge under such mystical symbols.† In fig. 3, the deity is represented as triune, a figure which may have reference to the triple attributes of the god; though it is not altogether impossible that the triplicity observed in several representations of this deity, may have reference to a tradition of some such imperfect discovery of Saturn's ring as Galileo effected.

The use of engraved cylinders by Chaldæan astronomers has already been noticed. Fig. 4, Plate XIV., represents a tablet stamped from such a cylinder. The figures in the background represent the sun, the crescent moon, a star, and *a ring*. The opening of the ring is rather less than the full opening of Saturn's ring, and the breadth of the ring corresponds with the breadth we may imagine the ring to have had three or four thousand years ago. The figure of a ring is met with in other engraved tablets, the breadth being about the same in all, but the opening varying. In a cylinder, represented at page 343 of Layard's 'Nineveh and Babylon,' the sun and a bird (probably a constellation) appear above a ring nearly closed; the principal figures of this cylinder represent Dagon the Fish-god, another deity, and a crowned figure in the act of adoration before an object resembling figures 2 and 3, Plate XIV., but the human figure and ring are replaced by an open eye. It may be noticed that when Saturn is viewed with a telescope of small power, the rings being open to their full extent, he presents an appearance somewhat resembling an open eye: the resemblance is, at least, sufficiently close to attract the notice of an imaginative and poetical race like the Chaldæans.

It is probable that the researches of travellers in the East will, before

* It is not altogether impossible that some of the inner satellites have been formed from outer rings about Saturn within the interval of time mentioned.

† The mysterious precept of Pythagoras, 'Thou shalt not represent the deity within a ring,' probably has reference to such figures. His travels, therefore, may be presumed to have extended into Assyria and Babylonia. Like many others of his precepts, this one probably refers to religious observances and rites that had come under his notice during his travels, but which were unintelligible to him.

long, afford us more accurate information of the history, language, and arts of Assyria and Babylonia than we now possess. Thus a meaning may be found in symbols and inscriptions which are at present unintelligible; and we may thence obtain some idea of the methods of observation employed by Chaldæan astronomers, of the manner in which they recorded, and of the system by which they explained, astronomical phenomena.

# NOTE B.

## LAPLACE'S NEBULAR THEORY.

This world was once a fluid haze of light,
Till toward the centre set the starry tides,
And eddied into suns, that wheeling cast
The planets. TENNYSON.

Τὸν πάντα κόσμον ἐποίησεν ὁ δημιουργὸς οὐ χερσὶν ἀλλὰ λόγῳ.
MERCURIUS TRISMEGISTUS.

No part of the solar system affords so striking an argument in favour of Laplace's Nebular Hypothesis as the Saturnian system of rings. A brief examination of some objections that have been urged against that theory will therefore not be out of place in the present work. The hypothesis itself is so well known, that it is unnecessary to enter into any lengthened description of it.

The planetary system presents certain points of uniformity for which the law of gravity does not account; thus:—The planets revolve in the same direction about the sun, in orbits nearly circular, and nearly in one plane; they also rotate on their axes, and their satellites (excepting those of Uranus, and possibly Neptune's satellite) revolve, in the same direction; all the known asteroids also revolve in the same direction. That this uniformity is not the effect of chance, will appear from a single example:—The probability that the 83 known asteroids, if projected in the same plane but otherwise at random, should all revolve in one direction, is less than 1 in 4,835,700,000,000,000,000,000,000.

The uniformity here considered may, undoubtedly, result from design in original creation; but, on the other hand, the idea that the solar system has been *developed* under the operation of uniform laws, is no more opposed to just conceptions of the wisdom and power of the Creator, than is the idea of the development of a tree or of an animal.* The solar system and

* This view has been objected against as atheistical; but the objection is founded on the atheistical assumption that the minor developments referred to are *not* parts of the scheme of the Almighty. It is also urged that to suppose the human race so insignificant a part of the universe, as it would appear (to our conceptions), from the

the whole portion of space falling within the range of human observation, form, necessarily, infinitely minute parts of the space over which the operations of the Almighty Mind extend—that is, of infinite space; and the time within which our system has been created, and during which it will continue to exist, form necessarily infinitely minute parts of the time during which the operations of the Almighty Mind have been and will be in action—that is of eternity. We must, then, seek infinitely farther back for the operation of a First Cause, than merely to the origin of our system, vast as it may appear to merely human conceptions. We are therefore at liberty to seek or adopt any theory which explains the existing state of our system by the operation of uniform laws.*

Laplace conceived that the solar system may have been formed by the gradual cooling and condensation of a vast rotating nebulous globe; that in the process of contraction successive rings were thrown off, to form in one case a zone of small planets, but in general to break up and form each a single globe; that in the formation of such globes a similar process was repeated, ending in the formation of satellites, and in a single case of what we now know to be a ring of small satellites.

Modern science is opposed to the idea of a vast nebulous globe, maintained in a state of extreme tenuity by intense heat.† But, on the other hand, the laws of Thermo-dynamics supply a satisfactory explanation of the original process of formation. If we conceive the distribution imagined by Chladni (see Explanation of Astronomical Terms, *Meteoric Stones*) to extend throughout the interstellar spaces, then all the results suggested by Laplace would follow from the agglomeration of vast numbers of cosmical bodies, gathered from vast distances ‡ under the influence of their own attractions, or of the attractions of a central body; and the heat generated by the loss of *vis viva* at the formation of each planet or satellite, would be

above view, is to suppose the infinite wisdom and goodness undoubtedly displayed towards the human race bestowed on an unworthy object; an objection founded on the erroneous notion that we are to conceive otherwise of the infinite wisdom and goodness of the Almighty than as bestowed on each the minutest (or to our view the most insignificant) of His works, and on each the minutest interval of time.

* See Bacon's 'Advancement of Learning,' Book I., § 1, and Book III., chapter iv.; and Hooker's 'Ecclesiastical Polity,' Book I., chapters ii. and iii. Compare Nichol's 'Cyclopædia of the Physical Sciences,' Article 'Nebular Hypothesis;' and Whewell's 'Astronomy and General Physics,' (Bridgewater Treatise.)

† It was in illustration of this part of his theory that Laplace referred to Herschel's Nebular Theory. (See Explanation of Astronomical Terms, *Nebular Theory, Herschel's.*) It is a mistake to suppose that the overthrow of the latter theory carries with it Laplace's Nebular Theory, the main points of which are in no way connected with Herschel's.

‡ Distance is an important element in such a process. See Explanation of Astronomical Terms, *Vis Viva.*

sufficient to account for the obvious signs that the planets, and, in a less degree the satellites, were originally in a state of intense heat.'*

It has been considered by some that the motions of the satellites of Uranus are altogether opposed to the theory of Laplace. It has even been stated that Laplace himself, had he lived to the present day, would have abandoned his theory as untenable on this account alone—which will hardly appear probable when we remember that Laplace survived the elder Herschel more than four years, and published the first and fifth editions of his 'Système du Monde' nine and thirty-seven years, respectively, after the discovery of two of the satellites of Uranus.† In fact, the motions of these satellites are not so utterly opposed to the theory as might at first sight be supposed. Satellites travel, in general, nearly in the equatorial planes of their primaries, and these planes have very various inclinations to the ecliptic.‡ Assuming that the inclination in the case of Uranus was originally very nearly 90°, it is conceivable that external disturbing causes § (to which Uranus must have been exposed for a longer time than any planet within his orbit), may have carried the inclination *to and beyond* the right angle: so that, instead of saying that the satellites of Uranus move

* It is possible that in such considerations we may find an explanation of the peculiarities of form and rotation observed in the moon. (See Note C.)

† It has been argued that Laplace 'considered his scheme a mere conjecture' (see 'Science and Scripture,' by Professor Young, p. 14). It is true that Laplace presented his hypothesis *as* an hypothesis, and not as a scientific doctrine; speculations on past processes traceable only in their results must always be imperfect and uncertain; or, to use Laplace's own words, 'everything not resulting from observation or calculation must,' to a certain extent, 'inspire distrust.' But it is also true that Laplace formed a high estimate of the probability of the hypothesis. He speaks of it as 'une hypothèse qui me paraît *résulter, avec une grande vraisemblance* des phénomènes précédents; mais que je présente avec la défiance que doit inspirer tout ce qui n'est point un résultat de l'observation ou du calcul.' Professor Young mistranslates the closing words of the passage into 'that distrust which should inspire everything which is not the result of observation or calculation,' possibly gathering from this singular sentence his idea that Laplace attached a low value to the Nebular Theory.

‡ It may be remarked that Laplace's theory, as originally presented, offers no satisfactory explanation of this diversity of inclination. In the successive collisions through which each globe may be conceived to have been formed (on the altered theory suggested above) we appear to have a sufficiently plausible explanation of the peculiarity in question.

§ We appear to have an indication of the operation of such causes in the peculiar distribution of the perihelia of the planetary orbits alluded to at p. 37 (note). If we suppose, for instance, that our system had passed through a region of the interstellar spaces in which cosmical bodies were distributed with a density varying according to some uniform or tolerably uniform law, and that such passage occupied an interval of time in which the most distant members of the system completed several revolutions; then it is certain that after the passage the aphelia of all the orbits would be found on that side of the system which had passed through the most densely-crowded part of the region.

in a retrograde manner in a plane inclined 78° 58′ to the ecliptic, we might more correctly say that they move in a direct manner in a plane inclined 101° 2′ to the ecliptic. The same considerations apply to the case of Neptune's satellite, increased distance aiding us to interpret what (on the assumption we are considering) would be increased disturbance of inclination. Owing to Neptune's immense distance, however, and his slow motion in his orbit, it is not probable that the direction in which his satellite moves, can have been satisfactorily established in the short interval that elapsed between the discovery of the satellite and the announcement of its retrograde motion. * In any case 'we do not require absolute affirmation or negation, even in universal propositions; if the exceptions be singular or rare, it is sufficient for our purpose.'†

The theory of Laplace is perfectly reconcilable with the Scripture account of creation. It is only necessary to assume that in the first chapter of Genesis the sacred penman has recorded a series of visions, in which was presented to him all that the Almighty saw fit to reveal to mankind of the former condition of our globe. That Moses, like Jeremiah, Ezekiel, Daniel, and Zechariah, received inspired knowledge specially by means of visions, seems suggested by the injunction that the ark and its appurtenances should be made 'after their pattern which was showed' him—or, more correctly, which he 'was caused to see'—'in the mount.'‡ The words of the second verse of the Bible seem to confirm this view:—'The earth was without form and void:' the earth must have had form of some kind, regular or irregular, and though the expression may be interpreted to signify merely that the earth was without regular form, it seems little likely that reference is here intended to the present spheroidal form of the earth; but assuming that Moses describes a vision that passed before him, the words which immediately follow explain the true meaning of the expression:—'The earth was,' that is appeared, 'without form and void,' because 'darkness was upon the face of the deep.' Now, if the earth in 'tracts of fluent heat began'—as would follow from the theory we are considering, and as geological evidence appears clearly to establish—the whole of the waters now forming our oceans must have been suspended round the earth in the form of a dense vaporous envelope through which no ray of the sun's light could pene-

* It is clearly only by the change in the position of the primary that the direction of a satellite's motion can be determined.

† Bacon's 'Novum Organum,' Book II., aphor. xxxiii.

‡ Exodus xxv. 9 and 40; xxvi. 30; and xxvii. 8; see also Numbers viii. 4; Acts vii. 44; and Hebrews viii. 5. Compare the mode in which David received the pattern;—'all this,' he says, 'the Lord made me understand in writing by his hand upon me' (1 Chron. xxviii. 11–19).

trate.* But in the course of ages, the heat of the earth's globe would diminish until it became insufficient to maintain masses so vast in the form of vapour: then light—but not as yet the source of light—began to penetrate the earth's cloudy envelope:—

> Forthwith light
> Ethereal, first of things, quintessence pure,
> Sprang from the deep, and from her native east
> To journey through the aery gloom began,
> Sphered in a radiant cloud.

We are thus able to explain the recurrence of day and night before the appearance of the sun, without having recourse to the Aurora Borealis, to successive electric flashes, to sustained disturbance of the æther pervading space, or to any of the other contrivances that have been invented to explain away the difficulty.

With the further diminution of the overhanging cloud-masses, a firmament,

> Expanse of liquid, pure,
> Transparent, elemental air,

began to appear between the waters on the earth's surface and the vaporous envelope. And next, the dry land appeared, heaved up by volcanic action following the precipitation of such vast masses of water on the as yet lately formed crust of the earth.‡ The land thus upheaved became covered with dense forests and abundant vegetation, nourished by internal heat, while

> From the earth a dewy mist
> Went up and water'd all the ground.

* Venus and Mercury appear to be still surrounded with such dense vaporous envelopes; the true surfaces of these planets have probably never been seen, and, possibly, have never yet received a ray of the sun's light.

† It seems clear that the word (*rokiā*), translated firmament (the στερέωμα of the Septuagint), means merely the variable transparent expanse above the earth at any time [Compare Genesis i. verses 16, 17 and 20.] It seems equally clear that by 'the waters above the firmament' clouds are signified. During many ages after the change recorded in Genesis i. verses 6 and 7, and even after the appearance of the heavenly bodies, the 'waters above the firmament' must have constituted an important part of all the waters of our globe. The notion that the waters above the firmament are waters above the stellar spaces is too absurd to need serious refutation. Compare Genesis vii. 11; viii. 1–3; Job xxvi. 8–11, and xxxviii. 8-11; and Proverbs viii. 23–29.

‡ It may be remarked that nearly all the active volcanoes on our globe are found near the sea. Even the volcano Pe-schan, noted by Remusat as an exception to this rule, is found near a region probably at no very distant date covered by an ocean of vast extent. From Capes Blanco and Verd to the sea of Okotsk the traces of such an ocean run in an uninterrupted series, which includes the deserts of Sahara, Arabia, Shamo, and the Russian steppes; the Mediterranean, Black, Caspian, and Aral seas; and the lakes Bal-kash, Isse-kul (not very far from Pe-schan), and Bai-kal. Probably the ranges of mountains running across Central Africa and

At length the heavenly bodies appear. First the mid-day sun breaks through the cloudy envelope still surrounding the earth; not until many ages have elapsed appears

The moon
Globose; then ev'ry magnitude of stars.*

In this manner may be explained those passages in the Scripture account of creation, which (literally interpreted) appear most opposed to Laplace's Theory, and, I think, to any rational conceptions of the former state of our earth or of the solar system. Into the other difficulties which attend the literal interpretation of that account, or, on the other hand, into the singular correspondence exhibited between the features of the successive days of Creation, considered as visions, and the main features of the successive geological epochs, I do not propose to enter: on these points, the reader is referred to works especially treating of those subjects.†

It may be noticed in conclusion, that in Saturn's ring-system we seem to see the processes conceived by Laplace going on before our eyes; so that it is not impossible that in the course of time we may obtain evidence founded on 'observation and calculation' of the truth of that theory which Laplace despaired of seeing established on a firmer foundation than that of 'strong probability.'

Asia formed the southern limits of a vast northern ocean, a series of promontories pointing northwards marking the outline of a vast southern continent.

* Miller (see following note) fails to notice the correspondence between the order in which the heavenly bodies are mentioned (Genesis i. 14–18), and the order in which they must successively have appeared. He describes the stars as appearing before the sun.

† See Miller's 'Testimony of the Rocks:' Lectures III. and IV.; and authors referred to by him in those lectures.

# NOTE C.

## HABITABILITY OF THE MOON.

The question of the moon's habitability—interesting to astronomers on its own account—acquires an additional interest if we consider that on its solution depends the opinion we shall form of the habitability of the important secondary systems attending on Saturn, Jupiter, and Uranus. I propose to consider in this note some points connected with the inquiry.

The physical conditions and peculiarities of the moon are undoubtedly in striking contrast to those prevailing on the earth. The lunar year consists of little more than twelve lunar days, each day lasting more than four of our weeks. Our seasons, due to an inclination of 23 degrees, are also very different from the lunar seasons due to an inclination of 1½ degrees; if, indeed, we can apply the term seasons to intervals in which the sun rises and sets only three times. Again, our earth (considered as a satellite of the moon) is altogether invisible to three-sevenths of the moon's surface; to the remaining four-sevenths the earth does not rise and set as the moon does to us, but moves within narrow limits round a fixed point on the celestial concave, such motions being the exact converse of the lunar librations; the earth also passes through all her phases in a lunar day and night, the half set of phases passed through in the lunar night varying for each point of the moon's surface.

That the moon has not an atmosphere corresponding in extent and density to our own is undoubted; it has not been considered so certain, however, that the moon's surface is absolutely devoid of atmospheric envelope.

The first and most obvious argument against the presence of a lunar atmosphere, is that the lunar disc, even when examined with the most powerful telescopes, exhibits no indication of clouds. Owing to the slowness of the moon's rotation we should hardly expect that belts of clouds would be formed, as on the swiftly rotating planets Saturn and Jupiter, but irregularly dispersed clouds, even if not separately visible, must produce effects very easily traceable from the earth. The distinctness of the outlines of mountains, plains, and valleys, on the moon's surface, would vary with the aggregation and dispersion (due to variations of temperature) of clouds and mists about them. No such changes are observable: as long as the clearness of our own atmosphere remains unchanged, the irregularities of the lunar surface are seen with unvarying distinctness. It appears reasonable, then, to conclude that the visible lunar hemisphere is either devoid cf air or of water.

Secondly, if the moon were surrounded by an atmosphere, even of limited extent, the effects of refraction could not fail to be traced in the occultations of stars. The refractive effects of the atmosphere surrounding Saturn are, as we have seen, traceable from the earth, which is removed fully 3,500 times as far from Saturn as from the moon—a disproportion in the distances that would compensate an immense disproportion in the extent and density of the atmospheric envelopes surrounding the two bodies.*

Lastly, there is not the slightest trace of a twilight-circle on the moon, nor do the horns of the new moon extend beyond the semicircle. When it is considered that Venus, though removed so much farther than the moon, and though she is one of the most difficult objects of telescopic observation in the heavens, distinctly presents both these phenomena, their absence in the case of the moon appears the more remarkable. If the moon had an atmosphere, even of small extent and density, the powerful telescopes that have been directed towards her could not have failed to exhibit the phenomena considered.

On the other hand, arguments are not wanting in support of the hypothesis

* The stars are not always instantaneously occulted by the moon. Some disappear by sudden diminutions of brilliancy (as the star κ Cancri)—a phenomenon that may be accounted for by supposing such stars to be close double or multiple stars; others, after disappearing, reappear for a brief interval—a phenomenon that appears to indicate the existence of vast irregularities upon the moon's surface. But the phenomena that would result from the presence of a lunar atmosphere are altogether different. Thus, suppose an observer on the moon to witness a central occultation of a star by the earth:—The star as it entered (apparently) the confines of our atmosphere would move more and more slowly; instead of appearing as a point it would assume the form of a circular arc gradually extending farther and farther round the earth's disc; and when actually behind the centre of the earth, the star would appear as a circle of light concentric with the outline of the earth's disc. Passing beyond this point the star would present similar appearances in reverse order. That even in such a central passage a star would not be actually occulted, is clear from the consideration that the horizontal refraction of the earth's atmosphere is upwards of 33′, which would be doubled for an object seen beyond the earth from the moon; but the earth's semi-diameter seen from the moon subtends an arc of only 57′ 6″. Since the moon's semi-diameter viewed from the earth never exceeds 16′ 45″ it is evident that an atmospheric envelope of much less extent than that of the earth would suffice to render the occultation of a star by the moon impossible.

In the article referred to at page 177, note *, Professor Challis omits to notice the phenomena considered above. It seems clear, however, that they would be the most marked phenomena attending an occultation, if the moon had an atmosphere. In a similar manner it may be shown that the phenomena attending an eclipse of the sun would be very different from those actually presented, if the moon had an atmosphere.

that the moon has an atmosphere. In the first place, we might infer from the analogy of our earth, and of the larger planets, that all the members of the solar system are surrounded by atmospheres of greater or less density and extent. In the second place, the traces of past volcanic action on the lunar surface, leave little doubt that while such action went on the moon must have had an atmosphere capable of supporting combustion; and further, must have been enveloped by the gases distributed during tremendous and long-continued eruptions.

An attempt has been made to reconcile these contradictory evidences by the hypothesis that an atmosphere originally surrounding the visible lunar surface has been attracted to the opposite hemisphere.

The moon's centre of gravity is undoubtedly nearer to us than her centre of figure. In the first place, we have in such a displacement the only possible explanation of the peculiarity of the moon's rotation referred to at p. 50.* Secondly, Professor Hansen has proved that an observed discrepancy between the actual lunar inequalities and the results of the theoretical examination of the lunar motions, is removed, if the centre of gravity of the moon is assumed to be 33½ miles farther from the earth than the centre of figure. This result has been confirmed by the comparison of photographic pictures of the moon, taken at the times of her extreme eastern and western librations. In the year 1862, M. Gussew, Director of the Imperial Observatory at Wilna, carefully examined two such pictures taken by Mr. Delarue. The result of the examination may be thus stated:—The outer parts of the visible lunar disc belong to a sphere having a radius of 1,082 miles, the central parts to a sphere having a radius of 1,063 miles; the centre of the smaller sphere is about 79 miles nearer to us than the centre of the larger; the line joining the centres is inclined at an angle of about 5° to the line from the earth at the epoch of mean libration; thus the central

* The question of the moon's rotation has frequently aroused controversy. Bentley and Keill disputed over it in 1690, and so recently as 1855 the columns of the daily press were occupied with its discussion. The question is altogether a verbal one. The moon's motions may be described as being compounded of a motion of revolution around the earth and a motion of rotation in the same time about an axis through the moon's centre (the moon not being a spheroid it is incorrect to speak of 'the moon's own axis'). Now, if it were not for certain irregularities we might simply say that the moon rotates about an axis near the earth, just as a globe rigidly attached to an arm moving on a central stem would be said to rotate about the stem, though to an eye from which stem and arm were concealed the globe would appear to revolve around a centre and rotate in the same time about its axis. In the case of the moon's motions no such simple rotation exists; but it is a question whether the lunar movements would not be expressed more simply, and (taking dynamical considerations into account) more accurately, by saying that the moon rotates about an axis near the earth, and that this axis is subject to such and such motions, than by the mode of expression generally adopted.

part of the moon's disc is about 60 miles nearer to us than it would be if the moon were a sphere of the dimensions indicated by the disc's outline. If we suppose the invisible part of the moon's surface to belong to the larger sphere, and the density of the moon's substance uniform, it would follow from this conformation, that the centre of gravity of the moon is about 30 miles nearer to the earth than is the centre of the larger sphere—that is, than is the centre of the moon's apparent figure.

But although the moon's centre of gravity is thus displaced, it is very doubtful whether we have in such displacement a satisfactory explanation of the observed peculiarities of the lunar disc. The visible hemisphere in all probability was originally clothed with an atmosphere and partially covered by oceans.* Now, it is hardly conceivable that a displacement of the moon's centre of gravity should be followed by the departure even of all the inelastic fluids from the nearer to the further hemisphere, far less of all the elastic atmospheric envelope. Assuming, however, that the atmosphere had thus been displaced, and the fluids dissipated by evaporation *in vacuo* from all the depressions on the visible lunar surface, it is inconceivable that no traces should be visible of the atmosphere and the oceans thus collected on the further lunar hemisphere. Even if the exact half of the moon's surface were invisible to us, some of the oceans would extend into the cavities and depressions visible round the edge of the lunar disc, and the atmosphere would be traceable (by its effect in occultations) completely round that edge; and when we consider that owing to the moon's librations only three-sevenths of the lunar surface are actually hidden from us, we are compelled to reject the notion that the distribution of air and water on the moon's surface is such as has been suggested.

It appears to me that the simplest of all the phenomena presented by the moon—namely, her *colour*—will serve to guide us to an explanation of the contradictions we are considering. Imagine our earth stripped of air and water, and all verdure destroyed from its surface: what would be the appearance of such a globe removed to the distance of the moon? Bathed in the sun's light it would doubtless be a brilliant object, but its brilliancy would differ altogether from the silvery effulgence of the moon. The various strata which rise to the surface at different parts of the earth might not, in general, be separately visible; but the commingling of the colours that mark such strata would certainly produce warmer tints than are observed on any part of the lunar disc. The vast deserts, steppes, llanos, savannahs, and prairies of the earth would be distinctly visible as streaks and patches of uniform colour. The icy polar regions and the

* That oceans once covered parts of the visible lunar hemisphere seems evidenced by the traces of past volcanic action upon the moon's surface. See note ‡, p. 205.

snow-covered mountain ranges would alone reflect the kind of light that we receive from the moon.*

But now let us imagine our globe subjected to another change. We have plain evidence that the climate of the earth was in past ages far warmer than at present. Animals and plants now found only in the tropics were found in the temperate zones, and many forms of life existed on the earth for which even the tropics would now form but a bleak and unsuitable residence. It seems reasonable to conclude that this change of climate is due to the loss of internal heat by slow radiation, and that the change is still proceeding.† Now, imagine this change to proceed until the whole of the water on the earth's surface should be frozen. Then if this dismal globe were removed to the moon's distance, its brilliancy would no longer present a very marked contrast to the lunar light. The frozen surface of the ocean would present precisely such vast level tracts as the so-called lunar seas; ‡ the glacial regions on land would resemble the rough and mountainous districts of the lunar surface; while, if we conceive the continents on our earth gradually covered with snow as the process we have imagined went on, they would correspond exactly to the vast tracts of brilliant white so conspicuous upon the lunar disc.

It is obvious that some of the difficulties before considered disappear if we suppose all fluids on the moon's surface to be frozen. As the atmosphere would in such a case be perfectly free from clouds or mists, all the irregularities of the lunar surface would be distinctly visible, and such distinctness would not be liable to any perceptible variations. Yet, if the lunar atmosphere bore any proportion to the atmosphere of the earth as regards extent, the outlines of the lunar irregularities would be softer than

* Even at the immense distances to which the planets are removed the colours of their surfaces can be traced whenever the actual surfaces are visible, as in the cases of Mars, Jupiter, and Saturn. When a dense atmosphere supports heavy masses of vapour the light reflected is brilliantly white, as in the cases of Mercury and Venus, and the white belts on Saturn and Jupiter. Such whiteness, however, obviously differs altogether from the surface colours of the moon.

† Such a change would be accompanied by a gradual diminution in the dimensions of the earth, and it has been argued that since the period of the earth's rotation has not perceptibly altered during three or four thousand years, no such change can be taking place. It may be remarked, however, that periods of ten or twenty thousand years are but as seconds when compared with the interval necessary to effect perceptible changes of climate in the manner considered. That the dimensions of the earth were once far greater than at present seems evidenced by the cumbrous forms of the animals and reptiles that inhabited it of old. Such animals could have moved with freedom and activity only under the diminished attraction of gravity resulting from greater dimensions of the earth's globe.

‡ Independently of their colour these level spaces in the moon seem scarcely explicable on any other supposition than that they are frozen seas, for while absolutely level they are clearly solid. In every respect save in fluidity they correspond to our terrestrial system of oceans.

they actually appear. The clearness, also, of such an atmosphere would only serve to render the phenomena attending an occultation more distinctly visible; and we have seen that such phenomena would be sufficiently marked.

Let us imagine, however, the effects of a further diminution of temperature. It is well known that when subjected to a loss of heat sufficiently great, many gases, elementary and compound, are reduced successively to the liquid and solid forms. Hitherto no process has enabled the chemist to convert oxygen, nitrogen, or hydrogen, into either the liquid or solid forms; * but there is no reason for supposing that they form exceptions to the general rule, that, under suitable variations of temperature, all substances in nature may assume any one of the three forms—gaseous, liquid, and solid. Now, it is probable that the variations of temperature with which we are familiar include but a small part of the range of possible variations. Thus it is conceivable that a planet parting with its heat by slow radiation might after the lapse of many ages have lost so much heat that all the gases upon its surface would be condensed to the liquid or solid forms. The length of time required to effect such changes would depend on the mass of the planet's globe—a large planet would obviously require a longer time to part with its internal warmth than a small planet. What relation such time would bear to the mass of a planet could not easily be determined, but it is certain that some such relation exists. Now, on Laplace's hypothesis of the development of the solar system, the moon was formed before the earth; and the mass of the moon is little more than $\frac{1}{80}$th part of the earth's mass. Thus it is conceivable that the moon's mass may have become so intensely cold that the atmospheric envelope once clothing it has been condensed into the liquid, and thence into the solid form. It need not necessarily be assumed, however, that all the gases on the moon have been thus solidified. Small seas of liquefied gases may exist upon the moon's surface; and, again, some of the phenomena that have been supposed to indicate the presence of an atmosphere may be due to gaseous envelopes of small extent still uncondensed. We may imagine, for instance, that hydrogen would resist an intensity of cold that would liquefy or solidify all other gases. On these points we can only form vague conjectures, since as yet the more important gases have defied all attempts at liquefaction or solidification.

* A gas may be converted into the liquid and solid forms either by loss of temperature or by pressure. A combination of both processes is generally adopted to condense a gas into the liquid form; then part of this liquid being allowed rapidly to resume the gaseous form, the remainder is solidified, owing to the loss of latent heat.

# APPENDIX II.

## TABLE I.

*Certain Solar Elements.*

| | | |
|---|---|---|
| Greatest apparent diameter (viewed from the earth) | | 32′ 36″·41 |
| Mean apparent diameter (viewed from the earth) | | 32′ 3″·64 |
| Least apparent diameter (viewed from the earth) | | 31′ 31″·79 |
| Greatest apparent diameter (viewed from Saturn) | | 3′ 33″·61 |
| Mean apparent diameter (viewed from Saturn) | | 3′ 21″·66 |
| Least apparent diameter (viewed from Saturn) | | 3′ 10″·98 |
| Equatorial horizontal parallax at mean distance from the earth | 8″·5776 | 8″·9159 |
| Volume (earth's as 1) | 1,415,225 | 1,260,160 |
| Mass (earth's as 1) | 354,936 | 316,047 |
| Density (earth's as 1) | 0·250 | 0·250 |
| Diameter in miles | 888,646 | 854,928 |
| Gravity at equator | 28·7 | 27·6 |
| In one second of time bodies fall in feet | 462·07 | 444·54 |

## TABLE II.

*Certain Elements of the Earth, Jan. 1st,* 1865.

| | |
|---|---|
| Mean distance from sun in miles (sun's equatorial parallax 8″·5776) | 95,274,000 |
| Mean distance (sun's equatorial parallax 8″·9159) | 91,659,000 |
| Greatest distance (mean distance 1) | 1·0167605 |
| Least distance (same unit) | 0·9832395 |
| Mean sidereal revolution (mean solar days) | 365ᵈ 6ʰ 9ᵐ 9ˢ·6 |
| Mean tropical revolution (mean solar days) | 365ᵈ 5ʰ 48ᵐ 46ˢ·6 |
| Mean anomalistic revolution (mean solar days) | 365ᵈ 6ʰ 13ᵐ 49ˢ·3 |
| Earth's motion in perihelio in a mean solar day | 1° 1′ 9″·1 |
| Earth's mean motion in a mean solar day | 0° 59′ 8″·9 |
| Earth's motion in aphelio in a mean solar day | 0° 57′ 11″·8 |
| Earth's mean motion in a mean sidereal day | 0° 59′ 59″·0 |

| | |
|---|---|
| Eccentricity of orbit | 0·0167605 |
| Annual variation of same (decrease) | 0·0000004164 |
| Density (water's as 1) | 5·6747 |
| Polar diameter in miles | 7,898 |
| Mean diameter in miles | 7,916 |
| Equatorial diameter in miles | 7,924 |
| Bodies fall in one second of time in feet | 16·1 |
| Centrifugal force at equator | ·00345 |

## TABLE III.

*Elements of Saturn, Jan. 1st, 1865.*

| | |
|---|---|
| Greatest distance from the sun (earth's mean distance as 1) | 10·072533 |
| Mean distance from the sun (same unit) | 9·538850 |
| Least distance from the sun (same unit) | 9·005167 |
| Eccentricity of orbit (semi-major-axis as 1) | 0·0559484 |
| Annual variation of same (decrease) | 0·000003125 |
| Sidereal revolution in days | 10759·2197106 |
| Synodical revolution in days (at epoch) | 377·767 |
| Mean synodical revolution | 378·092 |
| Longitude of the perihelion | 90° 23′ 36″·4 |
| Annual variation of same (increase) | 19″·31 |
| Same variation referred to ecliptic (increase) | 69″·41 |
| Longitude of the ascending node [ν]* | 112° 29′ 18″·20 |
| Annual variation of same (decrease) | 19″·54 |
| Same variation referred to the ecliptic (increase) | 30″·56 |
| Inclination of orbit to the ecliptic [i′.] | 2° 29′ 26″·15 |
| Annual variation of same (decrease) | 0″·15 |
| Daily motion in orbit, in perihelio | 2′ 15″·3 |
| Mean daily motion in orbit | 2′ 0″·6 |
| Daily motion in orbit, in aphelio | 1′ 48″·2 |
| Inclination of axis to the plane of Saturn's orbit | 63° 10′ 32″·13 |
| Annual variation of same (decrease) | 0″·321 |
| Inclination of axis to the ecliptic | 61° 49′ 38″·05 |
| Annual variation of same (increase) | 0″·350 |
| Time of rotation on axis | 10h 29m 17s |
| Apparent equatorial diameter, at mean distance from earth | 17″·05 |
| Same, Saturn in opposition, in perihelio | 20″·31 |
| Same, Saturn in opposition, at mean distance from sun | 19″·04 |
| Same, Saturn in opposition, in aphelio | 17″·92 |

* The symbols thus bracketed refer to the corresponding symbols in Tables VII. and VIII.

| | | |
|---|---|---|
| Same, Saturn in conjunction, in perihelio | | 16″·25 |
| Same, Saturn in conjunction, at mean distance from sun | | 15″·43 |
| Same, Saturn in conjunction, in aphelio | | 14″·68 |
| Light received at perihelion (earth's at mean distance being 1) | | 0·01233150 |
| Light received at mean distance (same unit) | | 0·01099021 |
| Light received at aphelion (same unit) | | 0·00985651 |
| Greatest distance from the sun in miles | 959,650,000 | 923,238,000* |
| Mean distance from the sun in miles | 908,804,000 | 874,321,000 |
| Least distance from the sun in miles | 857,958,000 | 825,404,000 |
| Equatorial diameter in miles | 75,100 | 72,250 |
| Polar diameter in miles | 68,270 | 65,680 |
| Polar diameter (equatorial diameter as 1) | ·909 | ·909 |
| Compression | $\frac{1}{11}$ | $\frac{1}{11}$ |
| Equatorial diameter (earth's mean diameter being 1) | 9·4871 | 9·1271 |
| Polar diameter (same unit) | 8·6246 | 8·2974 |
| Volume (earth's being 1) | 776·432 | 691·362 |
| Mass or weight (earth's being 1) | 102·683 | 91·433 |
| Density (earth's being 1) | 0·132251 | 0·132251 |
| Same (density of water as 1) | 0·750482 | 0·750482 |
| Surface (unit 1,000,000 square miles) | 16,655 | 15,415 |
| Same (earth's being 1) | 84·54 | 78·25 |
| Weight of a terrestrial pound, or gravity, at pole | 1·239 | 1·192 |
| Weight of a terrestrial pound at equator | 1·041 | 1·002 |
| Centrifugal force at equator | 0·1706 | 0·1641 |
| Bodies fall in one second of time, in feet, at pole | 19·95 | 19·19 |
| Bodies fall in one second, in feet, at equator | 16·76 | 16·12 |

## TABLE IV.

*Elements of Saturn's Rings, Jan. 1st,* 1865.

| | |
|---|---|
| Longitude of ascending node of ring on the ecliptic [☊] | 167° 43′ 28″·93 |
| Annual variation of same (decrease) | 3″·638 |
| Same variation referred to the ecliptic (increase) | 46″·462 |
| Longitude of rising node of ring on Saturn's orbit [λ′] | 171° 43′ 35″·06 |
| Annual variation of same (decrease) | 3″·134 |
| Same variation referred to the ecliptic (increase) | 46″·966 |
| Latitude of rising node of ring on Saturn's orbit [β′] | 2° 8′ 25″·88N |
| Annual variation of same (increase) | 0″·235 |
| Inclination of ring's plane to the ecliptic [ι] | 28° 10′ 21″·95 |
| Annual variation of same (decrease) | 0″·350 |

* The first column corresponds to an equatorial horizontal solar parallax of 8″·5776 the second to an equatorial horizontal solar parallax of 8″·9159.

| | | |
|---|---|---|
| Inclination of ring's plane to the plane of Saturn's orbit [I′] . . . . . . . . | | 26° 49′ 27″·87 |
| Annual variation of same (increase) . . . . | | 0″·321 |
| Arc from rising node of Saturn's orbit on the ecliptic to rising node of ring's plane on Saturn's orbit [☊′] . | | 59° 15′ 42″·49 |
| Annual variation of same (increase) . . . . | | 16″·395 |
| Annual precession of the rising node of ring's plane on Saturn's orbit (or, annual precession of the vernal equinox of Saturn's northern hemisphere) . . . . | | 3″·145 |
| Complete revolution of either equinox in years, about . | | 412,080 |
| Exterior diameter of the outer ring (in miles) . | 173,500 | 166,920* |
| Interior diameter of the outer ring . . | 153,500 | 147,670 |
| Exterior diameter of the inner ring . . | 150,000 | 144,310 |
| Interior diameter of the inner ring . . | 113,400 | 109,100 |
| Interior diameter of the dark ring . . | 95,400 | 91,780 |
| Breadth of the outer bright ring . . . | 10,000 | 9,625 |
| Breadth of the division between the rings . . | 1,750 | 1,680 |
| Breadth of the inner bright ring . . . | 18,300 | 17,605 |
| Breadth of the dark ring . . . . . | 9,000 | 8,660 |
| Breadth of the system of bright rings . . | 30,050 | 28,910 |
| Breadth of the entire system of rings . . | 39,050 | 37,570 |
| Space between the planet and the inner edge of the dark ring . . . . . . | 10,150 | 9,760 |

* The first column corresponds to an equatorial horizontal solar parallax of 8″·5776; the second to an equatorial horizontal solar parallax of 8″·9159

# TABLE V.—*Elements of Saturn's Satellites.*

| No. | Name | Sidereal Revolution | | Distance from Saturn's centre | | | | Eccentricity | Longitude of Peri-Saturnium | Mean Longitude | Epoch |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | In days, hours, &c. | In days | In mean Radii of Saturn | Calculated from Titan's by Kepler's third law | In miles—Sun's Equatorial Horizontal Parallax | | | | | |
| | | | | | | 8″·5776 | 8″·9159 | | | | |
| | | d. h. m. s. | | | | | | | ° ′ ″ | ° ′ ″ | |
| I. | Mimas | 0 22 37 27·9 | 0·942684 | 3·3607 | 3·3607 | 119,725 | 115,335 | 0·06889 | 104 42 | 264 16 36 | 1789·705 |
| II. | Enceladus | 1 8 53 6·7 | 1·370216 | 4·3125 | 4·3123 | 153,630 | 148,000 | uncertain | — | 67 56 25 | 1789·705 |
| III. | Tethys | 1 21 18 25·7 | 1·887798 | 5·3396 | 5·3394 | 190,225 | 183,250 | 0·0051 | 184 36 | 158 31 | 1836·308 |
| IV. | Dione | 2 17 41 8·9 | 2·736908 | 6·8398 | 6·8395 | 243,670 | 224,740 | 0·02 | 42 30 | 327 40 48 | 1836·0 |
| V. | Rhea | 4 12 25 10·8 | 4·517486 | 9·5528 | 9·5525 | 340,320 | 327,840 | 0·02269 | 95 | 353 44 | 1836·0 |
| VI. | Titan | 15 22 41 25·2 | 15·945431 | 22·1450 | 22·1450 | 788,915 | 759,990 | 0·029223 | 244 35 30 | 137 21 24 | 1830·0 |
| VII. | Hyperion | 21 7 7 40·8 | 21·297000 | 26·7834 | 26·8574 | 954,160 | 919,170 | 0·115 | 295 | 32 | 1849·0 |
| VIII. | Japetus | 79 7 54 40·4 | 79·329634 | 64·3590 | 64·5370 | 2,292,790 | 2,208,720 | 0·025 | 130 | 269 37 48 | 1790·0 |

# TABLE VI.—*Elements of the Planets.**

| Name | Symbol | Distance from the Sun | | | Eccentricity | Sidereal Revolution | Synodical Revolution | Longitude of the Perihelion | Annual Variation | Longitude of Ascending Node | Annual Variation |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Mean | Greatest | Least | | | | | | | |
| | | | | | | Days | Days | ° ′ ″ | ″ | ° ′ ″ | ″ |
| Mercury | ☿ | 0·3870984 | 0·4666927 | 0·3075041 | 0·2056178 | 87·9692824 | 115·877 | 74 57 27·0 | + 5·81 | 46 23 55·0 | −10·07 |
| Venus | ♀ | 0·7233317 | 0·7282636 | 0·7183998 | 0·0068183 | 224·7007754 | 583·920 | 124 14 25·6 | − 3·24 | 75 11 29·8 | −20·50 |
| Earth | ⊕ | 1·0000000 | 1·0167751 | 0·9832249 | 0·0167751 | 365·2563744 | — | 100 11 27·0 | +11·24 | — | — |
| Mars | ♂ | 1·523691 | 1·6657795 | 1·3816025 | 0·0932528 | 686·9794561 | 779·936 | 333 6 38·4 | +15·46 | 48 16 18·0 | −25·22 |
| Zone of Asteroids | | | | | | | | | | | |
| Jupiter | ♃ | 5·202767 | 5·453663 | 4·951871 | 0·0482235 | 4332·5848032 | 398·867 | 11 45 32·8 | + 6·65 | 98 48 37·8 | −15·90 |
| Saturn | ♄ | 9·538850 | 10·073278 | 9·004422 | 0·0560265 | 10759·2197106 | 378·090 | 89 54 41·2 | +19·31 | 112 16 34·2 | −19·54 |
| Uranus | ♅ | 19·18239 | 20·07630 | 18·28848 | 0·0466006 | 30686·8205556 | 369·656 | 168 5 24 | + 2·28 | 73 8 47·8 | −36·05 |
| Ne,tune | ♆ | 30·03627 | 30·29816 | 29·77438 | 0·0087193 | 60126·722 | 367·488 | 47 17 58 | — | 130 10 12·3 | — |

| Name | Inclination of Orbit | Annual Variation | Mean daily Motion | Mean hourly Motion | Compression | Time of Rotation | Diameter | | Volume | Mass | Density | Light at | | Gravity | Bodies fall in one Sec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Mean apparent | In miles | | | | Perihelion | Aphelion | | |
| | ° ′ ″ | ″ | ′ ″ | Miles | | h. m. s. | ″ | | | | | | | | Feet |
| Sun | — | — | — | — | — | 607 48 | 1923·64 | 888,646 | 1,415,225 | 354,936 | 0·250 | — | — | 28·36 | 456·6 |
| Mercury | 7 0 13·3 | +0·18 | 245 32·6 | 100,000 | 1/180 | 24 5 28 | 6·69 | 3,089 | 0·0595 | 0·0729 | 1·225 | 10·58 | 4·59 | 0·48 | 7·7 |
| Venus | 3 23 31·4 | +0·07 | 96 7·8 | 80,000 | -- | 23 21 21 | 17·10 | 7,896 | 0·9960 | 0·9101 | 0·908 | 1·94 | 1·91 | 0·90 | 14·5 |
| Earth | — | — | 59 8·3 | 68,000 | 1/300 | 23 56 4 | — | 7,926 | 1·0000 | 1·0000 | 1·000 | 1·034 | 0·967 | 1·00 | 16·1 |
| Mars | 1 51 5·7 | −0·01 | 31 26·7 | 55,000 | 1/50 | 24 37 22 | 5·8 | 4,070 | 0·1364 | 0·1324 | 0·972 | 0·524 | 0·360 | 0·49 | 7·9 |
| Zone of Asteroids | | | | | | | | | | | | | | | |
| Jupiter | 1 18 42·4 | −0·23 | 4 59·3 | 30,000 | 1/17 | 9 55 26 | 38·4 | 92,164 | 1491·0 | 338·718 | 0·227 | 0·0408 | 0·0336 | 2·45 | 39·4 |
| Saturn | 2 29 29·9 | −0·15 | 2 0·6 | 22,000 | 1/10 | 10 29 17 | 17·1 | 75,070 | 772·0 | 101·364 | 0·131 | 0·0123 | 0·0099 | 1·09 | 17·6 |
| Uranus | 0 46 29·2 | +0·03 | 0 42·4 | 15,600 | 1/9 | — | 4·1 | 36,216 | 86·5 | 14·251 | 0·167 | 0·0027 | 0·0025 | 0·76 | 12·3 |
| Neptune | 1 46 59·0 | — | 0 21·6 | 12,700 | — | . — | 2·4 | 33,610 | 76·6 | 18,900 | 0·321 | 0·0011 | 0·0011 | 1·36 | 21·8 |

* The elements of Neptune for January 1st, 1854; the others for January 1st, 1840.

# TABLE VII.

*For Determining the Appearance of Saturn's Rings, &c.**

| January 1st | N | Difference | Log. tan I | Difference | ☊ | i | I | ω |
|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | | | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| 1860 | 125 35 15·76 | 2 10·23 | 9·0970522 | 2564 | 167 39 36·62 | 28 10 23·70 | 7 7 38·45 | 23 27 27·070 |
| 1861 | 125 37 25·99 | 2 10·31 | 9·0967958 | 2565 | 167 40 23·08 | 28 10 23·35 | 7 7 23·45 | 23 27 26·594 |
| 1862 | 125 39 36·30 | 2 10·39 | 9·0965393 | 2566 | 167 41 9·54 | 28 10 23·00 | 7 7 8·45 | 23 27 26·118 |
| 1863 | 125 41 46·69 | 2 10·48 | 9·0962827 | 2567 | 167 41 56·01 | 28 10 22·65 | 7 6 53·45 | 23 27 25·642 |
| 1864 | 125 43 57·17 | 2 10·57 | 9·0960260 | 2568 | 167 42 42·47 | 28 10 22·30 | 7 6 38·46 | 23 27 25·166 |
| 1865 | 125 46 7·74 | 2 10·66 | 9·0957692 | 2568 | 167 43 28·93 | 28 10 21·95 | 7 6 23·48 | 23 27 24·690 |
| 1866 | 125 48 18·40 | 2 10·76 | 9·0955124 | 2569 | 167 44 15·39 | 28 10 21·60 | 7 6 8·50 | 23 27 24·214 |
| 1867 | 125 50 29·16 | 2 10·86 | 9·0952555 | 2570 | 167 45 1·85 | 28 10 21·25 | 7 5 53·53 | 23 27 23·738 |
| 1868 | 125 52 40·02 | 2 10·97 | 9·0949985 | 2570 | 167 45 48·32 | 28 10 20·90 | 7 5 38·56 | 23 27 23·262 |
| 1869 | 125 54 50·99 | 2 11·08 | 9·0947415 | 2571 | 167 46 34·78 | 28 10 20·55 | 7 5 23·60 | 23 27 23·786 |
| 1870 | 125 57 2·07 | 2 11·19 | 9·0944844 | 2571 | 167 47 21·24 | 28 10 20·20 | 7 5 8·65 | 23 27 22·310 |
| 1871 | 125 59 13·26 | 2 11·31 | 9·0942273 | 2572 | 167 48 7·70 | 28 10 19·85 | 7 4 53·70 | 23 27 21·834 |
| 1872 | 126 1 24·57 | 2 11·44 | 9·0939701 | 2572 | 167 48 54·16 | 28 10 19·50 | 7 4 38·76 | 23 27 21·358 |
| 1873 | 126 3 36·01 | 2 11·57 | 9·0937129 | 2573 | 167 49 40·63 | 28 10 19·15 | 7 4 23·83 | 23 27 20·882 |
| 1874 | 126 5 47·58 | 2 11·70 | 9·0934556 | 2573 | 167 50 27·09 | 28 10 18·80 | 7 4 8·90 | 23 27 20·406 |
| 1875 | 126 7 59·28 | 2 11·83 | 9·0931983 | 2573 | 167 51 13·55 | 28 10 18·45 | 7 3 53·98 | 23 27 19·930 |
| 1876 | 126 10 11·11 | 2 11·96 | 9·0929410 | 2573 | 167 52 0·01 | 28 10 18·10 | 7 3 39·07 | 23 27 19·454 |
| 1877 | 126 12 23·07 | 2 12·09 | 9·0926837 | 2573 | 167 52 46·47 | 28 10 17·75 | 7 3 24·17 | 23 27 18·978 |
| 1878 | 126 14 35·16 | 2 12·22 | 9·0924264 | 2574 | 167 53 32·94 | 28 10 17·40 | 7 3 9·27 | 23 27 18·502 |
| 1879 | 126 16 47·38 | 2 12·35 | 9·0921690 | 2574 | 167 54 19·40 | 28 10 17·05 | 7 2 54·38 | 29 27 18·026 |
| 1880 | 126 18 59·73 | 2 12·48 | 9·0919116 | 2574 | 167 55 5·86 | 28 10 16·70 | 7 2 39·49 | 23 27 17·550 |
| 1881 | 126 21 12·21 | 2 12·61 | 9·0916542 | 2574 | 167 55 52·32 | 28 10 16·35 | 7 2 24·61 | 23 27 17·074 |
| 1882 | 126 23 24·82 | 2 12·74 | 9·0913968 | 2574 | 167 56 38·78 | 28 10 16·00 | 7 2 9·74 | 23 27 16·598 |
| 1883 | 126 25 37·56 | 2 12·87 | 9·0911394 | 2574 | 167 57 25·25 | 28 10 15·65 | 7 1 54·88 | 23 27 16·122 |
| 1884 | 126 27 50·43 | 2 13·00 | 9·0908820 | 2574 | 167 58 11·71 | 28 10 15·30 | 7 1 40·03 | 23 27 15·646 |
| 1885 | 126 30 3·43 | 2 13·13 | 9·0906246 | 2574 | 167 58 58·17 | 28 10 14·95 | 7 1 25·19 | 23 27 15·170 |
| 1886 | 126 32 16·56 | 2 13·26 | 9·0903670 | 2575 | 167 59 44·63 | 28 10 14·60 | 7 1 10·35 | 23 27 14·694 |
| 1887 | 126 34 29·82 | 2 13·39 | 9·0901095 | 2575 | 168 0 31·09 | 28 10 14·25 | 7 0 55·52 | 23 27 14·218 |
| 1888 | 126 36 43·21 | 2 13·52 | 9·0898520 | 2575 | 168 1 17·56 | 28 10 13·90 | 7 0 40·70 | 23 27 13·742 |
| 1889 | 126 38 56·73 | 2 13·65 | 9·0895945 | 2575 | 168 2 4·02 | 28 10 13·55 | 7 0 25·89 | 23 27 13·266 |
| 1890 | 126 41 10·38 | 2 13·78 | 9·0893370 | 2575 | 168 2 50·48 | 28 10 13·20 | 7 0 11·09 | 23 27 12·790 |
| 1891 | 126 43 24·16 | 2 13·91 | 9·0890795 | 2575 | 168 3 36·94 | 28 10 12·85 | 6 59 56·29 | 23 27 12·314 |
| 1892 | 126 45 38·07 | 2 14·04 | 9·0888220 | 2575 | 168 4 23·40 | 28 10 12·50 | 6 59 41·50 | 23 27 11·838 |
| 1893 | 126 47 52·11 | 2 14·17 | 9·0885645 | 2575 | 168 5 9·87 | 28 10 12·15 | 6 59 26·72 | 23 27 11·362 |
| 1894 | 126 50 6·28 | 2 14·30 | 9·0883070 | 2576 | 168 5 56·33 | 28 10 11·80 | 6 59 11·95 | 23 27 10·886 |
| 1895 | 126 52 20·58 | 2 14·43 | 9·0880494 | 2576 | 168 6 42·79 | 28 10 11·45 | 6 58 57·18 | 23 27 10·410 |
| 1896 | 126 54 35·01 | 2 14·56 | 9·0877918 | 2576 | 168 7 29·25 | 28 10 11·10 | 6 58 42·42 | 23 27 9·934 |
| 1897 | 126 56 49·57 | 2 14·70 | 9·0875342 | 2576 | 168 8 15·71 | 28 10 10·75 | 6 58 27·67 | 23 27 9·458 |
| 1898 | 126 59 4·27 | 2 14·84 | 9·0872766 | 2576 | 168 9 2·18 | 28 10 10·40 | 6 58 12·93 | 23 27 8·982 |
| 1899 | 127 1 19·11 | 2 14·98 | 9·0870190 | 2576 | 168 9 48·64 | 28 10 10·05 | 6 57 58·20 | 23 27 8·506 |
| 1900 | 127 3 34·09 | | 9·0867614 | | 168 10 35·10 | 28 10 9·70 | 6 57 43·48 | 23 27 8·030 |

* See Note p. 216.

# TABLE VIII.

*For Calculating the Elevation of the Sun above the Plane of the Rings, &c.**

| January 1st | ν | ☊′ | Log. sin I′ | Difference | i′ | λ′ | β′ | I′ |
|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | | | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| 1860 | 112 26 44·40 | 59 14 20·21 | 9·6544178 | 14 | 2 29 26·90 | 171 39 40·23 | 2 8 24·70 | 26 49 26·27 |
| 1861 | 112 27 15·96 | 59 14 36·91 | 9·6544192 | 13 | 2 29 26·75 | 171 40 27·20 | 2 8 24·94 | 26 49 26·59 |
| 1862 | 112 27 46·52 | 59 14 53·30 | 9·6544205 | 14 | 2 29 26·60 | 171 41 14·16 | 2 8 25·17 | 26 49 26·91 |
| 1863 | 112 28 17·08 | 59 15 9·70 | 9·6544219 | 13 | 2 29 26·45 | 171 42 1·13 | 2 8 25·41 | 26 49 27·23 |
| 1864 | 112 28 47·64 | 59 15 26·09 | 9·6544232 | 14 | 2 29 26·30 | 171 42 48·09 | 2 8 25·64 | 26 49 27·55 |
| 1865 | 112 29 18·20 | 59 15 42·49 | 9·6544246 | 13 | 2 29 26·15 | 171 43 35·06 | 2 8 25·88 | 26 49 27·87 |
| 1866 | 112 29 48·76 | 59 15 58·88 | 9·6544259 | 14 | 2 29 26·00 | 171 44 22·03 | 2 8 26·12 | 26 49 28·19 |
| 1867 | 112 30 19·32 | 59 16 15·28 | 9·6544273 | 13 | 2 29 25·85 | 171 45 8·99 | 2 8 26·35 | 26 49 28·51 |
| 1868 | 112 30 49·88 | 59 16 31·67 | 9·6544286 | 13 | 2 29 25·70 | 171 45 55·96 | 2 8 26·59 | 26 49 28·83 |
| 1869 | 112 31 20·44 | 59 16 48·07 | 9·6544299 | 14 | 2 29 25·55 | 171 46 42·92 | 2 8 26·82 | 26 49 29·15 |
| 1870 | 112 31 51·00 | 59 17 4·46 | 9·6544313 | 13 | 2 29 25·40 | 171 47 29·89 | 2 8 27·06 | 26 49 29·48 |
| 1871 | 112 32 21·56 | 59 17 20·86 | 9·6544326 | 14 | 2 29 25·25 | 171 48 16·86 | 2 8 27·30 | 26 49 29·80 |
| 1872 | 112 32 52·12 | 59 17 37·25 | 9·6544340 | 13 | 2 29 25·10 | 171 49 3·82 | 2 8 27·53 | 26 49 30·12 |
| 1873 | 112 33 22·68 | 59 17 53·65 | 9·6544353 | 13 | 2 29 24·95 | 171 49 50·79 | 2 8 27·77 | 26 49 30·44 |
| 1874 | 112 33 53·24 | 59 18 10·04 | 9·6544366 | 14 | 2 29 24·80 | 171 50 37·75 | 2 8 28·00 | 26 49 30·76 |
| 1875 | 112 34 23·80 | 59 18 26·44 | 9·6544380 | 13 | 2 29 24·65 | 171 51 24·72 | 2 8 28·24 | 26 49 31·08 |
| 1876 | 112 34 54·36 | 59 18 42·83 | 9·6544393 | 14 | 2 29 24·50 | 171 52 11·69 | 2 8 28·47 | 26 49 31·40 |
| 1877 | 112 35 24·92 | 59 18 59·23 | 9·6544407 | 13 | 2 29 24·35 | 171 52 58·65 | 2 8 28·71 | 26 49 31·72 |
| 1878 | 112 35 55·48 | 59 19 15·62 | 9·6544420 | 14 | 2 29 24·20 | 171 53 45·62 | 2 8 28·94 | 26 49 32·04 |
| 1879 | 112 36 26·04 | 59 19 32·02 | 9·6544434 | 13 | 2 29 24·05 | 171 54 32·58 | 2 8 29·18 | 26 49 32·36 |
| 1880 | 112 36 56·60 | 59 19 48·41 | 9·6544447 | 13 | 2 29 23·90 | 171 55 19·55 | 2 8 29·41 | 26 49 32·69 |
| 1881 | 112 37 27·16 | 59 20 4·81 | 9·6544460 | 14 | 2 29 23·75 | 171 56 6·52 | 2 8 29·65 | 26 49 33·01 |
| 1882 | 112 37 57·72 | 59 20 21·20 | 9·6544474 | 13 | 2 29 23·60 | 171 56 53·48 | 2 8 29·88 | 26 49 33·33 |
| 1883 | 112 38 28·28 | 59 20 37·60 | 9·6544487 | 14 | 2 29 23·45 | 171 57 40·45 | 2 8 30·12 | 26 49 33·65 |
| 1884 | 112 38 58·84 | 59 20 53·99 | 9·6544501 | 13 | 2 29 23·30 | 171 58 27·41 | 2 8 30·35 | 26 49 33·97 |
| 1885 | 112 39 29·40 | 59 21 10·39 | 9·6544514 | 13 | 2 29 23·15 | 171 59 14·38 | 2 8 30·58 | 26 49 34·29 |
| 1886 | 112 39 59·96 | 59 21 26·78 | 9·6544527 | 14 | 2 29 23·00 | 172 0 1·35 | 2 8 30·82 | 26 49 34·61 |
| 1887 | 112 40 30·52 | 59 21 43·18 | 9·6544541 | 13 | 2 29 22·85 | 172 0 48·31 | 2 8 31·05 | 26 49 34·93 |
| 1888 | 112 41 1·08 | 59 21 59·57 | 9·6544554 | 13 | 2 29 22·70 | 172 1 35·28 | 2 8 31·29 | 26 49 35·25 |
| 1889 | 112 41 31·64 | 59 22 15·97 | 9·6544567 | 14 | 2 29 22·55 | 172 2 22·24 | 2 8 31·52 | 26 49 35·57 |
| 1890 | 112 42 2·20 | 59 22 32·36 | 9·6544581 | 13 | 2 29 22·40 | 172 3 9·21 | 2 8 31·75 | 26 49 35·90 |
| 1891 | 112 42 32·76 | 59 22 48·76 | 9·6544594 | 14 | 2 29 22·25 | 172 3 56·18 | 2 8 31·99 | 26 49 36·22 |
| 1892 | 112 43 3·32 | 59 23 5·15 | 9·6544608 | 13 | 2 29 22·10 | 172 4 43·14 | 2 8 32·22 | 26 49 36·54 |
| 1893 | 112 43 33·88 | 59 23 21·55 | 9·6544621 | 13 | 2 29 21·95 | 172 5 30·11 | 2 8 32·46 | 26 49 36·86 |
| 1894 | 112 44 4·44 | 59 23 37·94 | 9·6544634 | 14 | 2 29 21·80 | 172 6 17·07 | 2 8 32·69 | 26 49 37·18 |
| 1895 | 112 44 35·00 | 59 23 54·34 | 9·6544648 | 13 | 2 29 21·65 | 172 7 4·04 | 2 8 32·92 | 26 49 37·50 |
| 1896 | 112 45 5·56 | 59 24 10·73 | 9·6544661 | 13 | 2 29 21·50 | 172 7 51·01 | 2 8 33·16 | 26 49 37·82 |
| 1897 | 112 45 36·12 | 59 24 27·13 | 9·6544674 | 14 | 2 29 21·35 | 172 8 37·97 | 2 8 33·39 | 26 49 38·14 |
| 1898 | 112 46 6·68 | 59 24 43·52 | 9·6544688 | 13 | 2 29 21·20 | 172 9 24·94 | 2 8 33·63 | 26 49 38·46 |
| 1899 | 112 46 37·24 | 59 24 59·92 | 9·6544701 | 14 | 2 29 21·05 | 172 10 11·90 | 2 8 33·86 | 26 49 38·78 |
| 1900 | 112 47 7·80 | 59 25 16·31 | 9·6544715 | | 2 29 20·90 | 172 10 58·87 | 2 8 34·09 | 26 49 39·11 |

* See Note p. 216.

# TABLE IX.

*Great Inequality of Saturn and Jupiter.*

| Years | Great Inequality of Jupiter | | | Great Inequality of Saturn | | |
|---|---|---|---|---|---|---|
| | Equation | Differences | | Equation | Differences | |
| | | First | Second | | First | Second |
| | ′ ″ | ′ ″ | ″ | ′ ″ | ′ ″ | ″ |
| 1550 | − 0 14·16 | | | + 0 48·79 | | |
| | | + 1 18·73 | | | − 3 14·07 | |
| 1560 | + 1 4·57 | | − 0·16 | − 2 25·28 | | + 0·51 |
| | | 1 18·57 | | | 3 13·56 | |
| 1570 | 2 23·14 | | 0·61 | 5 38·84 | | 1·43 |
| | | 1 17·96 | | | 3 12·13 | |
| 1580 | 3 41·10 | | 1·01 | 8 50·97 | | 2·33 |
| | | 1 16·95 | | | 3 9·80 | |
| 1590 | 4 58·05 | | 1·36 | 12 0·77 | | 3·14 |
| | | 1 15·59 | | | 3 6·66 | |
| 1600 | 6 13·64 | | 1·62 | 15 7·43 | | 4·02 |
| | | 1 13·97 | | | 3 2·64 | |
| 1610 | 7 27·61 | | 1·95 | 18 10·07 | | 4·83 |
| | | 1 12·02 | | | 2 57·81 | |
| 1620 | 8 39·63 | | 2·26 | 21 7·88 | | 5·57 |
| | | 1 9·76 | | | 2 52·24 | |
| 1630 | 9 49·39 | | 2·60 | 24 0·12 | | 6·39 |
| | | 1 7·16 | | | 2 45·85 | |
| 1640 | 10 56·55 | | 2·84 | 26 45·97 | | 7·12 |
| | | 1 4·32 | | | 2 38·73 | |
| 1650 | 12 0·87 | | 3·15 | 29 24·70 | | 7·88 |
| | | 1 1·17 | | | 2 30·85 | |
| 1660 | 13 2·04 | | 3·43 | 31 55·55 | | 8·55 |
| | | 0 57·74 | | | 2 22·30 | |
| 1670 | 13 59·78 | | 3·67 | 34 17·85 | | 9·13 |
| | | 0 54·07 | | | 2 13·17 | |
| 1680 | 14 53·85 | | 3·95 | 26 31·02 | | 9·89 |
| | | 0 50·12 | | | 2 3·28 | |
| 1690 | 15 43·97 | | 4·17 | 38 34·30 | | 10·40 |
| | | 0 45·95 | | | 1 52·88 | |
| 1700 | 16 29·92 | | 4·32 | 40 27·18 | | 10·95 |
| | | 0 41·63 | | | 1 41·93 | |
| 1710 | 17 11·55 | | 4·60 | 42 9·11 | | 11·47 |
| | | 0 37·03 | | | 1 30·46 | |
| 1720 | 17 48·58 | | 4·75* | 43 39·57 | | 11·95 |
| | | 0 32·28 | | | 1 18·51 | |
| 1730 | 18 20·86 | | 4·94 | 44 58·08 | | 12·25 |
| | | 0 27·34 | | | 1 6·26 | |
| 1740 | 18 48·20 | | 5·05 | 46 4·34 | | 12·61 |
| | | 0 22·29 | | | 0 53·65 | |
| 1750 | 19 10·49 | | 5·18 | 46 57·99 | | 12·96 |
| | | 0 17·11 | | | 0 40·69 | |
| 1760 | 19 27·60 | | 5·25 | 47 38·68 | | 13·21 |
| | | 0 11·86 | | | 0 27·48 | |
| 1770 | 19 39·46 | | 5·38 | 48 6·16 | | 13·26 |
| | | 0 6·48 | | | 0 14·22 | |
| 1780 | 19 45·94 | | 5·38 | 48 20·38 | | 13·37 |
| | | + 0 1·10 | | | − 0 0·85 | |
| 1790 | 19 47·04 | | 5·38 | 48 21·23 | | 13·52 |
| | | − 0 4·28 | | | + 0 12·67 | |
| 1800 | 19 42·76 | | 5·47 | 48 8·56 | | 13·45 |
| | | 0 9·75 | | | 0 26·12 | |
| 1810 | 19 33·01 | | 5·41 | 47 42·44 | | 13·37 |
| | | 0 15·16 | | | 0 39·49 | |
| 1820 | 19 17·85 | | 5·32 | 47 2·95 | | 13·19 |
| | | 0 20·48 | | | 0 52·68 | |
| 1830 | 18 57·37 | | 5·31 | 46 10·27 | | 13·03 |
| | | 0 25·79 | | | 1 5·71 | |
| 1840 | 18 31·58 | | 5·22 | 45 4·56 | | 12·64 |
| | | 0 31·01 | | | 1 18·35 | |
| 1850 | 18 0·57 | | 5·05 | 43 46·21 | | 12·43 |
| | | 0 36·06 | | | 1 30·78 | |
| 1860 | 17 24·51 | | 4·86 | 42 15·43 | | 11·93 |
| | | 0 40·92 | | | 1 42·71 | |
| 1870 | 16 43·59 | | 4·73 | 40 32·72 | | 11·53 |
| | | 0 45·65 | | | 1 54·24 | |
| 1880 | 15 57·94 | | 4·51 | 38 38·48 | | 10·86 |
| | | 0 50·16 | | | 2 5·10 | |
| 1890 | 15 7·78 | | 4·37 | 36 33·38 | | 10·39 |
| | | 0 54·53 | | | 2 15·49 | |
| 1900 | 14 13·25 | | | 34 17·89 | | |

# TABLE X.

*Passages of the Rings' Plane through the Sun between the years 1600 and 2000.*

| Year | Month | Day | Civil Time | Face of Ring illuminated during interval between passages | Phase at passage |
|---|---|---|---|---|---|
| | | | h. m. | | |
| 1612 | December | 28th | 1 P.M. | | Disappearance |
| | | | | Southern | |
| 1626 | September | 15th | 11 A.M. | | Reappearance |
| | | | | Northern | |
| 1642 | June | 18th | 5 A.M. | | Reappearance |
| | | | | Southern | |
| 1656 | March | 3rd | 7 P.M. | | Reappearance |
| | | | | Northern | |
| 1671 | December | 5th | 9 A.M. | | Disappearance |
| | | | | Southern | |
| 1685 | August | 20th | 12 noon | | Disappearance |
| | | | | Northern | |
| 1701 | May | 23rd | 7 P.M. | | Reappearance |
| | | | | Southern | |
| 1715 | February | 6th | 1 P.M. | | Reappearance |
| | | | | Northern | |
| 1730 | November | 7th | 11 A.M. | | Disappearance |
| | | | | Southern | |
| 1744 | July | 23rd | 6 P.M. | | Disappearance |
| | | | | Northern | |
| 1760 | April | 24th | 4 A.M. | | Reappearance |
| | | | | Southern | |
| 1774 | January | 7th | 1 A.M. | | Reappearance |
| | | | | Northern | |
| 1789 | October | 7th | 12 midn. | | Disappearance |
| | | | | Southern | |
| 1803 | June | 23rd | 10 A.M. | | Disappearance |
| | | | | Northern | |
| 1819 | March | 23rd | 7 P.M. | | Reappearance |
| | | | | Southern | |
| 1832 | December | 4th | 9 P.M. | | Reappearance |
| | | | | Northern | |
| 1848 | September | 3rd | 3 26 P.M. | | Reappearance |
| | | | | Southern | |
| 1862 | May | 18th | 8 30 A.M. | | Disappearance |
| | | | | Northern | |
| 1878 | February | 14th | 2 P.M. | | Disappearance |
| | | | | Southern | |
| 1891 | October | 29th | 1 A.M. | | Reappearance |
| | | | | Northern | |
| 1907 | July | 29th | 3 A.M. | | Reappearance |
| | | | | Southern | |
| 1921 | April | 10th | 4 A.M. | | Disappearance |
| | | | | Northern | |
| 1937 | January | 6th | 4 P.M. | | Disappearance |
| | | | | Southern | |
| 1950 | September | 19th | 1 P.M. | | Reappearance |
| | | | | Northern | |
| 1966 | June | 17th | 2 P.M. | | Reappearance |
| | | | | Southern | |
| 1980 | February | 28th | 3 P.M. | | Reappearance |
| | | | | Northern | |
| 1995 | November | 26th | 5 A.M. | | Disappearance |

TABLE XI.—*For Determining the Sun's Diurnal Path on the Saturnian Heavens, at the Saturnian Solstices and Equinoxes, the Appearance of the Rings, and the Eclipses of the Sun by the Rings, for different Latitudes on Saturn's Globe.*

Altitudes marked * are measured from the north point for northern, and from the south point for southern latitudes; all other altitudes, and all azimuths, are measured from the south point for northern, and from the north point for southern latitudes. The years in the last section of the table are years of 365·25 days.

| SATURNICENTRIC LATITUDE | 0° | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Azimuth of Sun, at sunrise or sunset, at* | | | | | | | | | | | | | | |
| Summer Solstice | 116°49′ | 116°56′ | 117°16′ | 117°51′ | 118°42′ | 119°52′ | 121°24′ | 123°26′ | 126° 5′ | 129°39′ | 134°35′ | 141°53 | 154°31′ | —[1] |
| Either Equinox | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90 0 | 90° 0′ |
| Winter Solstice | 63 11 | 63 4 | 62 44 | 62 9 | 61 18 | 60 8 | 58 36 | 56 34 | 53 55 | 50 21 | 45 25 | 38 7 | 25 29 | —[1] |
| *Meridian Altitude of the Sun, at* | | | | | | | | | | | | | | |
| Summer Solstice | 63 11* | 68 11* | 73 11* | 78 11* | 83 11* | 88 11* | 86 49 | 81 49 | 76 49 | 71 49 | 66 49 | 61 49 | 56 49 | 51 49[1] |
| Either Equinox | 90 0 | 85 0 | 80 0 | 75 0 | 70 0 | 65 0 | 60 0 | 55 0 | 50 0 | 45 0 | 40 0 | 35 0 | 30 0 | 25 0 |
| Winter Solstice | 63 11 | 58 11 | 53 11 | 48 11 | 43 11 | 38 11 | 33 11 | 28 11 | 23 11 | 18 11 | 13 11 | 8 11 | 3 11 | —1 49[1] |
| *Diurnal Arc traversed by the Sun, at* | | | | | | | | | | | | | | |
| Summer Solstice | 180 0 | 185 4 | 190 14 | 195 34 | 201 12 | 207 16 | 213 56 | 221 28 | 230 12 | 240 46 | 254 8 | 272 28 | 302 18 | 360 0[1] |
| Either Equinox | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 | 180 0 |
| Winter Solstice | 180 0 | 174 56 | 169 46 | 164 26 | 158 48 | 152 44 | 146 4 | 138 32 | 129 48 | 119 14 | 105 52 | 87 32 | 57 42 | 0 0[1] |
| *Length of Day, at* | | | | | | | | | | | | | | |
| Summer Solstice | 5h 14·6m | 5h 23·5m | 5h 32·6m | 5h 41·9m | 5h 51·7m | 5h 59·3m | 6h 14·0m | 6h 27·1m | 6h 42·4m | 7h 0·9m | 7h 24·3m | 7h 56·3m | 9h 48·5m | 10h 29·3m[1] |
| Either Equinox | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 | 5 14·6 |
| Winter Solstice | 5 14·6 | 5 5·8 | 4 56·7 | 4 47·4 | 4 37·6 | 4 30·0 | 4 15·3 | 4 2·2 | 3 46·9 | 3 28·4 | 3 5·0 | 2 33·0 | 1 40·8 | 0 0·0[1] |
| *Azimuth of Point of Intersection with Horizon, of* | | | | | | | | | | | | | | |
| Outer Edge of Outer Ring (A) | 90° 0′ | 88° 1′ | 85°59′ | 83°51′ | 81°36′ | 79° 8′ | 76°24′ | 73°16′ | 69°36′ | 65°13′ | 59°42′ | 52°22′ | 41°37′ | 20°59′ |
| Inner Edge of Outer Ring | 90 0 | 87 38 | 85 18 | 82 49 | 80 10 | 77 16 | 74 1 | 70 18 | 65 57 | 60 36 | 53 41 | 44 12 | 28 34 | —[2] |
| Outer Edge of Inner Ring | 90 0 | 87 34 | 85 9 | 82 35 | 79 50 | 76 50 | 73 29 | 69 39 | 65 8 | 59 33 | 52 22 | 42 17 | 24 57 | —[2] |
| Inner Edge of Inner Ring (A′) | 90 0 | 86 20 | 82 34 | 78 36 | 74 19 | 69 31 | 64 3 | 57 30 | 49 16 | 37 55 | 17 26 | —[2] | —[2] | —[2] |
| Inner Edge of Dark Ring | 90 0 | 84 42 | 79 13 | 73 21 | 66 50 | 59 17 | 50 0 | 37 28 | 13 55 | —[2] | —[2] | —[2] | —[2] | —[2] |
| *Arc of Horizon on either side of Meridian, covered by* | | | | | | | | | | | | | | |
| Outer Ring | —[2] | 0 23 | 0 41 | 1 2 | 1 26 | 1 52 | 2 23 | 2 58 | 3 39 | 4 37 | 6 1 | 8 10 | 13 3 | 20 59 |
| Division between Rings | —[2] | 0 4 | 0 9 | 0 14 | 0 20 | 0 26 | 0 32 | 0 39 | 0 49 | 1 3 | 1 19 | 1 55 | •3 37 | —[2] |
| Inner Ring | —[2] | 1 14 | 2 35 | 3 59 | 5 31 | 7 19 | 9 26 | 12 9 | 15 52 | 21 38 | 34 56 | 42 17 | 24 57 | —[2] |
| Dark Ring | —[2] | 1 38 | 3 21 | 5 15 | 7 29 | 10 14 | 14 3 | 20 2 | 35 21 | 37 55 | 17 26 | —[2] | —[2] | —[2] |
| System of Bright Rings | —[2] | 1 41 | 3 25 | 5 15 | 7 17 | 9 37 | 12 21 | 15 46 | 20 20 | 27 18 | 42 16 | 52 22 | 41 37 | 20 59 |
| Complete System of Rings | —[2] | 3 19 | 6 46 | 10 30 | 14 46 | 19 51 | 26 24 | 35 48 | 55 41 | 65 13 | 59 42 | 52 22 | 41 37 | 20 59 |
| *Altitude of Point of Intersection with Meridian, of* | | | | | | | | | | | | | | |
| Outer Edge of Outer Ring (B) | 90 0 | 81 52 | 73 48 | 65 52 | 58 9 | 50 41 | 43 29 | 36 35 | 30 0 | 23 43 | 17 44 | 12 2 | 6 37 | 1 26 |
| Inner Edge of Outer Ring | 90 0 | 81 4 | 72 15 | 63 39 | 55 24 | 47 30 | 39 58 | 32 52 | 26 11 | 19 52 | 13 56 | 8 10 | 3 2 | —[2] |
| Outer Edge of Inner Ring | 90 0 | 80 53 | 71 54 | 63 9 | 54 47 | 46 47 | 39 12 | 32 4 | 25 22 | 19 4 | 13 9 | 7 34 | 2 18 | —[2] |
| Inner Edge of Inner Ring (B′) | 90 0 | 77 0 | 64 28 | 53 13 | 43 1 | 33 57 | 25 55 | 18 47 | 12 22 | 6 33 | 1 14 | —[2] | —[2] | —[2] |
| Inner Edge of Dark Ring | 90 0 | 70 14 | 53 3 | 39 14 | 28 20 | 19 35 | 12 20 | 6 9 | 0 46 | —[2] | —[2] | —[2] | —[2] | —[2] |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Arc of Meridian covered by* | | | | | | | | | | | | | | |
| Outer Ring | —[3] | 0 48 | 1 3 | 2 13 | 2 45 | 3 11 | 3 31 | 3 43 | 3 49 | 3 51 | 3 48 | 3 42 | 3 35 | 1 26 |
| Division between Rings | —[3] | 0 11 | 0 21 | 0 30 | 0 37 | 0 43 | 0 46 | 0 48 | 0 49 | 0 48 | 0 47 | 0 46 | 0 44 | —[3] |
| Inner Ring | —[3] | 3 53 | 7 18 | 9 56 | 11 46 | 12 50 | 13 17 | 13 17 | 13 0 | 12 31 | 11 55 | 7 34 | 2 18 | —[3] |
| Dark Ring | —[3] | 6 46 | 11 33 | 13 59 | 14 41 | 14 22 | 13 35 | 12 38 | 11 36 | 6 33 | 1 14 | —[3] | —[3] | —[3] |
| System of Bright Rings | —[3] | 4 52 | 9 12 | 12 39 | 15 8 | 16 44 | 17 34 | 17 48 | 17 38 | 17 10 | 16 30 | 12 2 | 6 37 | 1 26 |
| Complete System of Rings | —[3] | 11 38 | 20 45 | 26 38 | 29 49 | 31 6 | 31 9 | 30 26 | 29 14 | 23 43 | 17 44 | 12 2 | 6 37 | 1 26 |
| *Arc of* | | | | | | | | | | | | | | |
| Outer Edge of Outer Ring } above Horizon. | 128 42 | 128 32 | 128 0 | 127 6 | 125 46 | 123 54 | 121 28 | 118 16 | 114 2 | 108 24 | 100 42 | 89 38 | 72 20 | 37 2 |
| Inner Edge of Outer Ring } above Horizon. | 121 24 | 121 12 | 120 36 | 119 32 | 117 58 | 115 48 | 112 54 | 109 6 | 104 2 | 97 12 | 87 40 | 73 24 | 48 8 | —[3] |
| Outer Edge of Inner Ring } above Horizon. | 119 54 | 119 42 | 119 4 | 117 58 | 116 20 | 114 8 | 111 8 | 107 12 | 101 56 | 94 50 | 84 52 | 69 44 | 41 54 | —[3] |
| Inner Edge of Inner Ring } above Horizon. | 97 4 | 96 44 | 95 46 | 94 4 | 91 32 | 88 0 | 83 10 | 76 34 | 67 12 | 53 0 | 24 58 | —[3] | —[3] | —[3] |
| Inner Edge of Dark Ring } above Horizon. | 76 8 | 75 42 | 74 16 | 71 46 | 67 58 | 62 28 | 54 30 | 42 8 | 16 8 | —[3] | —[3] | —[3] | —[3] | —[3] |
| Meridian Altitude of a Declination-Parallel through the point } A | 90 0 | 83 1 | 76 3 | 69 4 | 62 7 | 55 10 | 48 15 | 41 22 | 34 31 | 27 45 | 21 5 | 14 30 | 8 3 | 1 46 |
| Meridian Altitude of a Declination-Parallel through the point } A′ | 90 0 | 81 21 | 72 41 | 64 0 | 55 17 | 46 31 | 37 44 | 28 53 | 20 0[4] | 11 6[4] | 2 10[4] | —[3] | —[3] | —[3] |
| Azimuth of Point of Intersection with Horizon of a Decⁿ-Parˡ through the point. } B | 90 0 | 86 51 | 83 42 | 80 32 | 77 23 | 74 10 | 70 50 | 67 19 | 63 29 | 59 7 | 53 53 | 47 8 | 37 28 | 18 54 |
| Azimuth of Point of Intersection with Horizon of a Decⁿ-Parˡ through the point. } B′ | 90 0 | 81 58 | 74 21 | 67 24 | 61 8[4] | 55 19[4] | 49 41[4] | 43 50[4] | 37 9[4] | 28 26[4] | 13 4[4] | —[3] | —[3] | —[3] |
| Interval of Time from Autumnal Equinox to Commencement of Eclipses[5] | —[3] | 0y129·3d | 0y261·4d | 1y 29·1d | 1y162·6d | 1y298·0d | 2y 69·7d | 2y209·5d | 2y351·9d | 3y129·5d | 3y275·4d | 4y 57·8d | 4y208·0d | 4y358·7d |
| *Interval of Time during which the Sun is eclipsed* | | | | | | | | | | | | | | |
| Partially or Totally in the Morning and Evening | —[3] | 0 80·4 | 0 154·7 | 0 223·6 | 0 283·5 | 0 332·5 | 1 4·2 | 1 25·6 | 1 28·2 | 1 13·2 | 0 338·9 | 0 273·8 | 0 175·6 | 0 49·0 |
| Totally throughout the Day | —[3] | 0 31·3 | 0 72·5 | 0 128·7 | 0 210·7 | 0 336·9 | 1 169·7 | 2 240·5 | 3 118·2[6] | 2 355·5[6] | 2 249·2[6] | 2 166·6[6] | 2 114·6[6] | 2 90·5[6] |
| Partially or Totally in the Forenoon and Afternoon | —[3] | 0 298·3 | 1 226·5 | 2 179·6 | 4 206·7[6] | 3 261·4[6] | 2 254·6[6] | 1 22·7[6] | — | — | — | — | — | — |
| Partially or Totally in some part of the Day | —[3] | 1 44·8 | 2 351·7 | 3 166·7 | 5 335·7[6] | 5 200·3[6] | 5 63·3[6] | 4 288·8[6] | 4 146·4[6] | 4 3·5[6] | 3 222·9[6] | 3 75·2[6] | 2 290·2[6] | 2 139·5[6] |
| Totally throughout the Day, expressed in Saturnian Days | —[3] | 71·6 | 165·9 | 294·4 | 482·1[6] | 770·8[6] | 1224·0[6] | 2222·0[6] | 2771·5[6] | 2485·2[6] | 2241·9[6] | 2053·2[6] | 1934·0[6] | 1878·9[6] |

[1] The Arctic Circles fall in latitude 63° 10′ 32″; so that in latitude 65° there is no night at the Summer, no day at the Winter Solstice. The Tropics fall in latitude 26° 49′ 28″.

[2] With the assumed dimensions of the Rings and neglecting refraction, the Outer Edge of the Outer Ring / the Inner Edge of the Outer Ring / the Outer Edge of the Inner Ring / the Inner Edge of the Inner Ring / the Inner Edge of the Dark Ring is not visible in latitudes higher than 66° 25′ / 62° 59′ / 62° 16′ / 51° 13′ / 40° 45′ respectively.

[3] Assuming the Inner Edge of the Inner Ring alone visible, and its thickness 100 miles, it covers a zone on the celestial sphere, varying in width from 19′ at the zenith to 8′ at the horizon—the prime vertical being the central line of the zone. At each Equinox the Sun is totally eclipsed throughout the day for 9·6 days (22·0 Saturnian days); totally or partially eclipsed in the middle of the day for two equal periods of 20·35 days, preceding and following the former period; hence the Sun is partially or totally eclipsed for 50·3 days at each Equinox.

[4] The Sun does not reach the point {A′ / B′} in latitudes higher than {35° 52′ / 19° 50′}

[5] The diameter of the Sun's Disc is assumed to be 3½′ in this part of the table.

[6] These Eclipses extend to the Winter Solstice, so that the periods here given must be doubled to obtain the period during which these Eclipses actually continue. The other periods are also repeated (but after an interval) between the Winter Solstice and the Vernal Equinox.

## EXPLANATION OF THE TABLES.

TABLES I., II., and III. have been calculated for the most part from 'Mädler's Elements.' The dimensions of Saturn's globe, as of the rings in Table IV., have been selected for reasons mentioned in Chapter III. The gravity of Saturn has been calculated, for the equator and poles, from approximate formulæ; ('Todhunter's Analytical Statics,' art. 217.)

TABLE IV. The quantities ☊, $\lambda'$, $\beta'$, $i$, I, and ☊$'$ have been calculated from the formulæ given in the explanation of Tables VII. and VIII. The variations of all these quantities, except $\beta'$, are nearly uniform for long intervals of time, and have been determined from the values of the quantities for the years 1800 and 1900. The annual variation of $\beta'$ is not uniform for that interval, ranging from $0''\cdot239$ in the year 1800, to $0''\cdot234$ in the year 1900.

TABLE V. Except the 4th, 6th, 7th, and 8th columns, calculated by myself, the elements of Table V. are Mädler's, corrected in places from the best modern determinations.

TABLE VI. Except the column of mean hourly motions, this table is the same as Table XXXIII. of Loomis' 'Practical Astronomy.' I have added the hourly motion of Neptune to the hourly motions of the other planets, taken from Hind's 'Introduction to Astronomy.' The equatorial horizontal parallax of the sun at his mean distance is assumed to be $8''\cdot5776$ throughout this table.

TABLE VII. has been calculated by means of the following formulæ*:—

Let ☊ represent the mean longitude of the ring's ascending node in the ecliptic at time $t$.

$i$, the mean inclination of the plane of the ring to the ecliptic.

I, the mean inclination to the equator.

N, the mean position of the ascending node in the equator.

$\omega$, the obliquity of the ecliptic.

* See 'Nautical Almanac' for 1838, preface, p. viii.; and prefaces to later Nautical Almanacs.

Then adopting Bessel's determination of ☊ and $i$, viz.:—

$$\left.\begin{aligned}☊&=166^\circ\ 53'\ \ 8''\cdot9+46''\cdot462\ (t-1800)\\ i&=\ 28^\circ\ 10'\ 44''\cdot7-\ \ 0''\cdot350\ (t-1800)\end{aligned}\right\}^{*};$$

and assuming $\tan\phi=\tan i\cos ☊$;

$$\tan \mathrm{N}=\frac{\sin\phi}{\sin(\phi+\omega)}\tan ☊;\qquad \tan \mathrm{I}=\frac{\tan(\phi+\omega)}{\cos \mathrm{N}}.$$

The table is applied to the determination of the appearance of the rings, by means of the following formulæ:—

Let $a$ represent the major axis of the ring at the planet's mean distance.

$\left.\begin{array}{l}a', \text{ the apparent outer } \textit{major} \text{ axis of the outer ring}\\ a'', \text{ the apparent inner } \textit{major} \text{ axis of the inner ring}\end{array}\right\}$

$\left.\begin{array}{l}b', \text{ the apparent outer } \textit{minor} \text{ axis of the outer ring}\\ b'', \text{ the apparent inner } \textit{minor} \text{ axis of the inner ring}\end{array}\right\}$†

$\left.\begin{array}{l}l, \text{ the elevation of the earth above the plane of the rings}\\ l', \text{ the elevation of the sun above the plane of the rings}\end{array}\right\}$†

$p$, the inclination of the northern semi-minor axis of the ring to the circle of declination.†

$\left.\begin{array}{l}\alpha, \text{ the geocentric right ascension}\\ \delta, \text{ the geocentric declination}\\ \rho, \text{ the distance from the earth}\\ \lambda, \text{ the heliocentric longitude}\\ \beta, \text{ the heliocentric latitude}\\ r, \text{ the mean distance from the sun}\end{array}\right\}$ of the planet.

Then, adopting Bessel's value of $a$ and Bouvard's value of $r$:—

$$a=39''\cdot308;\ r=9\cdot54301;\ \log(a\,r)=2\cdot5741663;$$

and assuming $\tan \mathrm{Q}=\tan \mathrm{I}\sin(\alpha-\mathrm{N})$;

$$\tan p=-\frac{\sin \mathrm{Q}}{\cos(\mathrm{Q}-\delta)}\cot(\alpha-\mathrm{N});\ \tan l=\tan(\mathrm{Q}-\delta)\cos p;$$

$$\sin l'=\sin i\cos\beta\sin(\lambda-☊)-\cos i\sin\beta;$$

$$a'=\frac{a\,r}{\rho};\ a''=a'\times\cdot665;\ b'=a'\sin l;\ b''=b'\times\cdot665.$$

*Example.*—Required to determine $a', a'', b', b'', p$, and $l$ for June 22nd, 1865, at mean noon:—

* M. Maradi, in 1716, determined ☊ = 166° 20′; and λ′ (see Table VIII.) = 169° 48′ 30″: these values agree well with the values and variations given in the text.

† $b'$, $b''$, and $l$, are considered positive when the north surface of the ring is visible, otherwise negative; $l'$ is considered positive if the sun is above the northern surface of the ring, otherwise negative; $p$ is considered positive if the northern semi-minor axis of the ring is inclined to the *east* of the declination-circle, otherwise negative.

We obtain from Table VII.,

$$N=125^\circ\ 47'\ 9''\cdot 7\,;\ \text{and}\ \log\tan I=9\cdot 0956472$$

Again $\alpha=202^\circ\ 49'\ 21''\cdot 0$; and $\delta=6^\circ\ 46'\ 14''\cdot 5$ s.

Thus $\alpha-N=77^\circ\ 2'\ 11''\cdot 3$; and the calculation proceeds as follows:—

| | |
|---|---|
| | $\log\sin(\alpha-N)=9\cdot 9887877$ |
| | $\log\tan I=9\cdot 0956472$ |
| $Q=6^\circ\ 55'\ 31''$ | $\log\tan Q=9\cdot 0844349$ |
| $\delta=-6^\circ\ 46'\ 14''$ | $\log\sin Q=9\cdot 0812563$ |
| $Q-\delta=13^\circ\ 41'\ 45''$ | $\log\cot(\alpha-N)=9\cdot 3621013$ |
| | $19\cdot 4433576$ |
| | $\log\cos(Q-\delta)=9\cdot 9874723$ |
| $p=-1^\circ\ 38'\ 14''$ | $\log\sin p=8\cdot 4558853$ |
| | $\log\tan(Q-\delta)=9\cdot 3868543$ |
| | $\log\cos p=9\cdot 9998227$ |
| $l=+13^\circ\ 41'\ 26''$ | $\log\tan l=9\cdot 3866770$ |
| | $\log(a\,r)=2\cdot 5741663$ |
| | $\log\rho=0\cdot 9696225$ |
| $a'=40''\cdot 229$ | $\log a'=1\cdot 6045438$ |
| | $\log\sin l=9\cdot 3741578$ |
| $b'=+\ 9''\cdot 521$ | $\log b'=0\cdot 9787016$ |

$$a''=40''\cdot 229\times\cdot 665=26''\cdot 753$$
$$b''=+\ 9''\cdot 521\times\cdot 665=+\ 6''\cdot 332$$

These values correspond very well with those given in the 'Nautical Almanac' for 1865, p. 486, viz.:—

$$p=-\ 1^\circ\ 38'\cdot 4\,;\ a'=40''\cdot 23\,;\ a''=26''\cdot 75\,;$$
$$l=+13^\circ\ 41'\cdot 4\,;\ b'=+\ 9''\cdot 52\,;\ b''=+\ 6''\cdot 33.$$

The value of $l'$ may be more conveniently calculated from Table VIII. than from the formula given in the 'Nautical Almanac.'

Table VIII. The elements in this table are those referred to under the same symbols in Table IV. The value of $l'$ may be determined from these elements as follows:—

Let $\lambda$ be Saturn's heliocentric longitude.
$\beta$, Saturn's heliocentric latitude.

Then, assuming

$$\cos\psi=\cos(\lambda-\nu)\cos\beta\ ,$$

we obtain

$$\sin l'=\sin(\psi-\Omega')\sin I'.$$

Thus for June 22nd, 1865, at mean noon :—

We obtain from Table VII.

$\nu = 112^\circ 29' 32''\cdot 7$; ☊$' = 59^\circ 15' 50''\cdot 3$; and $\log \sin I' = 9\cdot 6544253$

Again $\lambda = 209^\circ\ 8' 34''\cdot 9$; and $\beta = 2^\circ 28' 19''\cdot 2$.

Thus $\lambda - \nu = 96^\circ 39'\ 2''\cdot 2$; and the calculation proceeds as follows :—

$\log \cos (\lambda - \nu) = 9\cdot 0637632$

$\log \cos \beta = 9\cdot 9995956$

$\psi = 96^\circ 38' 39''\cdot 8$ $\qquad \log \cos \psi = 9\cdot 0633588$

and $\psi -$ ☊$' = 37^\circ 22' 49''\cdot 5$ * $\qquad \log \sin (\psi -$ ☊$') = 9\cdot 7832633$

$\log \sin I' = 9\cdot 6544253$

$l' = +15^\circ 54' 3''\cdot 7$ $\qquad \log \sin l' = 9\cdot 4376886$

The 'Nautical Almanac' gives $l' = +15^\circ 54'\cdot 1$.

Mädler's values of $\nu$ and $i'$ have been adopted in Table VIII. Bouvard's values differ slightly from those of Mädler. Thus, for the year 1900, the values of $\nu$ and $i'$, according to Bouvard's Tables, are respectively $112^\circ 47' 20''\cdot 7$ and $2^\circ 29' 20''\cdot 4$. In Table VIII., $\nu$ and $i'$ are respectively $112^\circ 47' 7''\cdot 8$ and $2^\circ 29' 20''\cdot 9$. Adopting Mädler's values, the formulæ for ☊$'$ and $I'$ are as follows :—

☊$' = 58^\circ 57' 56''\cdot 8 + 16''\cdot 395\ (t - 1800)$;

$I' = 26^\circ 49'\ 7''\cdot 0 +\ 0''\cdot 321\ (t - 1800)$.

Adopting Bouvard's values, we obtain the formulæ :—

☊$' = 58^\circ 58^\circ\ 1''\cdot 9 + 16''\cdot 201\ (t - 1800)$;

$I' = 26^\circ 49'\ 7''\cdot 2 +\ 0''\cdot 317\ (t - 1800)$.

The hour at which the ring's plane passes through the sun may also be determined from Table VIII. Thus, for the passage in May, 1862 :—

From the table we have for May, 1862, $\lambda' = 171^\circ 41' 32''\cdot 4$;

and Saturn's heliocentric longitude being $171^\circ 39' 47''\cdot 1$ at mean noon of May 17th, and his daily motion $2' 3''\cdot 4$, it is easily calculated that his heliocentric longitude was $171^\circ 41' 32''\cdot 4$ at 8h. 30m. A.M. of May 18th, 1862, at which time, accordingly, the plane of the rings passed through the sun. In a similar way, it may be shown, that the passage of the ring's plane through the sun in 1848, took place on September 3rd, at 3h. 24m. P.M. It may be mentioned that $\beta'$ deduced from the table is not necessarily the same as Saturn's heliocentric latitude at such a passage, since Saturn may be (through perturbations) above or below his mean orbital path.

* It may be mentioned that ($\psi -$ ☊$'$) is Argument XVIII. of Bouvard's Tables of Saturn.

TABLE IX. combines parts of Bouvard's Tables (XI. of Jupiter and Saturn), the French measures being converted into English. A word or two may be required in explanation of the table. Take the inequality of Saturn:— The equation of Saturn for 1550 is $+48''\cdot79$,—by this is to be understood that (so far as the great inequality is concerned*) Saturn at this time was $48''\cdot79$ in advance of his mean place in longitude; the equation for 1560 is negative,—or in 1560 Saturn was behind his mean place. At some intermediate date, then, he must have been in his mean place, and it is easily seen that this must have happened in the year 1552. From this time Saturn lagged more and more behind his mean place, until, between the years 1780 and 1790, his equation attained its greatest negative value. This happened about the beginning of the year 1786, Saturn being then about $48\frac{1}{2}'$ behind his mean place in longitude. From this point his distance from his mean place has been continually diminishing, being now about $41\frac{1}{2}'$. This diminution will continue till he reaches his mean place in the beginning of the 21st century; he will then pass in advance of his mean place, till in the middle of the 23rd century he will be about as much in advance as he was behind in 1786. Now, it is clear, from the consideration that Saturn was falling farther and farther behind his mean place from 1552 to 1786, that during all that time he was moving with a motion less than his mean motion; and further, it is clear that the same was the case before the year 1550; for before that year he had been gradually falling back to his mean place, from a place about $48\frac{1}{2}'$ in advance of it. Thus, from 1552 to 1786, and for about as long an interval before 1552, Saturn's period was greater than his mean period. And it is clear that Saturn's period has been less than his mean period from 1786 to the present time. It will continue less till the middle of the 23rd century. The column of first differences indicates these changes more clearly. It exhibits the amount of longitude lost or gained by Saturn in each decennial period. It appears that the loss was greatest when Saturn was near his mean place; from which we learn that Saturn moved more slowly at that time,—or, in other words, that his period was then greatest.† Similarly, we learn that Saturn's period gradually diminished, until in 1786 it passed through its mean value, and that it has continued to diminish ever since. Thus Saturn's period attained its maximum when the inequality vanished, and passed through its mean value when the inequality obtained its maximum negative value; and similarly Saturn's period attains its

* Throughout the explanation these words must be understood; other perturbations operate to diminish or increase the departure of Saturn from his mean place.

† By 'the period of a planet at any instant' is to be understood the period in which the planet would accomplish a sidereal revolution about the sun if undisturbed from that instant by the attractions of other bodies.

minimum value when the inequality vanishes, and returns to its mean value when the inequality attains its maximum positive value. Lastly, the column of second differences enables us to estimate the rate at which the period diminishes or increases. It appears from this column that the period varies most rapidly when it is passing through its mean value. The inequality of Jupiter may be interpreted in precisely the same manner.

If the two columns of third differences be drawn out, they will be found less regular than the columns of first and second differences appear to be. We have in this irregularity the first indication of the variation in the disturbing actions of the two planets, arising from the variable relations described in Chapter VI.

TABLE X. gives the days on which the plane of Saturn's ring has passed through the sun from the time of the discovery of the ring to the end of the 20th century. The passages have been calculated from those of September 3rd, 1848, and May 18th, 1862. The only corrections applied have been those due to the great inequality, and to the precession of the nodes of the ring's plane on Saturn's orbit. Thus, for the calculation of the ring's disappearance in the year 1789 :—This disappearance must be calculated from September 3rd, 1848. Saturn's mean sidereal period contains 10759·2197 days, but in each year the nodes of the ring regrede through an arc of 3″·145; and therefore in Saturn's sidereal period the nodes of the ring regrede through 1′ 32″·6* Now Saturn's mean daily motion at the part of his orbit in which the passages we are considering take place—in other words, at the autumnal equinox of his northern hemisphere—is 1′ 59″·0; thus the time occupied in passing over an arc of 1′ 32″·6 is 0·779d., and by this amount Saturn's mean sidereal period must be diminished to obtain his mean tropical period for the part of his orbit in question. The latter period is therefore 10758·441 days; and this interval contains 7 sets of four years (one year in each set being leap year), one common year, and 166d. 10h. 35m.; therefore two such periods contain 14 sets of four years, and three common years, wanting 32d. 2h. 50m. Now, if we calculate such an interval backwards from 3h. 26m. P.M. of September 3rd, 1848, remembering that 1800 is a common year, we arrive at October 5th, 1789, 6h. 16m. P.M. But from Table IX. we find that for September 3rd, 1848, the inequality is—43′ 56″·6, and for October 5th, 1789, the inequality is—48′ 21″·4; thus Saturn was 4′ 24″·8 farther behind his mean place in October, 1789, than he was in September, 1848, and therefore the ring's plane passed later through the sun by the time in which Saturn with a daily motion of 1′ 59″·0 passes over an arc of his orbit of 4′ 24″·8, or by 2d. 5h. 24m.; so that the corrected date is October 7th, 11h. 40m. P.M., the nearest hour to which is 12 midn., October 7th.

* Saturn's mean period contains 29·4578 tropical, or 29·4566 sidereal years.

For passages calculated from May 18th, 1862, the excess of Saturn's tropical revolution over 7 sets of four years and one common year, is 166d. 11h. 16m. This may easily be verified in the manner shown above, remembering that Saturn's mean daily motion near the vernal equinox of his northern hemisphere is $2' 3''\cdot4$.

The autumnal equinox of Saturn's northern hemisphere is continually receding from, and the vernal equinox approaching, the perihelion of the orbit, by an arc of $19''\cdot31 + 3''\cdot145$, or $22''\cdot455$ in each year, or by $11' 1''\cdot5$ in each mean Saturnian period. Owing to this variation Saturn's tropical periods, measured from the vernal equinox, successively increase by about 21 seconds, while those measured from the autumnal equinox successively diminish by about 20 seconds. Another small correction is due to the same variation of motion applied to the correction for the great inequality of Saturn.

The table can be applied to determine approximately the dates of the passages of the ring's plane through the earth. Thus, take the passages of the year 1789: the ring's plane passed through the sun on October 7th, when the earth therefore had passed the autumnal equinox by about 18°; thus the earth was between the autumnal equinox and E (fig. 1, Plate VIII.), very near the point at which $q'$ 7 $r'$ crosses the earth's orbit. At this moment Saturn was near the point M, but had not quite reached that point; for the nodal line of the ring's plane on Saturn's orbit advances $46''\cdot966$ yearly in longitude, and therefore in 1789 the point corresponding to M was nearer $m''$ by an arc of $58' 54''\cdot2$; and similarly the nodal line of the ring's plane on the ecliptic was very nearly in direction S R, but passed nearer $r''$ by an arc of $58' 16''\cdot3$. Thus it is very easily seen that before the passage of the plane of the ring through the sun the earth had passed twice through the ring's plane, both passages occurring when the earth was very near the point $e'$, or in Midsummer, 1789. During the short interval of time that elapsed between these passages the earth and sun were on opposite sides of the plane of the ring, which was therefore invisible. It was during the reappearance of the ring in the summer of 1789 that the elder Herschel determined the period of rotation of the outer parts of the ring. After the passage of the ring's plane through the sun on October 7th, 1789, the ring was invisible for more than three months, reappearing when the earth was very near the point E$'$, or early in February, 1790, after which the ring continued visible for 13½ years.

Table X. can also be applied to determine approximately the appearance of the ring viewed from the earth, and the elevation of the sun above the ring's plane. Thus:—We may assume without important error that Saturn moves with uniform angular velocity about the sun between the successive passages tabulated. On this assumption the arc swept out from P or M (fig. 1, Plate VIII.) at any given time : 180° : : the time elapsed since the preceding passage tabulated : the complete interval between that passage and the next.* Thus Saturn's position on his orbit is approximately determined. The earth's position in her orbit at the same time is determined in a similar manner. Thus we can determine the required elements by construction, marking down the positions of the two bodies in a figure (as fig. 1, Plate VIII.), and thence determining the angles at which the sun and earth are respectively elevated above the plane of the rings, as shown in Chapter IV., page 80, note*; or we may obtain the required angles by a simple trigonometrical calculation

TABLE XI. exhibits the general features of the Saturnian heavens for those latitudes within which any part of the rings can be seen. Of the manner in which this table is to be interpreted enough has been said in Chapter VII. The table has been calculated as follows:—

The azimuths of the sun at sunrise and sunset at the winter solstice in any latitude are obtained from the formula—

$$\cos \text{azimuth} = \sin I' \sec \phi;$$

where I′ is the obliquity of the Saturnian ecliptic (or 26° 49′ 28″) and $\phi$ the saturnicentric latitude. The supplement of the angle thus obtained is the corresponding azimuth at the summer solstice in latitude $\phi$. At the equinoxes the corresponding azimuth is 90° for all latitudes.

The meridian altitudes of the sun at the winter and summer solstices in any latitude are obtained by subtracting and adding, respectively, 26° 49′ to the meridian altitude at either equinox—that is, to the complement of the latitude.

The diurnal arc traversed by the sun at the winter solstice, in latitude $\phi$, is given by the formula—

$$\cos \tfrac{1}{2} (\text{diurnal arc}) = \tan I' \tan \phi;$$

360°, diminished by the arc thus obtained, is the corresponding arc at the summer solstice; also the nocturnal arc at the summer is equal to the diurnal arc at the winter solstice, and *vice versâ*. The length of the day follows at once from the length of the diurnal arc; since—

length of day : 10h. 29m. 17s. : : diurnal arc : 360°

The length of the day at the winter solstice is obviously equal to the length of the night at the summer solstice, and *vice versâ*.

For the appearance of the ring we have the following formulæ:—

* The interval from the passage in May, 1862, to the next passage (in February, 1878) being 5751½ days, Saturn's average daily angular velocity in this interval is 1′ 52″·7. The following interval (to the passage in October, 1891) contains 5004½ days; in this interval, therefore, Saturn's average daily motion is 2′ 9″·5.

Let $\phi$ represent the saturnicentric latitude.

$r$, the semi-diameter of an edge of a ring.

$s$, the equatorial semi-diameter of Saturn.

$s'$, the polar semi-diameter of Saturn.

$a$, the azimuth of the ring's edge where it crosses the horizon.

$\beta$, the altitude of the ring's edge where it crosses the meridian.

$\gamma$, the arc of the ring's edge above the horizon.

Put

$$\cot\theta=\frac{s}{s'}\cot\phi;\ k=s\sec\theta;\ l=k\sin^2\theta\operatorname{cosec}\phi.$$

Then

$$\tan a=\frac{\sqrt{(r+k)(r-k)}}{l},\qquad \cos\frac{\gamma}{2}=\frac{k}{s},$$

and

$$\tan\left(\frac{\pi}{4}-\frac{\phi}{2}\pm\beta\right)=\frac{r-k-l}{r-k+l}\tan\left(\frac{\pi}{4}-\frac{\phi}{2}\right);$$

the upper or lower sign to be taken in the last formula according as $(r-k)$ is greater or less than $l$. The latitude in which the edge of a ring disappears is obtained by putting $k=r$, whence we get

$$\cot\phi=\frac{s'}{\sqrt{(k+s)(k-s)}}.$$

The arcs of the horizon or meridian covered by a ring or division are obtained by taking the difference of the azimuths or altitudes, respectively, of the inner and outer edges of the ring or division.

The parts of the table referring to declination-parallels through the points A, A′, B and B′ have been calculated from the following formulæ:—

Let A represent the azimuth of the point A } for latitude $\phi$.
B, the meridian altitude of the point B } for latitude $\phi$.

P, the meridian altitude of a declination-parallel through A.

Q, the azimuth of the point of intersection with the horizon of a declination-parallel through B.

Then,—

$$\cos(\phi+P)=\cos A\cos\phi;$$

and

$$\cos Q\cos\phi=\cos(B+\phi).$$

And substituting A′ for A, and B′ for B, the same formulæ are used to determine the altitudes of corresponding parallels through A′ and B′.

For those latitudes within which the sun reaches the points A′ and B′, these parallels represent the sun's diurnal paths at that part of the Saturnian year in which the sun passes through the points A, A′, B, or B′. Thus we can determine the intervals contained in the last section of the table, as follows:—

Let L be the angle swept out by Saturn about the sun from the autumnal equinox, when the sun's meridian altitude is M, in latitude $\phi$.

Then,—

$$\sin L = \frac{\cos(\phi + M)}{\sin I'};$$

and, if we suppose Saturn to move uniformly with his mean motion, the interval from the equinox in which he would sweep out the angle L about the sun is $\frac{L}{\text{Saturn's mean daily motion}}$ days. For one degree of arc the time is 29·9 days, for one minute of arc the time is 0·5 d. From these formulæ and values the intervals of total and partial eclipse have been calculated—a suitable correction being introduced for the sun's apparent diameter (assumed to be 3′ 20″) viewed from Saturn. These results are not affected by the refraction of Saturn's atmosphere, for when the sun is eclipsed by any part of the ring refraction elevates the sun and that part of the ring by the same amount. The other elements tabulated are affected by the refraction of Saturn's atmosphere. The effects of such refraction are:—To increase altitudes and azimuths, especially the latter; to increase the diurnal arcs traversed by the sun, the lengths of the Saturnian days, and the arcs of the rings' edges above the horizon; and, in general, to diminish the arcs of the meridian and horizon covered by the rings; but when the outer edge only of a ring is visible the arcs of the meridian and horizon covered by that ring are increased by refraction.

## EXPLANATION OF ASTRONOMICAL TERMS USED IN THE BODY OF THE WORK.

*Aberration.*—An apparent displacement of any celestial object, due to the progressive motion of light. Aberration is caused in two ways—first, by the orbital motion of the earth, secondly, by the motion of the observed celestial object. Aberration due to the first cause is constant for all celestial objects except the moon. Aberration due to the second cause is common to all celestial objects, but varies with their distances and rates of motion: where these are known the aberration can be determined. The aberration of the fixed stars due to this cause is not determinable; but the distance between the real and apparent place of a star must in general be very great. For instance, light is 14 or 15 years travelling from Sirius to the earth, and in this interval Sirius no doubt travels many millions of miles.

The earth's axial rotation also causes a small change in the apparent positions of all celestial objects; this is called the *diurnal aberration.*

*Æther.*—See *Medium, Resisting.*

*Altitude.*—The angular distance of a heavenly object from the horizon, measured in the direction of a great circle passing through the object and the zenith. See *Azimuth.*

*Altitude and Azimuth Instrument.* See *Azimuth.*

*Amplitude.*—The distance of a celestial object at rising or setting, from the east or west points, respectively, of the horizon.

*Angular Velocity.*—The angular velocity of one body about another is the rate at which angles are described by the radius vector of the former body. See *Radius Vector.*

*Anomalistic Period.*—The time of revolution of a planet or satellite in reference to the line of apsides of its orbit. The period will vary according as the nearer or farther apse is chosen as the starting-point:—Thus, in the case of the earth the line of apsides is continually advancing, so that the anomalistic period exceeds the sidereal period by the time in which the earth passes over the annual arc of advance traversed by the line of apsides; and the earth will plainly pass over this arc more rapidly when near perihelion than when near aphelion.

*Ansæ.*—Handles; a term applied to the apparent projections formed by the ring on each side of Saturn's globe.

*Aphelion.*—The point in the orbit of a planet or comet which is farthest from the sun.

*Aposaturnium.*—The point in a satellite's orbit farthest from Saturn.

*Apparent Motion.*—The motion of the celestial bodies viewed from the earth. The term is sometimes applied to the daily motions of the celestial bodies caused by the diurnal rotation of the earth, at others to the motions of the celestial bodies on the sidereal sphere,—that is, among the fixed stars.

*Apsides, Line of.*—The imaginary line joining the apses of the orbit of a planet or satellite; more strictly, it is the line joining what would be the apses of the planet's path if the planet moved undisturbed through a complete revolution from the moment considered.

*Apse or Apsis.*—The point of the orbit of a planet or satellite at which it is farthest from, or nearest to the sun or primary, respectively; or, more correctly, the points of such orbits at which the direction of motion is at right angles to the line from the centre of motion.

*Arc of Progression.*—The arc passed over by a planet when its motion is direct, or in the order of the signs.

*Arc of Retrogradation.*—The arc passed over by a planet when its motion is retrograde, or contrary to the order of the signs.

*Aries.*—A constellation; but also the first sign of the Zodiac. The commencement of this sign is called *the first point of Aries:* it is the point in which the ecliptic and the equinoctial line intersect, the ecliptic passing from south to north of the equinoctial line. The sun's centre occupies this point at the vernal equinox of the northern hemisphere; and from this point longitudes are measured along the ecliptic in the order of the signs, and right ascensions upon the equator from west to east. The constellation Pisces at present occupies the sign Aries. At the vernal equinox the earth's heliocentric position corresponds to *the first point of Libra.*

*Ascending Node.*—See *Node.*

*Ascension, Right.*—The right ascension of a celestial body is the angle between two planes, one passing through the pole of the heavens and the body, the other through the pole of the heavens and the first point of Aries. The method of indicating the position of a celestial object by assigning its right ascension and declination may be thus illustrated:—Suppose a telescope so constructed as to be moveable about an axis directed to the pole of the heavens, and also about an axis at right angles to the former and to the axis of the telescope; let the telescope in the first place be directed to the first point of Aries: then the telescope may be directed towards any celestial object by two movements—the first round the polar axis of the instrument (from west to east), the second about the other axis (towards or from the north pole); the angle through which the telescope is swept about the former axis is the same as the *Right Ascension* of the object; the angle through which the telescope is swept about the latter axis is the same as the *declination* of the object; and the declination is *north* or *south* according as the latter motion of the telescope is *towards* or *from* the north pole. A telescope so mounted is called *an Equatorial.* (The refractive effects of the atmosphere are neglected in the preceding lines; slight corrections are due to those effects.)

*Asteroids.*—The minor planets which revolve between the orbits of Mars and Jupiter.

*Axis of a planet.*—The imaginary line upon which the planet rotates.

*Axis of an orbit.*—The major-axis of the orbit of a planet is the apsidal line; the minor-axis is a line at right angles to the former through its middle point.

*Axis of figure.*—If the surface of a body may be supposed to be produced by the revolution of a plane curve about a straight line, the body is said to have a *surface of revolution*, and the straight line is called the *axis of figure* of the body. Neglecting minor irregularities, the planets are such bodies, the generating curve in each being an ellipse, and the minor axis of such ellipse the axis of figure.

*Axis of rotation.*—Any straight line about which a body revolves in the same manner as it would if the parts of the body were rigidly connected with the line, is called the axis of rotation of the body. In the earth and planets the axis of rotation coincides with the axis of figure.

*Azimuth.*—The azimuth of a celestial body is the angle between two planes, one passing through the zenith and the body, the other passing through the zenith and the north and south points of the horizon. Azimuths are measured through 180°, and in general from the north or south point of the horizon according as the north or south pole of the heavens is elevated. The method of indicating the position of a celestial body by assigning its altitude and azimuth may be thus illustrated:—Suppose a telescope constructed so as to be moveable about a vertical axis and also about a horizontal axis at right angles to the axis of the telescope; let the telescope be directed in the first place towards the north point of the horizon: then the telescope may be directed towards any celestial object by two movements—the first about the vertical axis, the second about the horizontal axis; the first movement directs the telescope to the point directly below the object, and the angle through the telescope is swept is the same as the *azimuth* of the object measured from the north point; the second movement raises the telescope till the object is in the field of view, and the angle through which the telescope is swept is the same as the *altitude* of the object. A telescope so mounted is called an *altitude and azimuth instrument.* (The altitude alone is affected by the refraction of the atmosphere. Compare *Ascension, Right.*)

*Belts.*—A name applied to the faintly coloured streaks crossing the discs of Saturn and Jupiter.

*Circle of the celestial sphere.*—A circle in which any plane meets the imaginary sphere called the celestial sphere. Planes passing through the centre of this sphere meet its surface in *great circles*—as the *ecliptic*, *prime vertical*, *meridian*, and the like: planes not passing through the centre meet the sphere in *small circles*; these are sometimes termed *parallels*. See *Parallel*. Where the word *circle* is combined with another term, as declination, latitude, or the like, the circle referred to is a circle on which the element mentioned is measured. Thus, a *declination-circle* is a circle passing through the poles of the heavens, on which, therefore, declinations are measured.

*Co-latitude.*—The complement of the latitude, or the angle by which the latitude falls short of 90°.

*Colure, Equinoctial.*—A great circle passing through the poles and the equinoctial points.

*Colure, Solstitial.*—A great circle passing through the poles and the solstitial points.

*Compression of a planet.*—The amount by which the polar axis falls short of an equatorial diameter. It is generally expressed by the ratio it bears to an equatorial diameter:—thus, if the compression of a planet is said to be $\frac{1}{18}$th, what is meant is that the excess of an equatorial over the polar diameter is equal to $\frac{1}{18}$th part of an equatorial diameter.

*Configuration.*—The relative positions of stars or other celestial bodies.

*Conjunction.*—Two bodies are said to be in conjunction upon the celestial sphere when they have the same longitude. When a planet is simply said to be *in conjunction*, it is to be understood that the planet is in conjunction with the sun. Since the planets are always near the ecliptic, a planet in conjunction with the sun has very nearly the same right ascension as the sun. The symbol expressing conjunction is ☌.

*Constellation.*—A number of stars included within an imaginary figure for the sake of easier identification.

*Culmination.*—The passage of a heavenly body across the celestial meridian of a place.

*Cycle.*—A period within which a series of celestial phenomena recurs.

*Cycle of Eclipses.*—See *Saros*.

*Declination.*—The angular distance of a celestial body from the equator, measured along a great circle passing through the body and the pole of the equator. See *Right Ascension*.

*Declination-circle.*—See *Circle*.

*Declination-parallel.*—See *Parallel.*

*Degree.*—Where the sexagesimal division of the circle is employed, a degree of arc is the 360th part of the circumference; a degree of angle is the 360th part of four right angles: the degree is divided into 60 minutes, the minute into 60 seconds, after which, in general, decimals are employed. In the centesimal division of the circle, a degree or grade is the 400th part of the circumference, and is divided into 100 minutes, the minutes into 100 seconds, and so on continually.

*Descending Node.*—See *Node.*

*Diameter, Apparent.*—The angle subtended by the diameter of a heavenly body, viewed from the earth.

*Disc.*—The visible surface of the sun, moon, or planets.

*Eccentricity of an orbit.*—The distance of the centre of an elliptic orbit from either focus. It is generally expressed by the ratio it bears to the mean distance or semi-major axis of the orbit:—thus, when the eccentricity of an orbit is said to be 0·01, what is meant is that the distance of the centre of the orbit from either focus is equal to $\frac{1}{100}$th part of the semi-major axis of the orbit.

*Eclipse.*—The concealment of a celestial body in the shadow of another body, or by the interposition of another. In speaking of the satellites of Jupiter or Saturn, the term eclipse is confined to the former class, a concealment due to the latter cause being called an *occultation.*

*Ecliptic.*—The great circle of the heavens along which (approximately) the sun's centre appears to move in the course of a year.

*Ecliptic, Obliquity of the.*—The angle between the planes of the equator and the ecliptic; or, in other words, the complement of the angle at which the earth's axis is inclined to the plane of the ecliptic.

*Elements of an orbit.*—Quantities whose determination defines the path of a celestial body in space.

*Ellipse.*—A closed curve produced by cutting a cone obliquely. If a cone be cut obliquely so that the resulting curve is open with one branch only, the curve is a *parabola*; if the resulting curve is open and has two branches, the curve is an *hyperbola.* In the last case, the cone must be what is commonly called *a double cone,* but is strictly speaking understood by the term *cone.*

*Elongation.*—The angular distance of a planet from the sun, or of a satellite from its primary, viewed from the earth.

*Epoch.*—The moment of time to which given numbers or quantities apply.

*Equation.*—Any number or quantity that has to be applied to the mean value of another number or quantity to obtain the true value.

*Equator.*—The equator of a planet is the circle in which a plane at right angles to the polar axis, and half-way between the poles, intersects the surface of the planet. This plane is called the plane of the planet's equator. The term is, in general, confined to the equator of the earth, and the intersection of the plane of the earth's equator with the celestial sphere, is called the *Celestial Equator*, and sometimes the *Equinoctial.*

*Equatorial Telescope.*—See *Right Ascension.*

*Equatorial Horizontal Solar Parallax.*—The angle subtended by the earth's equatorial semi-diameter from the sun when the earth is at her mean distance from the sun. From observations of the transits of Venus in the years 1761 and 1769, this angle has been estimated at $8''\cdot5776$. It appears probable that this value is too small, and therefore the sun's distance determined from it too great. The best modern observers determine the angle at about $8''\cdot9$; the value assumed in the body of this work is $8''\cdot9159$. The problem of the sun's distance awaits a more satisfactory solution from observations of the transits of Venus in the years 1874 and 1882. See *Parallactic Inequality, Moon's.*

*Equinoxes.*—The points in which the equator of a planet intersects the plane of the planet's orbit about the sun. The term is generally confined to the intersection of the ecliptic and the plane of the earth's equator. For the northern hemisphere, the point at which the ecliptic passes to the north of the equator, is called the *Vernal Equinox*; the opposite point, where the ecliptic passes to the south of the equator, is called the *Autumnal Equinox.* For the southern hemisphere, these terms are interchanged.

*Field of view.*—The part of the celestial sphere visible at any instant in a telescope. The greater the magnifying power applied to a telescope, the smaller is the field of view.

*Foci of an ellipse.*—Two points on the major axis of an ellipse equidistant from the centre, and whose distances from either extremity of the minor axis are equal to the semi-major axis of the ellipse. If two lines be drawn from the foci of an ellipse to any point of the curve, the sum of their lengths is equal to the major axis; they are also equally inclined to the tangent at that point.

*Geocentric.*—As supposed to be seen from the earth's centre.

*Geocentric Longitude and Latitude.*—See *Longitude* and *Latitude.*

*Heliocentric.*—As supposed to be seen from the centre of the sun.

*Hemisphere.*—One half (bounded by a great circle) of the surface of a sphere.

*Horizon, Sensible.*—The circle in which a tangent-plane to the earth at any point meets the celestial sphere, is called the *sensible horizon* of that point.

*Horizon, Rational.*—The circle in which a plane through the earth's centre, parallel to the plane of the sensible horizon of any point of the earth's surface, meets the celestial sphere, is called the *rational* or *true horizon* of that point.

*Inclination of an orbit.*—The angle at which the plane of the orbit is inclined to the plane of the ecliptic.

*Inequality, Great, of Saturn and Jupiter.*—A variation in the orbital motions of these planets, caused by their mutual disturbing attractions. See Chapter VI.

*Inequality, Parallactic, Moon's.*—See *Parallactic Inequality, Moon's.*

*Inferior Planet.*—A planet whose orbit lies within that of the earth.

*Latitude, Geocentric, of a heavenly body.*—The geocentric departure of the body from the ecliptic, measured along a great circle passing through the body and the poles of the ecliptic.

*Latitude, Geocentric, of a place on the earth.*—The *true* angular distance of the place from the equator; or, in other words, the angle between the vertical at the place and the vertical at the nearest point of the equator.

*Latitude, Geographical.*—The angular distance of a place from the equator, not corrected for the oblateness of the earth's form; or, in other words, the angle between two lines supposed to be drawn to the place and to the nearest point of the equator, from the centre of the earth.

*Latitude, Saturnicentric, of a place on Saturn.*—The *true* angular distance of a place from Saturn's equator.

*Libration of the Moon.*—An apparent oscillatory motion of the moon, whereby we are enabled to see rather more than half the moon's surface. The moon's motion of rotation being uniform, but her orbital motion not so, two lunes of the moon's surface become visible to us in turn. The ends of these lunes lie on that diameter of the lunar disc which is at right angles to the direction of the moon's motion; hence, their greatest breadths lie on the diameter which is in the direction of the moon's motion. This is called the *libration in longitude.* Again, the axis of the moon's rotation is not quite perpendicular to the plane of her orbit; thus, two other lunes become visible by turns, whose extremities lie on that diameter of the lunar disc which is in the direction of the moon's motion, so that their greatest breadths lie on the diameter at right angles to the former. This is called the *libration in latitude.* The spaces that become visible by the two librations have parts in common; the fringe of the moon's surface, thus rendered visible, is variable in breadth, being bounded by parts of four great circles of the moon's surface.

The *diurnal libration* is a less important libration, due to the earth's rotation on her axis.

*Line of Nodes.*—See *Nodes.*

*Longitude, Geocentric, of a heavenly body.*—The angular geocentric distance of the body from the first point of Aries, measured upon the ecliptic in the order of the signs.

*Longitude, Heliocentric, of a heavenly body.*—The angular heliocentric distance of the body from the first point of Aries, measured upon the ecliptic in the order of the signs.

*Longitude, Geographical, of a place.*—The angle between two planes through the axis of the earth, one passing through the place and the other through a fixed station. Geographical longitude is measured through 180° east and west of the fixed station. The plane through the axis of the earth and the place on the earth, intersects the earth's surface in an ellipse, which is called the *terrestrial meridian* of the place.

*Longitude of Perihelion.*—The heliocentric longitude of the perihelion of a planet's orbit. It is usually measured upon the ecliptic to the node, and thence along the orbit forwards or backwards, as the case requires; the sum or difference, respectively, of the arcs on the ecliptic and orbit, being taken as the longitude of the perihelion. A more satisfactory method would be to assign the heliocentric longitude and latitude of the perihelion-point.

*Lunation.*—See *Synodical Month.*

*Lune.*—Part of the surface of a sphere intercepted between two great circles; also a plane surface bounded by two circular arcs whose concavities are turned in the same direction.

*Major axis of an orbit.*—See *Axis of an orbit.*

*Mean distance.*—The mean between the greatest and least distances of a planet from the sun, or of a satellite from its primary. The mean distance is therefore equal to half the major axis of an orbit. The extremities of the minor axis of an orbit are at the mean distance from the focus. It may be remarked that the true average distance of a planet or satellite from the focus of its orbit, is somewhat greater than the semi-major axis of the orbit.

*Medium, Resisting.*—A diffused æthereal matter supposed to occupy the inter-planetary and interstellar spaces, resisting the motions of all bodies, and perceptibly modifying the motions of such bodies as comets. The same medium is supposed to occupy the spaces between the atoms composing solid bodies, 'passing as freely through the densest solid, as the air through a grove of trees.' It is in this medium or æther that the vibrations of light, heat, and electricity, are supposed to be propagated.

*Meridian, Celestial, of a place.*—The great circle of the heavens passing through the zenith and the poles. The *meridian altitude* of a celestial body is its altitude when crossing the meridian, or at the moment of *meridian transit.*

*Meridian, Terrestrial.*—See *Longitude, Geographical.*

*Meteoric Stones* or *Aerolites.*—Cosmical bodies which traverse the upper regions of the atmosphere or fall upon the surface of the earth, commonly called *shooting stars, falling stars,* or *fire-balls.* It has been noticed that on certain days in the year, these meteors appear in greater numbers than on others, occasionally falling in showers on these days. For an account of these periodic showers the reader is referred to Humboldt's Cosmos. The theory now generally accepted in explanation of these phenomena is that suggested by Chladni, that:—Vast numbers of small masses of solid matter are dispersed through the interplanetary spaces; they travel in irregular orbits under the influences of their mutual attractions and the attractions of the sun and planets; and when they approach a planet of powerful gravitation, they are attracted towards it, and may fall upon its surface. The modern science of thermo-dynamics explains the brilliancy of these meteoric stones, and the fact that, in general, they are dissipated into vapour in passing through the earth's atmosphere. These effects are due to the intense heat generated as the *vis viva* of a swiftly travelling meteoric body is destroyed by the resistance of the air.

*Milky Way.*—The nebula of which our sun is a member; it may be traced as an irregular luminous band extending completely round the heavens, and divided into two parts through a great portion of its length. It is probably similar in form to the great spiral nebulæ. The irregular nebulæ which lie near its borders—as the nebula in Orion, and the nebulæ in Argo—are, in all probability, distant outlying wisps of the Milky Way. The Magellanic Clouds may be connected with the Milky Way in a different manner:—The spiral nebulæ frequently exhibit amid their convolutions vast globular condensations; and it is conceivable that the Magellanic Clouds are such star-clusters connected with the Milky Way by star-streams too distant to be visible. The conformation of the great nebula in Andromeda, which is probably a spiral nebula seen from the side, seems to indicate that the whorls of spiral nebulæ may be separated by wide intervals from the mean plane of the spiral, a circumstance that would explain the distance at which the Magellanic Clouds are found from the borders of the Milky Way.

*Minor axis of an orbit.*—See *Axis of an orbit.*

*Month, Anomalistic.*—The period of the moon's revolution from perigee to perigee of her orbit.

*Month, Nodical.*—The period of the moon's passage from ascending to ascending, or from descending to descending node of her orbit.

*Month, Sidereal.*—The period in which the moon passes through the twelve signs of the Zodiac.

*Month, Synodical.*—The common *Lunar Month*, or *Lunation:*—viz., the period in which the moon goes through all her phases, as from new to new, or from full to full.

*Motion, Direct, of a planet.*—The apparent motion of a planet in the order of the signs.

*Motion, Proper, of a star.*—The apparent motion of a star due to the star's real motion in space.

*Motion, Relative.*—The change of position of one body with respect to another, when one or both are in motion.

*Motion, Retrograde, of a planet.*—The apparent motion of a planet contrary to the order of the signs.

*Nadir.*—The point of the celestial sphere vertically below the observer.

*Nebula.*—A collection of stars so closely congregated through effect of distance as to appear in ordinary telescopes as a cloudlike spot.

*Nebular Theory, Herschel's.*—A theory advanced by the elder Herschel, that certain classes of nebulæ consist of true nebulous matter, self-luminous, and spread 'in the manner of a cloud or fog through extensive regions of space.' Modern discoveries do not favour this supposition. It appears probable that, with sufficient telescopic power, all nebulæ would be resolvable into stars. Herschel considered the irresolvability of the great nebula in Orion (see *Milky Way*) a strong argument in favour of his hypothesis. Since Herschel's time, this nebula has been resolved by the Parsonstown reflector, and by the Harvard refractor. Herschel's hypothesis is frequently confounded with Laplace's Nebular Theory. Laplace drew an illustration of his theory, or rather of a part (probably erroneous) of his theory, from the views of Herschel; but, beyond this, the two theories are in no way connected.

*Nebular Theory, Laplace's.*—See Note B, Appendix I.

*Node.*—The points of intersection of any great circle on the celestial sphere with any other, are called the nodes of the former circle upon the latter: the point at which the former passes from north to south of the latter is called the *ascending node,*—its sign is ☊; the opposite point is called the *descending node,*—its sign is ☋; and the line joining the two nodes is called the *line of nodes* or the *nodal line.* The ecliptic is usually the circle of reference; so that, unless the contrary is expressed, the *ascending node of a planet's orbit* signifies the point at which the planet passes from the southern to the northern side of the ecliptic.

*Oblate spheroid.*—The solid figure generated by the revolution of an ellipse about its minor axis. If the ellipse revolve about its major axis, the figure generated is a *prolate spheroid.*

*Obliquity of the ecliptic.*—The inclination of the plane of the ecliptic to the plane of the equator.

*Occultation.*—See *Eclipse.*

*Opposition.*—Two heavenly bodies are said to be in opposition when their longitudes differ by 180°. See *Conjunction*: similar remarks apply in the case of opposition. The symbol expressing opposition is ☍.

*Orbit.*—The path of a planet or comet about the sun, or of a satellite about its primary.

*Parallactic Inequality, Moon's.*—An inequality in the moon's motion,—of some interest, as it has been applied to the determination of the sun's distance. If we suppose that S (fig. 5, plate X.) represents the earth, $P_1P_2P_3P_4$ the moon's orbit, and $Q_1$ the sun, then the investigation of P's motion about S (Q at rest) in pp. 134–140 would suffice to explain the inequality in the moon's motion known as *the moon's variation.* Now if $Q_1S$ be very great compared with $SP_2$, the ratio $Q_1P_2 : Q_1S$ is *very nearly* equal to the ratio $Q_1S : Q_1P_4$, but *not quite*; hence arises a small inequality in the effects of the moon's variation; this inequality is called the moon's parallactic inequality. It never affects the moon's longitude by more than 2′, and its period is that of a lunar synodical revolution.

*Parallax.*—An apparent change in the position of an object, caused by a change in the position of the observer.

*Parallel.*—A term sometimes applied to small circles of the celestial sphere, or of the earth's globe. The word *parallel* is used in combination with another term, as Declination, Latitude, or the like; and the small circle referred to is a circle for all points of which the element mentioned is constant. Thus, a *Declination-parallel* is a circle every point of which has the same declination, or is equi-distant from the poles of the celestial equator. Hence the plane of a Declination-parallel is parallel to the plane of the equator.

*Perihelion.*—That point in the orbit of a planet or comet which is nearest to the sun.

*Period, or Periodic Time.*—See *Revolution.*

*Perisaturnium.*—The point in a satellite's orbit nearest to Saturn.

*Perturbations.*—Variations of the motion of a heavenly body from the elliptic path it would describe about a central body, if undisturbed by the attractions of the other celestial bodies.

*Phase.*—The appearance at any moment of a celestial body subject to periodic changes of appearance.

*Planets, Minor.*—See *Asteroids.*

*Planets, Primary.*—The planets which revolve about the sun as centre.

*Planets, Secondary.*—The satellites which revolve about some of the primary planets.

*Pole of a great circle on the celestial sphere.*—The points in which a straight line through the centre of the circle, and at right angles to its plane, meets the celestial sphere.

*Precession of the Equinoxes.*—A slow retrograde motion of the equinoctial points upon the ecliptic.

*Prime Vertical.*—See *Vertical, Prime.*

*Quadrature.*—Two celestial bodies are said to be in quadrature when their longitudes differ by 90°.—See *Conjunction* and *Opposition.*—The symbol expressing Quadrature is □.

*Radius Vector.*—An imaginary line supposed to be drawn from a body to its centre of motion, and to accompany the body in its revolution about that centre.

*Reflecting Telescope.*—A telescope in which images of objects, formed by reflection in a polished mirror, are magnified by a lens or by a combination of lenses.

*Refracting Telescope.*—A telescope in which images of objects, formed by refraction through an object-glass, are magnified by a lens or by a combination of lenses.

*Refraction.*—A property by which rays of light are bent in passing through transparent media, or rather, by which the wave-fronts of light are made to travel in a new direction.

*Refraction, Atmospheric.*—Refraction of light by the atmosphere. Atmospheric refraction has the effect of making all bodies appear higher above the horizon than they really are;—near the horizon, by upwards of 33′; thence by an angle which at first diminishes rapidly, afterwards more slowly, till at the zenith it vanishes.

*Retrogradation.*—*Retrograde.*—See *Arc*, and *Motion.*

*Revolution, Time of.*—The period in which a body completes the circuit of its orbit about a centre. It is synonymous with the terms *period, periodic time.* For the different intervals of revolution in the case of the planets, see *Anomalistic, Sidereal, Synodical, and Tropical Revolution.* To these the *Nodical Revolution* may be added: it is the interval of passage from ascending to ascending, or from descending to descending node of the orbit; it will vary slightly with the node chosen as the starting-point, owing to the variations of planetary motions. For the lunar revolutions, see *Month.*

*Right Ascension.*—See *Ascension, Right.*

*Rotation.*—See *Axis of Rotation.*

*Sarôs.*—A Chaldæan period, now known as the *cycle of eclipses.* It contains 223 lunations, or rather more than 6585 days; in which period there are 238·992 anomalistic, 241·029 sidereal, and 241·999 nodical months.

*Satellite.*—A moon attending on a primary planet.

*Sidereal Revolution.*—The interval between the successive returns of a planet to the same heliocentric position among the fixed stars; or the period in which a planet, viewed heliocentrically, would appear to traverse the twelve signs of the Zodiac. The sidereal revolution of a planet is subject to slight variations, owing to the perturbing attractions of the other planets. The average value of many successive revolutions is called *the mean sidereal revolution* of the planet, and is its *true period.*

*Signs of the Zodiac.*—The twelve divisions of the Zodiac. Each division contains 30°. The names and symbols of the signs are as follows:—

| Spring signs. | Summer signs. | Autumn signs. | Winter signs. |
|---|---|---|---|
| *Aries* . . . ♈ | *Cancer* . . ♋ | *Libra* . . ♎ | *Capricornus*. ♑ |
| *Taurus* . . ♉ | *Leo* . . . ♌ | *Scorpio* . . ♏ | *Aquarius*. . ♒ |
| *Gemini* . . ♊ | *Virgo* . . ♍ | *Sagittarius*. ♐ | *Pisces*. . . ♓ |

The Zodiacal constellations are each removed one sign from the sign whose name they bear. Thus the constellation *Aquarius* falls on the sign *Pisces,* the constellation *Pisces* on the sign *Aries,* and so on.

*Solstices.*—The points of the ecliptic which are at the greatest distance north and south of the equator. The former point is called the *Winter Solstice,* the latter the *Summer Solstice.* When the sun is passing these points his daily change of declination is small, so that he appears for several days together to follow the same diurnal path.

*Southing.*—The meridian transit of a celestial body is called the southing of the body, if such transit takes place on the visible southern quadrant of the celestial meridian. See *Meridian.*

*Spheroid.*—See *Oblate Spheroid.*

*Stationary Points.*—The points of a planet's apparent path on the celestial sphere, at which progressive merges into retrograde, or retrograde into progressive motion.

*Superior Planet.*—A planet whose orbit lies outside that of the earth.

*Synodical Period, or Revolution.*—The period which elapses between successive conjunctions or successive oppositions of a superior planet; or between successive conjunctions of the same kind, of an inferior planet.

*Telescopic Objects.*—Objects not visible to the naked eye.

*Transit.*—See *Meridian, Celestial, of a place.*

*Transit of a Satellite.*—The passage of a satellite across the disc of its primary. The passage of the shadow of a satellite across the disc is called a *transit of the shadow.*

*Tropical Revolution.*—The revolution of a planet referred to the nodes of the plane of its equator upon the plane of its orbit. Thus a tropical revolution of the earth is the interval between successive passages of either equinox or of either solstice. The length of a tropical revolution varies with the point from which it is supposed to commence; for the precession of the equinoxes carries them forward in each sidereal year by a quantity very nearly uniform, whereas the motion of the earth is variable in different parts of its orbit. Thus, if the vernal or autumnal equinox is the commencement of a tropical year, the period has very nearly its mean value, since the earth at the equinoxes is very nearly at its mean distance from the sun; but the period estimated from the vernal is rather greater than the period estimated from the autumnal equinox, the motion of the earth being rather more rapid near the vernal than near the autumnal equinox, so that the sidereal year is diminished by a smaller quantity in the former than in the latter case. The *mean tropical revolution* of the earth, or the *mean tropical year*, is equal to the sidereal year diminished by the interval in which the earth would move with her mean velocity over an arc equal to the annual precession of the equinoxes. An application of the same principles in the case of Saturn will be found under the explanation of Table X.

*Vertical Circles.*—Circles passing through the zenith and nadir of the celestial sphere.

*Vertical, Prime.*—The vertical circle through the east and west points of the horizon.

*Vis Viva.*—The *vis viva* of a particle is the product of its mass into the square of its velocity. The *vis viva* of a system is the sum of the *vires vivæ* of the particles composing the system. It is sometimes stated that the *vis viva* of a system free from external influences is constant. This is not exactly true, as will appear from a simple illustration:—Let two perfectly cold bodies be supposed to start from rest under the influence of their mutual attractions only: they will approach with constantly increasing velocity; and thus the system, which at first had no *vis viva*, gradually acquires a larger and larger amount of *vis viva* till the bodies impinge, when more or less of the acquired *vis viva* will take the form of *heat.* Since the 'true living' force of a system (as distinguished from the technical *vis viva*) can never be diminished or increased save from external sources, it may be asked, 'What was the form in which this acquired *vis viva*, or its equivalent *heat*, originally appeared in the system?' Though the attraction of gravity between the two masses plainly operated to generate the *vis viva* acquired by the system, yet that attraction alone would not have produced such effect; since, if the two bodies had been originally in contact, the attraction of gravity would have subsisted between them, yet no *vis viva* would have accrued. The element of *distance* plainly has to be considered; and the answer to the above question is, that 'the *work done* in removing the two bodies from contact to the given distance is the "living force" of the system.' This living force may be variously distributed in *vis viva* and *change of distance*; or again may appear in the form of *heat*, which is merely the distribution of *vis viva* and *change of distance* among the molecules of the two bodies; but whatever forms the living force of the system may assume, it can never be diminished or increased but by the operation of external influences. The same is true of all systems, however varied or complex the relations they may present.

*Year, Anomalistic.*—The period of the anomalistic revolution of the earth about the sun. See *Anomalistic Revolution.*

*Year, Sidereal.*—The period of the sidereal revolution of the earth about the sun. See *Sidereal Revolution.*

*Year, Tropical.*—The period of the tropical revolution of the earth about the sun. See *Tropical Revolution.*

*Zenith.*—The point vertically above the observer's head; or, in other words, the pole of the horizon.

*Zenith Distance.*—The complement of the altitude of a heavenly body; or, in other words, the angular distance of a heavenly body from the zenith.

*Zodiac.*—A belt of the heavens extending 9° on each side of the *ecliptic.* Within this belt the sun, all the primary planets, and by far the larger number of the asteroids perform their revolutions.

*Zodiacal Light.*—A light in the form of a long triangle, observed in spring above the western horizon after sunset, and in autumn above the eastern horizon before sunrise. The light extends obliquely upwards (the base of the triangle being towards the horizon) to a distance from the sun's place varying from 40° to 100°. The breadth of this meteor at its base varies from 7° to about 40°. It probably consists of flights of small disconnected cosmical bodies travelling about the sun, as the satellites composing Saturn's ring travel about Saturn. The bodies composing the parts of this meteor nearest to the sun must travel with tremendous velocity, and as the *vis viva* of the system, and especially of its interior zones, gradually diminishes through the effects of collisions, flights of these bodies, becoming entangled in the solar atmosphere, must be dissipated into vapour through the effects of intense heat, and spreading over vast areas of the solar surface generate the light and heat distributed by the sun to the worlds that revolve around him.

THE END.

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